جـــامـــعـــة
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Linear Algebra and Vector Analysis
MATH 1120
Applied Engineering, Tutorial (4):
SOLUTION

Use the Gauss-Jordan method to solve each system of equations. For systems in two variables with infinitely many solutions, give the solution with y arbitrary; for systems in three variables with infinitely many solutions, give the solution with z arbitrary. See Examples 1–4.

15. $x + y = 5$
$x - y = -1$

16. $x + 2y = 5$
$2x + y = -2$

17. $3x + 2y = -9$
$2x - 5y = -6$

18. $2x - 3y = 10$
$2x + 2y = 5$

19. $6x + y - 5 = 0$
$5x + y - 3 = 0$

20. $2x - 5y - 10 = 0$
$3x + y - 15 = 0$

21. $2x - y = 6$
$4x - 2y = 0$

22. $3x - 2y = 1$
$6x - 4y = -1$

23. $3x - 4y = 7$
$-6x + 8y = -14$

24. $\frac{1}{2}x + \frac{3}{5}y = \frac{1}{4}$
$10x + 12y = 5$

25. $x + y - 5z = -18$
$3x - 3y + z = 6$
$x + 3y - 2z = -13$

26. $-x + 2y + 6z = 2$
$3x + 2y + 6z = 6$
$x + 4y - 3z = 1$

27. $x + y - z = 6$
$2x - y + z = -9$
$x - 2y + 3z = 1$

28. $x + 3y - 6z = 7$
$2x - y + z = 1$
$x + 2y + 2z = -1$

29. $x - z = -3$
$y + z = 9$
$x + z = 7$

SOLUTION:

15. $x + y = 5$
$x - y = -1$

This system has the augmented matrix

$$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 1 & -1 & -1 \end{array}\right].$$

$$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & -2 & -6 \end{array}\right] -1R1 + R2 \Rightarrow$$

$$\left[\begin{array}{cc|c} 1 & 1 & 5 \\ 0 & 1 & 3 \end{array}\right] -\tfrac{1}{2}R2 \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 3 \end{array}\right] -1R2 + R1$$

Solution set: $\{(2,3)\}$

16. $x + 2y = 5$
$2x + y = -2$

This system has the augmented matrix

$$\left[\begin{array}{cc|c} 1 & 2 & 5 \\ 2 & 1 & -2 \end{array}\right].$$

$$\left[\begin{array}{cc|c} 1 & 2 & 5 \\ 0 & -3 & -12 \end{array}\right] -2R1 + R2 \Rightarrow$$

$$\left[\begin{array}{cc|c} 1 & 2 & 5 \\ 0 & 1 & 4 \end{array}\right] -\tfrac{1}{3}R2 \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & -3 \\ 0 & 1 & 4 \end{array}\right] -2R2 + R1$$

Solution set: $\{(-3,4)\}$

17. $3x + 2y = -9$
$2x - 5y = -6$

This system has the augmented matrix

$$\left[\begin{array}{cc|c} 3 & 2 & -9 \\ 2 & -5 & -6 \end{array}\right].$$

$$\left[\begin{array}{cc|c} 1 & 7 & -3 \\ 2 & -5 & -6 \end{array}\right] -1R2 + R1 \Rightarrow$$

$$\left[\begin{array}{cc|c} 1 & 7 & -3 \\ 0 & -19 & 0 \end{array}\right] -2R1 + R2 \Rightarrow$$

$$\left[\begin{array}{cc|c} 1 & 7 & -3 \\ 0 & 1 & 0 \end{array}\right] -\tfrac{1}{19}R2 \Rightarrow$$

$$\left[\begin{array}{cc|c} 1 & 0 & -3 \\ 0 & 1 & 0 \end{array}\right] -7R2 + R1$$

