



Dr. Mohamed El-Shazly
CALCULUS FOR ENGINEERS
MATH 1110

Tutorial No. 9 Solution

Integral Basics

1- Evaluate the following integrals:

1-A-

$$\int_{-1}^3 (3x^2 - 2x + 1) dx.$$

■ $\int (3x^2 - 2x + 1) dx = x^3 - x^2 + x$. (We omit the arbitrary constant in all such cases.) So $\int_{-1}^3 (3x^2 - 2x + 1) dx = (x^3 - x^2 + x) \Big|_{-1}^3 = (3^3 - 3^2 + 3) - [(-1)^3 - (-1)^2 + (-1)] = 21 - (-3) = 24$.

1-B-

$$\int_1^4 (x^2 - 4x + 2) dx$$

1-C-

$$\int_0^2 (2x - x^3) dx$$

1-D-

$$\int_0^{\pi/4} \cos x dx.$$

$\int \cos x dx = \sin x$. Hence, $\int_0^{\pi/4} \cos x dx = \sin x \Big|_0^{\pi/4} = \sin \frac{\pi}{4} - \sin 0 = \frac{\sqrt{2}}{2} - 0 = \frac{\sqrt{2}}{2}$.

1-E-

$$\int_0^{\pi/3} \sec^2 x dx.$$

$\int \sec^2 x dx = \tan x$. Hence, $\int_0^{\pi/3} \sec^2 x dx = \tan x \Big|_0^{\pi/3} = \tan \frac{\pi}{3} - \tan 0 = \sqrt{3} - 0 = \sqrt{3}$.

1-F-

$$\int_1^{16} x^{3/2} dx.$$

| $\int x^{3/2} dx = \frac{2}{5} x^{5/2}$. Hence, $\int_1^{16} x^{3/2} dx = \left[\frac{2}{5} x^{5/2} \right]_1^{16} = \frac{2}{5} [(16)^{5/2} - (1)^{5/2}] = \frac{2}{5} [(\sqrt{16})^5 - (\sqrt{1})^5] = \frac{2}{5} (1024 - 1) = \frac{2}{5} (1023) = \frac{2046}{5}$.

1-G-

$$\int_4^5 \left(\frac{2}{\sqrt{x}} - x \right) dx.$$

| $\int \left(\frac{2}{\sqrt{x}} - x \right) dx = \int (2x^{-1/2} - x) dx = 4x^{1/2} - \frac{1}{2}x^2$. Hence, $\int_4^5 \left(\frac{2}{\sqrt{x}} - x \right) dx = \left(4x^{1/2} - \frac{1}{2}x^2 \right) \Big|_4^5 = [4\sqrt{5} - \frac{1}{2}(5)^2] - [4\sqrt{4} - \frac{1}{2}(4)^2] = (4\sqrt{5} - \frac{25}{2}) - (8 - 8) = 4\sqrt{5} - \frac{25}{2}$.

1-H-

$$\int_0^1 \sqrt{x^2 - 6x + 9} dx.$$

| $\sqrt{x^2 - 6x + 9} = \sqrt{(x-3)^2} = |x-3| = 3-x$ when $0 \leq x \leq 1$. So, the integral is $\int_0^1 (3-x) dx = (3x - \frac{1}{2}x^2) \Big|_0^1 = (3 - \frac{1}{2}) - (0 - 0) = \frac{5}{2}$.

2- In the following problems calculate the area under the graph of the function $f(x)$, above x-axis, and between the two indicated values a and b .

