

Student's Name	Student's ID	Group No.	Lecturer's Name

Question No.	I	II	III	IV	Total
Mark					

[I] Determine whether the following is **True** or **False**. [3 Points]

(1) The following system of equations is linear. ()

$$\begin{aligned} x + 2^4 y - z &= 0 \\ \sqrt{3} x - y + z &= 1 \end{aligned}$$

(2) For any $n \times n$ matrix A , the matrix AA^T is symmetric. ()

(3) The matrix $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is in the Reduced Row Echelon Form. ()

(4) Every elementary matrix is invertible. ()

(5) If A and B are $n \times n$ matrices, then $(A - B)(A + B) = A^2 - B^2$. ()

(6) If $D^3 = \begin{bmatrix} -8 & 0 \\ 0 & 27 \end{bmatrix}$, then $D^2 = \begin{bmatrix} 4 & 0 \\ 0 & 9 \end{bmatrix}$. ()

[II] Choose the correct answer. [5 Points]

(1) The values of c , if any, for which the matrix $A = \begin{bmatrix} c & -c & c \\ 1 & c & 1 \\ 0 & 0 & c \end{bmatrix}$ is invertible are

(a) $c \neq 0, 1$

(b) $c \neq 0, -1$

(c) $c = 0, -1$

(d) None of the previous

(2) If $A^T = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ and $p(x) = x^2 + 3$, then $p(A)$ equals

(a) $\begin{bmatrix} 6 & 8 \\ 4 & 14 \end{bmatrix}$

(b) $\begin{bmatrix} 9 & 4 \\ 8 & 14 \end{bmatrix}$

(c) $\begin{bmatrix} 8 & 4 \\ 9 & 14 \end{bmatrix}$

(d) None of the previous

(3) The values of x and y for which the matrix $\begin{bmatrix} x^2 & -1 & x^2 - 3 \\ -1 & 3 & 2y^2 + 5 \\ 1 & 7 & 2x - 5y \end{bmatrix}$ is symmetric are

(a) $x = \pm 1, y = \pm 2$

(b) $x = y = 0$

(c) $x = \pm 2, y = \pm 1$

(d) None of the previous

(4) For $\begin{bmatrix} 3 & 1 & 0 \\ -2 & -4 & 3 \\ 5 & 4 & -2 \end{bmatrix}$, the cofactor C_{32} equals

(a) -36

(b) 9

(c) -9

(d) None of the previous

(5) For any $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$, the system

$$\begin{aligned} 2x + y &= b_1 \\ 3x + 6y &= b_2 \end{aligned}$$

has

(a) No solution

(b) a unique solution

(c) Infinitely many solutions

(d) None of the previous

OVER

[III] Let $A = \begin{bmatrix} 1 & 3 & 1 & 1 \\ 2 & 5 & 2 & 2 \\ 1 & 3 & 8 & 9 \\ 1 & 3 & 2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ -2 & 3 & 0 \\ 2 & 4 & 1 \end{bmatrix}$. **Find** the following [7 Points]

(a) $\det(A)$ using **Row Reduction**;

(b) $(3B)^{-1}$;

(c) The matrix X for which $BX = \begin{bmatrix} 1 & 0 \\ -3 & 3 \\ 3 & 3 \end{bmatrix}$

OVER

[IV] [5 Points]

(a) **Solve** the following system

$$x + 2y - 3z = 6$$

$$2x - y + 4z = 1$$

$$x - y + z = 3$$

(b) **Can** the coefficient matrix of the previous system be written as a product of elementary matrices? **Justify** your answer.

Good Luck