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CALCULUS FOR ENGINEERS
MATH 1110

Tutorial No. 11 Solution

Double Integral

Evaluate the following iterated integral

1-

$$I = \int_2^3 \int_1^5 (x + 2y)\, dx\, dy.$$

Solution:

$$\int_1^5 (x + 2y)\, dx = \left(\tfrac{1}{2}x^2 + 2yx\right)\Big]_1^5$$

$$= \left(\tfrac{25}{2} + 10y\right) - \left(\tfrac{1}{2} + 2y\right) = 12 + 8y$$

Therefore, $\quad I = \int_2^3 (12 + 8y)\, dy =$

$$(12y + 4y^2)\Big]_2^3 = (36 + 36) - (24 + 16) = 32.$$

2-

$$I = \int_0^1 \int_{x^2}^{x^3} (x^2 + y^2)\, dy\, dx.$$

$$\int_{x^2}^{x^3} (x^2 + y^2)\, dy = \left(x^2 y + \tfrac{1}{3}y^3\right)\Big]_{x^2}^{x^3}$$

$$= \left(x^5 + \tfrac{1}{3}x^9\right) - \left(x^4 + \tfrac{1}{3}x^6\right).$$

Therefore, $\quad I = \int_0^1 \left(x^5 + \tfrac{1}{3}x^9 - x^4 - \tfrac{1}{3}x^6\right) dx =$

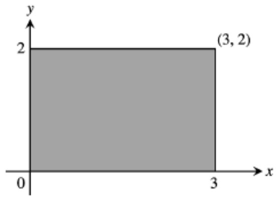
$$\tfrac{1}{6}x^6 + \tfrac{1}{30}x^{10} - \tfrac{1}{5}x^5 - \tfrac{1}{21}x^7\Big]_0^1 = \tfrac{1}{6} + \tfrac{1}{30} - \tfrac{1}{5} - \tfrac{1}{21} = -\tfrac{1}{21}.$$

3- For the following integrals sketch the region of integral and evalute it

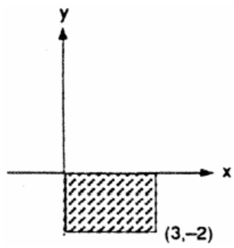
1. $\displaystyle\int_0^3\int_0^2 (4 - y^2)\,dy\,dx$
2. $\displaystyle\int_0^3\int_{-2}^0 (x^2y - 2xy)\,dy\,dx$
3. $\displaystyle\int_{-1}^0\int_{-1}^1 (x + y + 1)\,dx\,dy$
4. $\displaystyle\int_{\pi}^{2\pi}\int_0^{\pi} (\sin x + \cos y)\,dx\,dy$
5. $\displaystyle\int_0^{\pi}\int_0^{x} x\sin y\,dy\,dx$
6. $\displaystyle\int_0^{\pi}\int_0^{\sin x} y\,dy\,dx$
7. $\displaystyle\int_1^{\ln 8}\int_0^{\ln y} e^{x+y}\,dx\,dy$
8. $\displaystyle\int_1^2\int_y^{y^2} dx\,dy$
9. $\displaystyle\int_0^1\int_0^{y^2} 3y^3e^{xy}\,dx\,dy$
10. $\displaystyle\int_1^4\int_0^{\sqrt{x}} \frac{3}{2}e^{y/\sqrt{x}}\,dy\,dx$

Solution:

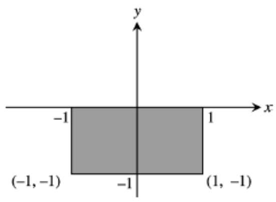
1. $\int_0^3\int_0^2 (4 - y^2)\,dy\,dx = \int_0^3 \left[4y - \frac{y^3}{3}\right]_0^2 dx = \frac{16}{3}\int_0^3 dx = 16$



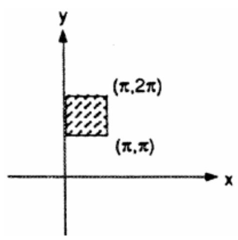
2. $\int_0^3\int_{-2}^0 ((x^2y - 2xy)\,dy\,dx = \int_0^3 \left[\frac{x^2y^2}{2} - xy^2\right]_{-2}^0 dx$
$= \int_0^3 (4x - 2x^2)\,dx = \left[2x^2 - \frac{2x^3}{3}\right]_0^3 = 0$



