Faculty of Engineering
Mechanical Engineering Department

Linear Algebra and Vector Analysis
MATH 1120

: Instructor
Dr. O. Phillips Agboola

- Matrix Equations

Matrix Equations

- We saw in Example 6 in Section 7.2 that a system of linear equations can be written as a single matrix equation.

Matrix Equations

- For example, the system

$$\begin{cases} x - 2y - 4z = 7 \\ 2x - 3y - 6z = 5 \\ -3x + 6y + 15z = 0 \end{cases}$$

is equivalent to the matrix equation

$$\begin{bmatrix} 1 & -2 & -4 \\ 2 & -3 & -6 \\ -3 & 6 & 15 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \\ 0 \end{bmatrix}$$

Coefficient Matrix

- If we let

$$A = \begin{bmatrix} 1 & -2 & -4 \\ 2 & -3 & -6 \\ -3 & 6 & 15 \end{bmatrix} \qquad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \qquad B = \begin{bmatrix} 7 \\ 5 \\ 0 \end{bmatrix}$$

  this matrix equation can be written as:

$$AX = B$$

  – The matrix *A* is called the coefficient matrix.

Matrix Equations

- We solve this matrix equation by multiplying each side by the inverse of *A*—provided the inverse exists.
- $$
\begin{aligned}
AX &= B \\
A^{-1}(AX) &= A^{-1}B \\
(A^{-1}A)X &= A^{-1}B \quad \text{Associative Property} \\
I_3X &= A^{-1}B \quad \text{Property of Inverses} \\
X &= A^{-1}B
\end{aligned}
$$

Matrix Equations

- In Example 4, we showed that:

$$A^{-1} = \begin{bmatrix} -3 & 2 & 0 \\ -4 & 1 & -\frac{2}{3} \\ 1 & 0 & \frac{1}{3} \end{bmatrix}$$

Matrix Equations

- So, from $X = A^{-1}B$, we have:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 & 2 & 0 \\ -4 & 1 & -\frac{2}{3} \\ 1 & 0 & \frac{1}{3} \end{bmatrix} \begin{bmatrix} 7 \\ 5 \\ 0 \end{bmatrix} = \begin{bmatrix} -11 \\ -23 \\ 7 \end{bmatrix}$$

  – Thus, $x = -11$, $y = -23$, $z = 7$ is the solution of the original system.

Matrix Equations

- We have proved that the matrix equation

$$AX = B$$

can be solved by the following method.

Solving a Matrix Equation

- Let:
  - A be a square n x n matrix that has an inverse A^{-1}.
  - X be a variable matrix, with n rows.
  - B be a known matrix, with n rows.

- Then, the solution of the matrix equation

$$AX = B$$

is given by:

$$X = A^{-1}B$$

E.g. 6—Solving a System Using a Matrix Inverse

$$\begin{cases} 2x - 5y = 15 \\ 3x - 6y = 36 \end{cases}$$

(a) Write the system of equations as a matrix equation.

(b) Solve the system by solving the matrix equation.

E.g. 6—Using a Matrix Inverse

Example (a)

- We write the system as a matrix equation of the form *AX* = *B*:

$$\begin{bmatrix} 2 & -5 \\ 3 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ 36 \end{bmatrix}$$

E.g. 6—Using a Matrix Inverse

Example (b)

- Using the rule for finding the inverse of a 2 x 2 matrix, we get:

$$A^{-1} = \begin{bmatrix} 2 & -5 \\ 3 & -6 \end{bmatrix}^{-1} = \frac{1}{2(-6)-(-5)3} \begin{bmatrix} -6 & -(-5) \\ -3 & 2 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} -6 & 5 \\ -3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & \frac{5}{3} \\ -1 & \frac{2}{3} \end{bmatrix}$$

E.g. 6—Using a Matrix Inverse

Example (b)

- Multiplying each side of the matrix equation by the inverse matrix, we get:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -2 & \frac{5}{3} \\ -1 & \frac{2}{3} \end{bmatrix} \begin{bmatrix} 15 \\ 36 \end{bmatrix} = \begin{bmatrix} 30 \\ 9 \end{bmatrix}$$

  – Thus, $x = 30$ and $y = 9$.

- Modeling with Matrix Equations

Applications

- Suppose we need to solve several systems of equations with the same coefficient matrix.
  - Then, converting the systems to matrix equations provides an efficient way to obtain the solutions.
  - We only need to find the inverse of the coefficient matrix once.