2-1

$$f(x) = \sin x, \quad a = \pi/6, \quad b = \pi/3.$$

$$A = \int_{\pi/6}^{\pi/3} \sin x dx = (-\cos x) \Big|_{\pi/6}^{\pi/3} = \left(-\cos \frac{\pi}{3} \right) - \left(-\cos \frac{\pi}{6} \right) = \left(-\frac{1}{2} \right) - \left(-\frac{\sqrt{3}}{2} \right) = \frac{\sqrt{3}-1}{2}.$$

2-2

$$f(x) = x^2 + 4x, \quad a = 0, \quad b = 3.$$

$$A = \int_0^3 (x^2 + 4x) dx = \left(\frac{1}{3}x^3 + 2x^2 \right) \Big|_0^3 = \left[\frac{1}{3}(3)^3 + 2(3)^2 \right] = 9 + 18 = 27.$$

2-3

$$f(x) = 1/\sqrt[3]{x}, \quad a = 1, \quad b = 8.$$

$$A = \int_1^8 \frac{1}{\sqrt[3]{x}} dx = \left[\frac{3}{2} x^{2/3} \right]_1^8 = \frac{3}{2} (8^{2/3} - 1^{2/3}) = \frac{3}{2} (4 - 1) = \frac{9}{2}.$$

2-4

$$f(x) = \sqrt{4x+1}, \quad a=0, \quad b=2.$$

| $A = \int_0^2 \sqrt{4x+1} dx$. To find $\int \sqrt{4x+1} dx$, let $u = 4x+1$, $du = 4 dx$. $\int \sqrt{4x+1} dx = \frac{1}{4} \int u^{1/2} du = \frac{1}{4} \cdot \frac{2}{3} u^{3/2} = \frac{1}{6} (\sqrt{4x+1})^3$. So, $A = \left[\frac{1}{6} (\sqrt{4x+1})^3 \right]_0^2 = \frac{1}{6} [(\sqrt{9})^3 - (\sqrt{1})^3] = \frac{1}{6} (27 - 1) = \frac{13}{3}$.

2-5

$$f(x) = x^2 - 3x, \quad a=3, \quad b=5.$$

$$\mathbf{|} \quad A = \int_3^5 (x^2 - 3x) dx = \left(\frac{1}{3} x^3 - \frac{3}{2} x^2 \right) \Big|_3^5 = \left[\frac{1}{3} (5)^3 - \frac{3}{2} (5)^2 \right] - \left[\frac{1}{3} (3)^3 - \frac{3}{2} (3)^2 \right] = \frac{25}{6} + \frac{9}{2} = \frac{26}{3}.$$

2-6

$$f(x) = \sin^2 x \cos x, \quad a=0, \quad b=\pi/2.$$

$$\mathbf{|} \quad A = \int_0^{\pi/2} \sin^2 x \cos x dx = \left[\frac{1}{3} \sin^3 x \right]_0^{\pi/2} = \frac{1}{3} [\sin^3 (\pi/2) - \sin^3 0] = \frac{1}{3}.$$

3- Find the following definite integral

3-1

$$\int_{-1}^1 \sqrt{3x^2 - 2x + 3} (3x - 1) dx.$$

| To find $\int \sqrt{3x^2 - 2x + 3} (3x - 1) dx$, let $u = 3x^2 - 2x + 3$, $du = (6x - 2) dx = 2(3x - 1) dx$. So, $\int \sqrt{3x^2 - 2x + 3} (3x - 1) dx = \frac{1}{2} \int \sqrt{u} du = \frac{1}{2} \cdot \frac{2}{3} u^{3/2} = \frac{1}{3} (3x^2 - 2x + 3)^{3/2}$. Hence, $\int_{-1}^1 \sqrt{3x^2 - 2x + 3} (3x - 1) dx = \left[\frac{1}{3} (3x^2 - 2x + 3)^{3/2} \right]_{-1}^1 = \frac{1}{3} [(3 - 2 + 3)^{3/2} - (3 + 2 + 3)^{3/2}] = \frac{1}{3} (8 - 16\sqrt{2}) = \frac{8}{3} (1 - 2\sqrt{2})$.