Solution set: $\{(-3,0)\}$

18. $2x-3y=10$
$2x+2y=5$

This system has the augmented matrix

$$\left[\begin{array}{cc|c} 2 & -3 & 10 \\ 2 & 2 & 5 \end{array}\right].$$

$$\left[\begin{array}{cc|c} 1 & -\frac{3}{2} & 5 \\ 2 & 2 & 5 \end{array}\right]\tfrac{1}{2}\text{R1} \Rightarrow \left[\begin{array}{cc|c} 1 & -\frac{3}{2} & 5 \\ 0 & 5 & -5 \end{array}\right]-2\text{R1}+\text{R2} \Rightarrow$$

$$\left[\begin{array}{cc|c} 1 & -\frac{3}{2} & 5 \\ 0 & 1 & -1 \end{array}\right]\tfrac{1}{5}\text{R2} \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & \frac{7}{2} \\ 0 & 1 & -1 \end{array}\right]\tfrac{3}{2}\text{R2}+\text{R1}$$

Solution set: $\left\{\left(\frac{7}{2},-1\right)\right\}$

19. $6x+y-5=0$
$5x+y-3=0$

Rewrite the system as $\begin{array}{l} 6x+y=5 \\ 5x+y=3 \end{array}$

The augmented matrix is $\left[\begin{array}{cc|c} 6 & 1 & 5 \\ 5 & 1 & 3 \end{array}\right].$

$$\left[\begin{array}{cc|c} 1 & \frac{1}{6} & \frac{5}{6} \\ 5 & 1 & 3 \end{array}\right]\tfrac{1}{6}\text{R1} \Rightarrow \left[\begin{array}{cc|c} 1 & \frac{1}{6} & \frac{5}{6} \\ 0 & \frac{1}{6} & -\frac{7}{6} \end{array}\right]-5\text{R1}+\text{R2} \Rightarrow$$

$$\left[\begin{array}{cc|c} 1 & \frac{1}{6} & \frac{5}{6} \\ 0 & 1 & -7 \end{array}\right]6\text{R2} \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -7 \end{array}\right]-\tfrac{1}{6}\text{R2}+\text{R1}$$

or

$$\left[\begin{array}{cc|c} 1 & 0 & 2 \\ 5 & 1 & 3 \end{array}\right]-1\text{R2}+\text{R1} \Rightarrow \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -7 \end{array}\right]-5\text{R1}+\text{R2}$$

Solution set: $\{(2,-7)\}$

20. $2x-5y-10=0$
$3x+y-15=0$

Rewrite the system as $\begin{array}{l} 2x-5y=10 \\ 3x+\ \ y=15. \end{array}$

The augmented matrix is $\left[\begin{array}{cc|c} 2 & -5 & 10 \\ 3 & 1 & 15 \end{array}\right].$

$$\left[\begin{array}{cc|c} 1 & -\frac{5}{2} & 5 \\ 3 & 1 & 15 \end{array}\right]\tfrac{1}{2}\text{R1} \Rightarrow$$

$$\left[\begin{array}{cc|c} 1 & -\frac{5}{2} & 5 \\ 0 & \frac{17}{2} & 0 \end{array}\right]-3\text{R1}+\text{R2} \Rightarrow$$

$$\left[\begin{array}{cc|c} 1 & -\frac{5}{2} & 5 \\ 0 & 1 & 0 \end{array}\right]\tfrac{2}{17}\text{R2} \Rightarrow$$

$$\left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 0 \end{array}\right]\tfrac{5}{2}\text{R2}+\text{R1}$$

or

$$\left[\begin{array}{cc|c} -1 & -6 & -5 \\ 3 & 1 & 15 \end{array}\right] -1R2+R1 \Rightarrow$$

$$\left[\begin{array}{cc|c} 1 & 6 & 5 \\ 3 & 1 & 15 \end{array}\right] -1R1 \Rightarrow$$

$$\left[\begin{array}{cc|c} 1 & 6 & 5 \\ 0 & -17 & 0 \end{array}\right] -3R1+R2 \Rightarrow$$

$$\left[\begin{array}{cc|c} 1 & 6 & 5 \\ 0 & 1 & 0 \end{array}\right] -\tfrac{1}{17}R2 \Rightarrow$$

$$\left[\begin{array}{cc|c} 1 & 0 & 5 \\ 0 & 1 & 0 \end{array}\right] -6R2+R1$$

Solution set: $\{(5,0)\}$

21. $2x - y = 6$
$4x - 2y = 0$

This system has the augmented matrix

$$\left[\begin{array}{cc|c} 2 & -1 & 6 \\ 4 & -2 & 0 \end{array}\right].$$

$$\left[\begin{array}{cc|c} 1 & -\tfrac{1}{2} & 3 \\ 4 & -2 & 0 \end{array}\right] \tfrac{1}{2}R1 \Rightarrow$$

$$\left[\begin{array}{cc|c} 1 & -\tfrac{1}{2} & 3 \\ 0 & 0 & -12 \end{array}\right] -4R1+R2$$

The second row of the augmented matrix corresponds to the equation $0x + 0y = -12$, which has no solution. Thus, the solution set is $\varnothing$.

22. $3x - 2y = 1$
$6x - 4y = -1$

This system has the augmented matrix

$$\left[\begin{array}{cc|c} 3 & -2 & 1 \\ 6 & -4 & -1 \end{array}\right].$$

$$\left[\begin{array}{cc|c} 1 & -\tfrac{2}{3} & \tfrac{1}{3} \\ 6 & -4 & -1 \end{array}\right] \tfrac{1}{3}R1 \Rightarrow$$

$$\left[\begin{array}{cc|c} 1 & -\tfrac{2}{3} & \tfrac{1}{3} \\ 0 & 0 & -3 \end{array}\right] -6R1+R2$$

The second row of the augmented matrix corresponds to the equation $0x + 0y = -3$, which has no solution. Thus, the solution set is $\varnothing$.

23. $3x-4y=7$
$-6x+8y=-14$

This system has the augmented matrix

$$\left[\begin{array}{cc|c} 3 & -4 & 7 \\ -6 & 8 & -14 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & -\frac{4}{3} & \frac{7}{3} \\ -6 & 8 & -14 \end{array}\right]\frac{1}{3}\text{R1} \Rightarrow$$

$$\left[\begin{array}{cc|c} 1 & -\frac{4}{3} & \frac{7}{3} \\ 0 & 0 & 0 \end{array}\right]6R1+R2$$

It is impossible to go further. The equation that corresponds to the first row in the final matrix is $x-\frac{4}{3}y=\frac{7}{3} \Rightarrow x=\frac{4}{3}y+\frac{7}{3}$

Solution set: $\left\{\frac{4}{3}y+\frac{7}{3}, y\right\}$

24. $\frac{1}{2}x+\frac{3}{5}y=\frac{1}{4}$
$10x+12y=5$

This system has the augmented matrix

$$\left[\begin{array}{cc|c} \frac{1}{2} & \frac{3}{5} & \frac{1}{4} \\ 10 & 12 & 5 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 1 & \frac{6}{5} & \frac{1}{2} \\ 10 & 12 & 5 \end{array}\right]2\text{R1} \Rightarrow$$

$$\left[\begin{array}{cc|c} 1 & \frac{6}{5} & \frac{1}{2} \\ 0 & 0 & 0 \end{array}\right]-10R1+R2$$

It is impossible to go further. The equatio that corresponds to the first row in the fin matrix is $x+\frac{6}{5}y=\frac{1}{2} \Rightarrow x=-\frac{6}{5}y+\frac{1}{2}$

