**Faculty of Engineering**
**Mechanical Engineering Department**

# Linear Algebra and Vector Analysis
# MATH 1120


**: Instructor**
**Dr. O. Phillips Agboola**

- **Gauss-Jordan Elimination**

# Putting in Reduced Row-Echelon Form

- If we put the augmented matrix of a linear system in reduced row-echelon form, then we don't need to back-substitute to solve the system.
  - To put a matrix in reduced row-echelon form, we use the following steps.
  - We see how the process might work for a 3 x 4 matrix.

# Putting in Reduced Row-Echelon Form—Step 1

- Use the elementary row operations to put the matrix in row-echelon form.

$$\begin{bmatrix} 1 & \_ & \_ & \_ \\ 0 & 1 & \_ & \_ \\ 0 & 0 & 1 & \_ \end{bmatrix}$$

# Putting in Reduced Row-Echelon Form—Step 2

- Obtain zeros above each leading entry by adding multiples of the row containing that entry to the rows above it.

$$\begin{bmatrix} 1 & \_ & \_ & \_ \\ 0 & 1 & \_ & \_ \\ 0 & 0 & 1 & \_ \end{bmatrix} \rightarrow \begin{bmatrix} 1 & \_ & 0 & \_ \\ 0 & 1 & 0 & \_ \\ 0 & 0 & 1 & \_ \end{bmatrix}$$

# Putting in Reduced Row-Echelon Form—Step 2

- Begin with the last leading entry and work up.

$$\begin{bmatrix} 1 & \_ & \_ & \_ \\ 0 & 1 & \_ & \_ \\ 0 & 0 & 1 & \_ \end{bmatrix} \to \begin{bmatrix} 1 & \_ & 0 & \_ \\ 0 & 1 & 0 & \_ \\ 0 & 0 & 1 & \_ \end{bmatrix} \to \begin{bmatrix} 1 & 0 & 0 & \_ \\ 0 & 1 & 0 & \_ \\ 0 & 0 & 1 & \_ \end{bmatrix}$$

# Gauss-Jordan Elimination

- Using the reduced row-echelon form to solve a system is called Gauss-Jordan elimination.
  - We illustrate this process in the next example.

# E.g. 4—Solving Using Reduced Row-Echelon Form

- Solve the system of linear equations, using Gauss-Jordan elimination.

$$\begin{cases} 4x + 8y - 4z = 4 \\ 3x + 8y + 5z = -11 \\ -2x + y + 12z = -17 \end{cases}$$

  – In Example 3, we used Gaussian elimination on the augmented matrix of this system to arrive at an equivalent matrix in row-echelon form.

# E.g. 4—Solving Using Reduced Row-Echelon Form

- We continue using elementary row operations on the last matrix in Example 3 to arrive at an equivalent matrix in reduced row-echelon form.

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & 1 & 4 & -7 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

# E.g. 4—Solving Using Reduced Row-Echelon Form

- $\xrightarrow[R_1+R_3\to R_1]{R_2-4R_3\to R_2} \begin{bmatrix} 1 & 2 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$

$$\xrightarrow{R_1-2R_2\to R_1} \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}$$

# E.g. 4—Solving Using Reduced Row-Echelon Form

- We now have an equivalent matrix in reduced row-echelon form.
- The corresponding system of equations is:

$$\begin{cases} x = -3 \\ y = 1 \\ z = -2 \end{cases}$$

  – Hence, we immediately arrive at the solution (–3, 1, –2).