3-2

$$\int_{-1}^2 \sqrt{x+2} x^2 dx.$$

| Let $u = x + 2$, $x = u - 2$, $du = dx$. When $x = -1$, $u = 1$, and, when $x = 2$, $u = 4$. Then, by change of variables, $\int_{-1}^2 \sqrt{x+2} x^2 dx = \int_1^4 \sqrt{u} (u - 2)^2 du = \int_1^4 \sqrt{u} (u^2 - 4u + 4) du = \int_1^4 (u^{5/2} - 4u^{3/2} + 4u^{1/2}) du = \left[\frac{2}{7} u^{7/2} - 4(\frac{2}{5}) u^{5/2} + 4(\frac{2}{3}) u^{3/2} \right]_1^4 = 2u^{3/2} (\frac{1}{7} u^2 - \frac{4}{5} u + \frac{4}{3}) \Big|_1^4 = 2[8(\frac{16}{7} - \frac{16}{5} + \frac{4}{3}) - (\frac{1}{7} - \frac{4}{5} + \frac{4}{3})] = 2(\frac{127}{7} - \frac{124}{5} + \frac{28}{3}) = \frac{562}{105}$.

3-3

$$\int_2^5 \sqrt{x^3 - 4} x^5 dx.$$

| Let $u = x^3 - 4$, $x^3 = u + 4$, $du = 3x^2 dx$. When $x = 2$, $u = 4$, and, when $x = 5$, $u = 121$. Then $\int_2^5 \sqrt{x^3 - 4} x^5 dx = \frac{1}{3} \int_4^{121} \sqrt{u}(u + 4) du = \frac{1}{3} \int_4^{121} (u^{3/2} + 4u^{1/2}) du = \frac{1}{3} (\frac{2}{5} u^{5/2} + \frac{8}{3} u^{3/2}) \Big|_4^{121} = \frac{2}{45} u^{3/2} (3u + 20) \Big|_4^{121} = \frac{2}{45} [(11)^3(383) - 8(32)] = \frac{113,226}{5}$.

3-4

$$\int_3^{15} \sqrt[3]{x^2 - 9} x^3 dx.$$

| Let $u = x^2 - 9$, $x^2 = u + 9$, $du = 2x dx$. Then $\int_3^{15} \sqrt[3]{x^2 - 9} x^3 dx = \frac{1}{2} \int_0^{216} u^{1/3}(u + 9) du = \frac{1}{2} \int_0^{216} (u^{4/3} + 9u^{1/3}) du = \frac{1}{2} (\frac{3}{7} u^{7/3} + 9 \cdot \frac{3}{4} u^{4/3}) \Big|_0^{216} = \frac{3}{56} u^{4/3} (4u + 63) \Big|_0^{216} = \frac{3}{56} \{(6)^4[4(216) + 63] - 0\} = \frac{349,522}{7}$.

3-5

$$\int_0^1 \frac{x}{(2x^2 + 1)^3} dx.$$

| Let $u = 2x^2 + 1$, $du = 4x dx$. Then $\int_0^1 \frac{x}{(2x^2 + 1)^3} dx = \frac{1}{4} \int_1^3 u^{-3} du = -\frac{1}{8} u^{-2} \Big|_1^3 = -\frac{1}{8} (\frac{1}{9} - 1) = \frac{1}{9}$.

3-6

$$\int_0^8 \frac{x}{(x + 1)^{3/2}} dx.$$

| Let $u = x + 1$, $x = u - 1$, $du = dx$. Then $\int_0^8 \frac{x}{(x + 1)^{3/2}} dx = \int_1^9 \frac{u - 1}{u^{3/2}} du = \int_1^9 (u^{-1/2} - u^{-3/2}) du = (2u^{1/2} + 2u^{-1/2}) \Big|_1^9 = 2[(3 + \frac{1}{3}) - (1 + 1)] = \frac{8}{3}$.

3-7

$$\int_1^2 \frac{x^7 - 2x + 1}{4x^3} dx.$$

| $\int_1^2 \frac{x^7 - 2x + 1}{4x^3} dx = \frac{1}{4} \int_1^2 (x^4 - 2x^{-2} + x^{-3}) dx = \frac{1}{4} (\frac{1}{5} x^5 + 2x^{-1} - \frac{1}{2} x^{-2}) \Big|_1^2 = \frac{1}{4} [(\frac{32}{5} + 1 - \frac{1}{8}) - (\frac{1}{5} + 2 - \frac{1}{2})] = \frac{1}{4} (\frac{31}{5} - 1 + \frac{3}{8}) = \frac{223}{160}$.