Solution set: $\left\{-\frac{6}{5}y+\frac{1}{2}, y\right\}$

25. $\begin{array}{l} x+\ \ y-5z=-18 \\ 3x-3y+\ z=\ \ 6 \\ x+3y-2z=-13 \end{array}$; This system has the augmented matrix $\left[\begin{array}{ccc|c} 1 & 1 & -5 & -18 \\ 3 & -3 & 1 & 6 \\ 1 & 3 & -2 & -13 \end{array}\right]$.

$$\left[\begin{array}{ccc|c} 1 & 1 & -5 & -18 \\ 0 & -6 & 16 & 60 \\ 1 & 3 & -2 & -13 \end{array}\right]-3\text{R1}+\text{R2} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -5 & -18 \\ 0 & -6 & 16 & 60 \\ 0 & 2 & 3 & 5 \end{array}\right]-1\text{R1}+\text{R3} \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -5 & -18 \\ 0 & 1 & -\frac{8}{3} & -10 \\ 0 & 2 & 3 & 5 \end{array}\right]-\frac{1}{6}\text{R2}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -\frac{7}{3} & -8 \\ 0 & 1 & -\frac{8}{3} & -10 \\ 0 & 2 & 3 & 5 \end{array}\right]-1\text{R2}+\text{R1} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -\frac{7}{3} & -8 \\ 0 & 1 & -\frac{8}{3} & -10 \\ 0 & 0 & \frac{25}{3} & 25 \end{array}\right]-2\text{R2}+\text{R3} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -\frac{7}{3} & -8 \\ 0 & 1 & -\frac{8}{3} & -10 \\ 0 & 0 & 1 & 3 \end{array}\right]\frac{3}{25}\text{R3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & -\frac{8}{3} & -10 \\ 0 & 0 & 1 & 3 \end{array}\right]\frac{7}{3}\text{R3}+\text{R1} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 3 \end{array}\right]\frac{8}{3}\text{R3}+\text{R2}$$

Solution set: $\{(-1,-2,3)\}$

26. $\begin{array}{l} -x+2y+6z=2 \\ 3x+2y+6z=6 \\ x+4y-3z=1 \end{array}$; This system has the augmented matrix $\left[\begin{array}{ccc|c} -1 & 2 & 6 & 2 \\ 3 & 2 & 6 & 6 \\ 1 & 4 & -3 & 1 \end{array}\right]$.

$$\left[\begin{array}{ccc|c} 1 & -2 & -6 & -2 \\ 3 & 2 & 6 & 6 \\ 1 & 4 & -3 & 1 \end{array}\right]-1\text{R1} \Rightarrow \left[\begin{array}{ccc|c} 1 & -2 & -6 & -2 \\ 0 & 8 & 24 & 12 \\ 1 & 4 & -3 & 1 \end{array}\right]-3\text{R1}+\text{R2} \Rightarrow \left[\begin{array}{ccc|c} 1 & -2 & -6 & -2 \\ 0 & 8 & 24 & 12 \\ 0 & 6 & 3 & 3 \end{array}\right]-1\text{R1}+\text{R3}$$

$$\left[\begin{array}{ccc|c} 1 & -2 & -6 & -2 \\ 0 & 1 & 3 & \frac{3}{2} \\ 0 & 6 & 3 & 3 \end{array}\right]\frac{1}{8}\text{R2} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 3 & \frac{3}{2} \\ 0 & 6 & 3 & 3 \end{array}\right]2\text{R2}+\text{R1} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 3 & \frac{3}{2} \\ 0 & 0 & -15 & -6 \end{array}\right]-6\text{R2}+\text{R3}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 3 & \frac{3}{2} \\ 0 & 0 & 1 & \frac{2}{5} \end{array}\right]-\frac{1}{15}\text{R3} \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & \frac{3}{10} \\ 0 & 0 & 1 & \frac{2}{5} \end{array}\right]-3\text{R3}+\text{R2}$$

Solution set: $\left\{\left(1,\frac{3}{10},\frac{2}{5}\right)\right\}$

27. $\begin{array}{l} x+\ y-\ z=\ 6 \\ 2x-\ y+\ z=-9 \\ x-2y+3z=\ 1 \end{array}$; This system has the augmented matrix $\left[\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 2 & -1 & 1 & -9 \\ 1 & -2 & 3 & 1 \end{array}\right]$.