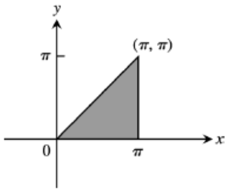
3. $\int_{-1}^0\int_{-1}^1 (x + y + 1)\,dx\,dy = \int_{-1}^0 \left[\frac{x^2}{2} + yx + x\right]_{-1}^1 dy$
$= \int_{-1}^0 (2y + 2)\,dy = \left[y^2 + 2y\right]_{-1}^0 = 1$



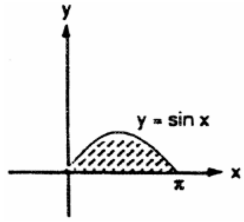
4. $\int_{\pi}^{2\pi}\int_0^{\pi} (\sin x + \cos y)\,dx\,dy = \int_{\pi}^{2\pi} [(-\cos x) + (\cos y)x]_0^{\pi}\,dy$
$= \int_{\pi}^{2\pi} (\pi\cos y + 2)\,dy = [\pi\sin y + 2y]_{\pi}^{2\pi} = 2\pi$



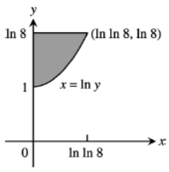
5. $\int_0^{\pi}\int_0^{x}(x \sin y)\,dy\,dx = \int_0^{\pi}[-x \cos y]_0^x\,dx$
$= \int_0^{\pi}(x - x \cos x)\,dx = \left[\frac{x^2}{2} - (\cos x + x \sin x)\right]_0^{\pi}$
$= \frac{\pi^2}{2} + 2$



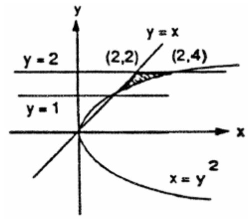
6. $\int_0^{\pi}\int_0^{\sin x} y\,dy\,dx = \int_0^{\pi}\left[\frac{y^2}{2}\right]_0^{\sin x} dx = \int_0^{\pi}\frac{1}{2}\sin^2 x\,dx$
$= \frac{1}{4}\int_0^{\pi}(1 - \cos 2x)\,dx = \frac{1}{4}\left[x - \frac{1}{2}\sin 2x\right]_0^{\pi} = \frac{\pi}{4}$



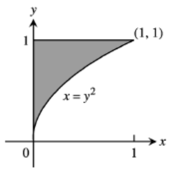
7. $\int_1^{\ln 8}\int_0^{\ln y} e^{x+y}\,dx\,dy = \int_1^{\ln 8}[e^{x+y}]_0^{\ln y}\,dy = \int_1^{\ln 8}(ye^y - e^y)\,dy$
$= [(y-1)e^y - e^y]_1^{\ln 8} = 8(\ln 8 - 1) - 8 + e$
$= 8 \ln 8 - 16 + e$



8. $\int_1^2\int_y^{y^2} dx\,dy = \int_1^2(y^2 - y)\,dy = \left[\frac{y^3}{3} - \frac{y^2}{2}\right]_1^2$
$= \left(\frac{8}{3} - 2\right) - \left(\frac{1}{3} - \frac{1}{2}\right) = \frac{7}{3} - \frac{3}{2} = \frac{5}{6}$



9. $\int_0^1\int_0^{y^2} 3y^3 e^{xy}\,dx\,dy = \int_0^1[3y^2 e^{xy}]_0^{y^2}\,dy$
$= \int_0^1\left(3y^2 e^{y^3} - 3y^2\right)dy = \left[e^{y^3} - y^3\right]_0^1 = e - 2$



10. $\int_1^4\int_0^{\sqrt{x}} \frac{3}{2} e^{y/\sqrt{x}}\,dy\,dx = \int_1^4\left[\frac{3}{2}\sqrt{x}\,e^{y/\sqrt{x}}\right]_0^{\sqrt{x}} dx$
$= \frac{3}{2}(e - 1)\int_1^4\sqrt{x}\,dx = \left[\frac{3}{2}(e-1)\left(\frac{2}{3}\right)x^{3/2}\right]_1^4 = 7(e - 1)$