



Faculty of Engineering Mechanical Engineering Department

Linear Algebra and Vector Analysis MATH 1120

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Gaussian Elimination

 In general, to solve a system of linear equations using its augmented matrix, we use elementary row operations to arrive at a matrix in a certain form.

– This form is described as follows.

Putting in Row-Echelon Form

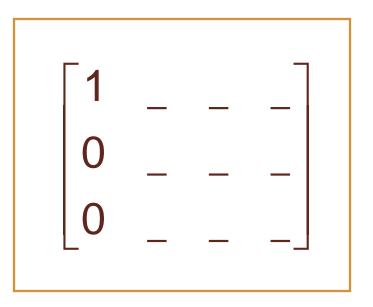
 We now discuss a systematic way to put a matrix in row-echelon form using elementary row operations.

 We see how the process might work for a 3 x 4 matrix.

Putting in Row-Echelon Form—Step 1

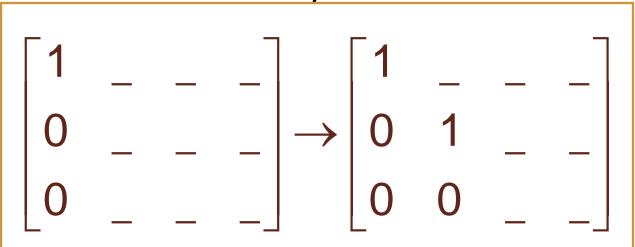
• Start by obtaining 1 in the top left corner.

• Then, obtain zeros below that 1 by adding appropriate multiples of the first row to the rows below it.



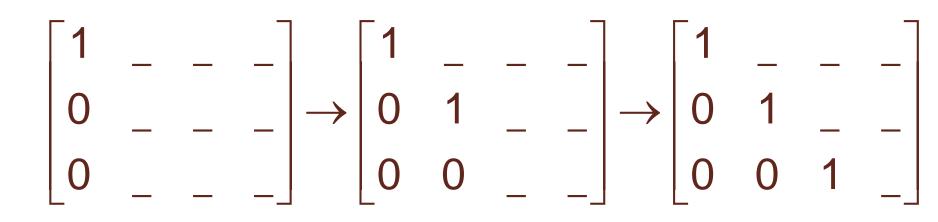
Putting in Row-Echelon Form—Steps 2 & 3

- Next, obtain a leading 1 in the next row.
- Then, obtain zeros below that 1.
 - At each stage, make sure every leading entry is to the right of the leading entry in the row above it.
 - Rearrange the rows if necessary.



Putting in Row-Echelon Form—Step 4

• Continue this process until you arrive at a matrix in row-echelon form.



Gaussian Elimination

 Once an augmented matrix is in row-echelon form, we can solve the corresponding linear system using back-substitution.

 This technique is called Gaussian elimination, in honor of its inventor, the German mathematician C. F. Gauss.

Solving a System Using Gaussian Elimination

• To solve a system using Gaussian elimination, we use:

- 1. Augmented matrix
- 2. Row-echelon form

3. Back-substitution

Solving a System Using Gaussian Elimination

1. Augmented matrix

– Write the augmented matrix of the system.

- 2. Row-echelon form
 - Use elementary row operations to change the augmented matrix to row-echelon form.

Solving a System Using Gaussian Elimination

3. Back-substitution

Write the new system of equations
 that corresponds to the row-echelon form
 of the augmented matrix and solve by
 back-substitution.

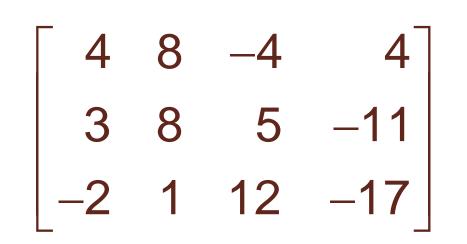
Example: 3—Solving a System Using Row-Echelon Form

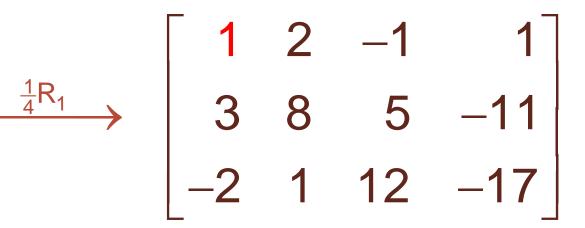
• Solve the system of linear equations using Gaussian elimination.

 $\begin{cases} 4x + 8y - 4z = 4 \\ 3x + 8y + 5z = -11 \\ -2x + y + 12z = -17 \end{cases}$

- We first write the augmented matrix of the system.
- Then, we use elementary row operations to put it in row-echelon form.

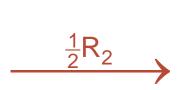
Example:3—Solving a System Using **Row-Echelon Form**







E.g. 3—Solving a System Using Row-
Echelon Form
$$\begin{array}{c} 1 & 2 & -1 & 1 \\ \hline R_2 - 3R_1 \rightarrow R_2 \\ R_3 + 2R_1 \rightarrow R_3 \end{array} \qquad \left[\begin{array}{ccc} 1 & 2 & -1 & 1 \\ 0 & 2 & 8 & -14 \\ 0 & 5 & 10 & -15 \end{array} \right]$$



 $\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 5 & 10 & -15 \end{bmatrix}$

E.g. 3—Solving a System Using Row-
Echelon Form

$$\begin{array}{c}
1 & 2 & -1 & 1\\
1 & 2 & -1 & 1\\
0 & 1 & 4 & -7\\
0 & 0 & -10 & 20\end{array}$$

$$\begin{array}{c}
-\frac{1}{10}R_{3} \\
-\frac{1}{10}R_{3} \\
0 & 1 & 4 & -7\\
0 & 1 & 4 & -7\\
0 & 0 & 1 & -2\end{array}$$

Example:3—Solving a System Using Row-Echelon Form

- We now have an equivalent matrix in row-echelon form.
- The corresponding system of equations is:

 $\begin{cases} x+2y-z=1\\ y+4z=-7\\ z=-2 \end{cases}$

We use back-substitution to solve the system.

Example: 3—Solving a System Using Row-Echelon Form

$$y + 4(-2) = -7$$

•
$$x + 2(1) - (-2) = 1$$

• $x = -3$

– The solution of the system is: