

EXERCISE SHEET #1

MATH 111 and MATH 106

Note: These exercises will be discussed in the tutorial lecture, but you are advised to solve the exercises at the end of each section in the book.

Chapter 4

Section 4.1

In problems 1 – 7, find the most general antiderivative:

1. $\int dx$;
2. $\int x dx$
3. $\int a dx$, where a is a constant;
4. $\int (ax + b) dx$, where a and b are constants;
5. $\int (1 + x + x^2 + x^3 + x^4) dx$;
6. $\int \frac{x^3 + x^2 - x}{x^{3/2}} dx$;
7. $\int (2 \sin x - 3 \cos x) dx$.

In problems 8 – 12, the derivative of a function and one point on its graph are given. Find the function:

8. $\frac{dy}{dx} = x^3 + x^2 - 3$, $(1, 5)$;
9. $\frac{dy}{dx} = 2x(x + 1)$, $(2, 0)$;
10. $\frac{dy}{dx} = \sqrt[3]{x} + x - \frac{1}{3\sqrt[3]{x}}$, $(-1, -8)$;
11. $\frac{dy}{dx} = 13x^{15/18} - 3$, $(1, 14)$;

12. $\frac{dy}{dx} = \cos x, \quad \left(\frac{\pi}{6}, 4\right).$

Section 4.2

In problems 1 – 4, evaluate the given sums:

1. $\sum_{k=1}^8 3^k;$

2. $\sum_{k=0}^6 1;$

3. $\sum_{i=2}^5 \frac{i}{i+1};$

4. $\sum_{j=5}^7 \frac{2j+3}{j-2};$

In problems 5 – 12, write each sum using \sum notation:

5. $1 + 2 + 4 + 8 + 16;$

6. $1 - 3 + 9 - 26 + 81 - 243;$

7. $\frac{2}{3} + \frac{3}{4} + \frac{4}{5} + \frac{5}{6} + \cdots + \frac{n}{n+1};$

8. $1 - \frac{1}{2!} + \frac{1}{3!} - \frac{1}{4!} + \frac{1}{5!} - \frac{1}{6!} + \frac{1}{7!};$

9. $1 + x^3 + x^6 + x^9 + x^{12} + x^{15} + x^{18} + x^{21};$

10. $-1 + \frac{1}{a} - \frac{1}{a^2} + \frac{1}{a^3} - \frac{1}{a^4} + \frac{1}{a^5} - \frac{1}{a^6} + \frac{1}{a^7} - \frac{1}{a^8} + \frac{1}{a^9};$

11. $\frac{1}{32} \left(\frac{1}{32}\right)^2 + \frac{1}{32} \left(\frac{2}{32}\right)^2 + \frac{1}{32} \left(\frac{3}{32}\right)^2 + \cdots + \frac{1}{32} \left(\frac{32}{32}\right)^2;$

12. $0.1 \sin(0.1) + 0.1 \sin(0.2) + 0.1 \sin(0.3) + \cdots + 0.1 \sin(1).$

In problems 13 – 15, three expressions are given, two of them being equal. Identify the one that does not equal the other two:

$$13. \sum_{k=0}^7 (2k+1), \quad \sum_{i=1}^{15} i, \quad \sum_{j=2}^9 (2j-3);$$

$$14. \sum_{k=1}^7 k^2, \quad \sum_{j=0}^6 (7-j)^2, \quad \sum_{i=1}^7 (7-i)^2;$$

$$15. \left(\sum_{k=7}^{11} k \right)^4, \quad \sum_{m=-11}^{-7} m^4, \quad \sum_{n=7}^{11} n^4.$$

Section 4.3

In problems 1 – 4, approximate the area under the curve, on the given interval, using n rectangles and the evaluation rules (a) left endpoints, (b) midpoint, (c) right endpoint:

$$1. y = 3x + 2, \quad [0, 3];$$

$$2. y = x + x^2, \quad [0, 1]$$

$$3. y = \frac{1}{2}x^2, \quad [0, 2];$$

$$4. y = 3x^3, \quad [-1, 0].$$

Section 4.4

In problems 1 – 3, answer *without computing the integrals*:

$$1. \text{ Explain why } \int_2^6 \frac{x}{x+6} dx \leq \int_2^6 \frac{x}{10} dx;$$

$$2. \text{ Is } \int_1^2 x dx \text{ greater or smaller than } \int_1^2 \sqrt{x} dx?;$$

3. Show that $\int_0^1 \sqrt{1+x^3} dx$ lies between 1 and $\frac{5}{4}$.

In problems 4 – 9, find upper and lower bounds for the given integrals:

4. $\int_1^4 4\sqrt{x} dx;$

5. $\int_1^8 7x^{1/3} dx;$

6. $\int_1^9 \frac{1}{\sqrt{x}} dx;$

7. $\int_2^3 (x^2 + x^3) dx;$

8. $\int_1^{100} \frac{1}{x} dx;$

9. $\int_0^1 \frac{1}{1+x^2} dx.$

In problems 10 – 12, find the value of c that satisfies the conclusion of the Integral Mean Value Theorem on the given interval:

10. $f(x) = 3\sqrt{x+1}; [-1, 8], \int_{-1}^8 f(x) dx = 54.$

11. $f(x) = x^2, [1, 4], \int_1^4 f(x) dx = 21.$

12. $f(x) = 3x^2 - 2x + 3, [-1, 3], \int_{-1}^3 f(x) dx = 32.$

In problems 13 – 14, express as one integral:

13. $\int_c^e f(x) dx + \int_a^b f(x) dx - \int_c^b f(x) dx - \int_d^d f(x) dx;$

14. $\int_a^d f(x) dx - \int_t^b f(x) dx - \int_g^g f(x) dx - \int_m^b f(x) dx + \int_t^a f(x) dx.$

Section 4.5

In problems 1 – 13, calculate the following integrals:

1. $\int_0^4 7dx;$

2. $\int_1^4 (7t - 3)dt;$

3. $\int_0^5 s^3 ds;$

4. $\int_{-1}^1 x^3 dx;$

5. $\int_3^0 (2z + 4)dz;$

6. $\int_{-1}^1 |x|dx;$

7. $\int_a^b x^2 dx.$

8. $\int_1^3 (x^3 + 3x + 5)dx;$

9. $\int_a^b (c_1 x^2 + c_2 x + c_3)dx;$

10. $\int_0^1 (1 + x^8 + x^{16} + x^{32})dx;$

11. $\int_{-a}^a x^{2n+1} dx$, where n is a positive integer and a is a real number;

12. $\int_2^3 (x - 1)(x + 2)dx;$

$$13. \int_0^1 (x^{3/2} - x^{2/3})(x^{4/3} - x^{3/4})dx.$$

In problems 14–17, calculate the derivative $F'(x)$. Then, evaluate $F'(x_0)$, for the given x_0 :

$$14. F(x) = \cos x \int_3^{\sin x} \frac{dt}{1+t^3}, \quad x_0 = 0;$$

$$15. F(x) = \int_{-x^3}^x \frac{s^2}{s^2+5} ds, \quad x_0 = 3;$$

$$16. F(x) = \ln|x| \int_{x^2+1}^0 \frac{\sqrt{u-1}}{\sqrt{u+1}} du, \quad x_0 = 1;$$

$$17. F(x) = \int_{2e^x}^x \frac{1+2t-3t^2}{t^3+t^{7/9}} dt, \quad x_0 = 1.$$

18. Let $f(x) = -\frac{1}{x^2}$ and $F(x) = \frac{1}{x}$. Are the following statements true or false?

- (a) $F'(x) = f(x)$;
- (b) $\int f(x)dx = F(x)$;
- (c) $\int f(x)dx = F(x) + \text{constant}$;
- (d) $\int_{-1}^1 f(x)dx = F(1) - F(-1)$;
- (e) $\int_{-1}^1 f(x)dx$ does not exist.

$$19. \text{ Compute } \frac{d}{dx} \int_3^x \left(\frac{d}{dt} \cos t \right) dt.$$

$$20. \text{ Prove that } \int_0^x \frac{t}{\sqrt{1+t^2}} dt + \frac{d}{dx} \int_x^{\tan x} \sqrt{1+t^2} dt = \sec^3 x - 1, \quad -\frac{\pi}{2} < x < \frac{\pi}{2}.$$

Section 4.6

Evaluate the integrals:

1. $\int \sqrt{2x+1}dx;$

2. $\int \frac{x}{\sqrt{1-4x^2}}dx;$

3. $\int e^{5x}dx;$

4. $\int \sqrt{1+x^2}x^5dx;$

5. $\int_1^2 \frac{dx}{(3-5x)^2};$

6. Suppose that f is continuous on $[-a, a]$.

(a) Show that, if f is even, then $\int_{-a}^a f(x)dx = 2 \int_0^a f(x)dx;$

(b) Show that, if f is odd, then $\int_{-a}^a f(x)dx = 0;$

(c) Evaluate $\int_{-2}^2 (x^6 + 1)dx;$

(d) Evaluate $\int_{-1}^1 \frac{\tan x}{1+x^2+x^4}dx.$

Evaluate the integrals:

7. $\int e^x \sin e^x dx;$

8. $\int x^3(2+x^4)^5 dx;$

9. $\int (x+1)\sqrt{2x+x^2}dx;$

10. $\int \frac{(\ln x)^2}{x} dx;$

$$11. \int (1 + \tan x)^5 \sec^2 x dx;$$

$$12. \int \frac{\sin 2x}{1 + \cos^2 x} dx;$$

$$13. \int_0^{\pi/2} \cos x \sin(\sin x) dx;$$

$$14. \int_0^4 \frac{x}{\sqrt{1+2x}} dx;$$

$$15. \int_0^a x \sqrt{x^2 + a^2} dx \quad (a > 0).$$

Section 4.7

In exercises 1 – 4, compute Midpoint, Trapezoidal and Simpson's rule approximations by hand, for $n = 4$:

$$1. \int_0^1 (x^3 + x) dx;$$

$$2. \int_1^3 \frac{1}{x} dx;$$

$$3. \int_0^1 5x^4 dx;$$

$$4. \int_0^1 \cos x dx.$$

Section 4.8

In exercises 1 – 4, solve each equation for x :

$$1. 2 \ln x = 1;$$

2. $\ln(5 - 2x) = -3$;
3. $\ln x + \ln(x - 1) = 1$;
4. $\ln(\ln x) = 1$.
5. $4xe^{-x^2} = |x|$.

