Faculty of Engineering
Mechanical Engineering Department

Linear Algebra and Vector Analysis
MATH 1120

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Gaussian Elimination

- In general, to solve a system of linear equations using its augmented matrix,
  we use elementary row operations to arrive at a matrix in a certain form.

  - This form is described as follows.

Putting in Row-Echelon Form

- We now discuss a systematic way to put a matrix in row-echelon form using elementary row operations.
  - We see how the process might work for a 3 x 4 matrix.

Putting in Row-Echelon Form—Step 1

- Start by obtaining 1 in the top left corner.

- Then, obtain zeros below that 1 by adding appropriate multiples of the first row to the rows below it.

$$\begin{bmatrix} 1 & \_ & \_ & \_ \\ 0 & \_ & \_ & \_ \\ 0 & \_ & \_ & \_ \end{bmatrix}$$

Putting in Row-Echelon Form—Steps 2 & 3

- Next, obtain a leading 1 in the next row.
- Then, obtain zeros below that 1.

  – At each stage, make sure every leading entry is to the right of the leading entry in the row above it.

  – Rearrange the rows if necessary.

$$\begin{bmatrix} 1 & \_ & \_ & \_ \\ 0 & \_ & \_ & \_ \\ 0 & \_ & \_ & \_ \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \_ & \_ & \_ \\ 0 & 1 & \_ & \_ \\ 0 & 0 & \_ & \_ \end{bmatrix}$$

Putting in Row-Echelon Form—Step 4

- Continue this process until you arrive at a matrix in row-echelon form.

$$\begin{bmatrix} 1 & \_ & \_ & \_ \\ 0 & \_ & \_ & \_ \\ 0 & \_ & \_ & \_ \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \_ & \_ & \_ \\ 0 & 1 & \_ & \_ \\ 0 & 0 & \_ & \_ \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \_ & \_ & \_ \\ 0 & 1 & \_ & \_ \\ 0 & 0 & 1 & \_ \end{bmatrix}$$

Gaussian Elimination

- Once an augmented matrix is in row-echelon form, we can solve the corresponding linear system using back-substitution.
  - This technique is called Gaussian elimination, in honor of its inventor, the German mathematician C. F. Gauss.

Solving a System Using Gaussian Elimination

- To solve a system using Gaussian elimination, we use:

1. Augmented matrix
2. Row-echelon form
3. Back-substitution

Solving a System Using Gaussian Elimination

1. Augmented matrix
   - Write the augmented matrix of the system.
2. Row-echelon form
   - Use elementary row operations to change the augmented matrix to row-echelon form.

Solving a System Using Gaussian Elimination

3. Back-substitution

   – Write the new system of equations that corresponds to the row-echelon form of the augmented matrix and solve by back-substitution.

Example: 3—Solving a System Using Row-Echelon Form

- Solve the system of linear equations using Gaussian elimination.

$$\begin{cases} 4x + 8y - 4z = 4 \\ 3x + 8y + 5z = -11 \\ -2x + y + 12z = -17 \end{cases}$$

  - We first write the augmented matrix of the system.
  - Then, we use elementary row operations to put it in row-echelon form.

Example:3—Solving a System Using Row-Echelon Form

- $$\begin{bmatrix} 4 & 8 & -4 & 4 \\ 3 & 8 & 5 & -11 \\ -2 & 1 & 12 & -17 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{4}R_1} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 3 & 8 & 5 & -11 \\ -2 & 1 & 12 & -17 \end{bmatrix}$$

E.g. 3—Solving a System Using Row-Echelon Form

- $$\xrightarrow[R_3+2R_1\to R_3]{R_2-3R_1\to R_2} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 2 & 8 & -14 \\ 0 & 5 & 10 & -15 \end{bmatrix}$$

$$\xrightarrow{\frac{1}{2}R_2} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 5 & 10 & -15 \end{bmatrix}$$

E.g. 3—Solving a System Using Row-Echelon Form

- $$\xrightarrow{R_3 - 5R_2 \to R_3} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & -10 & 20 \end{bmatrix}$$

$$\xrightarrow{-\frac{1}{10}R_3} \begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

Example:3—Solving a System Using Row-Echelon Form

- We now have an equivalent matrix in row-echelon form.
- The corresponding system of equations is:

$$\begin{cases} x + 2y - z = 1 \\ y + 4z = -7 \\ z = -2 \end{cases}$$

  - We use back-substitution to solve the system.

Example: 3—Solving a System Using Row-Echelon Form

- $y + 4(-2) = -7$
- $y = 1$
- $x + 2(1) - (-2) = 1$
- $x = -3$

  – The solution of the system is:

  $(-3, 1, -2)$