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 0 & -3 & 3 & -21 \\ 1 & -2 & 3 & 1 \end{array}\right] -2R1+R2 \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 0 & -3 & 3 & -21 \\ 0 & -3 & 4 & -5 \end{array}\right] -1R1+R3 \Rightarrow \left[\begin{array}{ccc|c} 1 & 1 & -1 & 6 \\ 0 & 1 & -1 & 7 \\ 0 & -3 & 4 & -5 \end{array}\right] -\tfrac{1}{3}R2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 7 \\ 0 & -3 & 4 & -5 \end{array}\right] -1R2+R1 \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 7 \\ 0 & 0 & 1 & 16 \end{array}\right] 3R2+R3 \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 23 \\ 0 & 0 & 1 & 16 \end{array}\right] R3+R2$$

Solution set: $\{(-1,23,16)\}$

28. $\begin{array}{l} x+3y-6z=\ 7 \\ 2x-\ y+\ z=\ 1 \\ x+2y+2x=-1 \end{array}$; This system has the augmented matrix $\left[\begin{array}{ccc|c} 1 & 3 & -6 & 7 \\ 2 & -1 & 1 & 1 \\ 1 & 2 & 2 & -1 \end{array}\right]$.

$$\left[\begin{array}{ccc|c} 1 & 3 & -6 & 7 \\ 0 & -7 & 13 & -13 \\ 1 & 2 & 2 & -1 \end{array}\right] -2R1+R2 \Rightarrow \left[\begin{array}{ccc|c} 1 & 3 & -6 & 7 \\ 0 & -7 & 13 & -13 \\ 0 & -1 & 8 & -8 \end{array}\right] -1R1+R3 \Rightarrow \left[\begin{array}{ccc|c} 1 & 3 & -6 & 7 \\ 0 & -1 & 8 & -8 \\ 0 & -7 & 13 & -13 \end{array}\right] R3 \leftrightarrow R$$

$$\left[\begin{array}{ccc|c} 1 & 3 & -6 & 7 \\ 0 & 1 & -8 & 8 \\ 0 & -7 & 13 & -13 \end{array}\right] -1R2 \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 18 & -17 \\ 0 & 1 & -8 & 8 \\ 0 & -7 & 13 & -13 \end{array}\right] -3R2+R1 \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 18 & -17 \\ 0 & 1 & -8 & 8 \\ 0 & 0 & -43 & 43 \end{array}\right] 7R2+R3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 18 & -17 \\ 0 & 1 & -8 & 8 \\ 0 & 0 & 1 & -1 \end{array}\right] -\tfrac{1}{43}R3 \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & -8 & 8 \\ 0 & 0 & 1 & -1 \end{array}\right] -18R3+R1 \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right] 8R3+R2$$

Solution set: $\{(1,0,-1)\}$

29. $\begin{array}{l} x-z=-3 \\ y+z=9 \\ x+z=7 \end{array}$; This system has the augmented matrix $\left[\begin{array}{ccc|c} 1 & 0 & -1 & -3 \\ 0 & 1 & 1 & 9 \\ 1 & 0 & 1 & 7 \end{array}\right]$.

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & -3 \\ 0 & 1 & 1 & 9 \\ 0 & 0 & 2 & 10 \end{array}\right] -1R1+R3 \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & -1 & -3 \\ 0 & 1 & 1 & 9 \\ 0 & 0 & 1 & 5 \end{array}\right] \tfrac{1}{2}R3$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 1 & 9 \\ 0 & 0 & 1 & 5 \end{array}\right] R3+R1 \Rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 5 \end{array}\right] -1R3+R2$$

Solution set: $\{(2,4,5)\}$