In exercises 6 – 8, find the exact value of each expression:

6. $\ln \frac{1}{e}$;
7. $\ln(\ln e^{e^{10}})$;
8. $e^{-2 \ln 5}$.

In exercises 9 – 11, express the given quantity as a single logarithm:

9. $\log 5 + 5 \log 3$;
10. $\ln(a + b) + \ln(a - b) - 2 \ln c$;
11. $\ln(1 + x^2) + \frac{1}{2} \ln x - \ln \sin x$.

In exercises 12 – 15, evaluate the integrals:

12. $\int_3^{10} \frac{x}{x^2 - 4} dx$;
13. $\int \left(\frac{1 - x}{x} \right)^2 dx$;
14. $\int \frac{t^2}{1 + t^3} dt$;

$$15. \int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx;$$

$$16. \int \frac{1}{\sqrt{x}(\sqrt{x} + 1)} dx.$$

In exercises 17 – 19, evaluate the derivative:

$$17. \frac{d}{dx} \log \left(\frac{2x^3}{x^4 + 2} \right);$$

$$18. \frac{d}{dx} \ln \sqrt{x^4 + 3};$$

$$19. \frac{d}{dx} \ln(x(x + 1)^{7/2} \sin^2 3x);$$

$$20. \frac{d}{dx} \{ 5^{3x-4} + \log_3 \left| \frac{1-x^2}{2-4x^3} \right| \}.$$

Hyperbolic and Inverse Hyperbolic functions

Evaluate the integrals;

$$1. \int \sinh 4x dx;$$

$$2. \int x^2 \cosh 2x dx;$$

$$3. \int \operatorname{csch} 3x \coth 3x dx;$$

$$4. \int x^2 \cosh(x^3 + 4) dx;$$

$$5. \int \cosh 2x \sinh 3x dx;$$

$$6. \int \cosh^2 3x dx;$$

$$7. \int \operatorname{sech}^2 7x dx;$$

8. $\int e^2 \sinh 3x dx;$
9. $\int \operatorname{sech} x dx;$
10. $\int_3^4 \frac{1}{\sqrt{x^2 - 4}} dx;$
11. $\int_1^2 \frac{1}{\sqrt{x^2 + 2x}} dx;$
12. $\int_0^1 \frac{1}{\sqrt{x^2 + 2x + 5}} dx;$
13. $\int_{-1}^1 \frac{1}{\sqrt{x^2 + 6x + 8}} dx;$
14. $\int_4^5 \frac{x + 1}{\sqrt{x^2 - 9}} dx;$
15. $\int_1^2 \frac{1}{\sqrt{x^2 + x}} dx;$
16. $\int_{-1}^0 \frac{1}{\sqrt{2x^2 + 4x + 7}} dx;$
17. $\int_1^2 \frac{1}{x\sqrt{4 - x^2}} dx.$
18. $\int \tanh^2 x dx;$
19. $\int \sinh^4 x dx;$
- In problems 20 – 27 find $f'(x)$,
20. $f(x) = \sqrt{x} \tanh \sqrt{x};$
21. $f(x) = \frac{\coth x}{\cot x};$
22. $f(x) = \sinh^2(\cos x);$
23. $f(x) = \sinh^{-1} e^x;$
24. $f(x) = \operatorname{sech}^{-1} \sqrt{1 - x};$

25. $f(x) = \sqrt{\cosh^{-1} x}$;

26. $f(x) = \tanh^{-1}(x^2 - 1)$;

27. $f(x) = \coth^{-1}\left(\frac{x}{x+1}\right)$.

In problems 28–35, verify the formula:

28. $\sinh 2x = 2 \sinh x \cosh x$;

29. $\cosh 2x = \cosh^2 x + \sinh^2 x$

30. $\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$;

31. $\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$;

32. $\tanh(x + y) = \frac{\tanh x + \tanh y}{1 + \tanh x \tanh y}$;

33. $\cosh x + \sinh x = e^x$;

34. $(\cosh x + \sinh x)^n = \cosh nx + \sinh nx$, for every positive integer n ;

35. $(\cosh x - \sinh x)^n = \cosh nx - \sinh nx$, for every positive integer n .

Chapter 6:

Calculus, Early Transcendental Functions pages 509-560

6.1: 7,12,14,20,22,29,30,32,35,39.

6.2: 15,18,28,30,33,36,39,40.

6.3: 15,17,20,21,24,26,29,30,33.

6.4: 4,7,8,17,21,24,28,30,32,37.

6.6: 6,8,11,16,26,32,49,52.