King Saud University | MATH 151 (Discrete Mathematics)

Department of Mathematics | Final Exam

2nd Semester 1436-1437 H | Duration: 3 Hours

Name:

Sequence Number:

Teacher's Name:

Section:

Question	Mark
Question I	
Question II	
Question III	
Question IV	
Question V	
Total	

Question	1	2	3	4	5	6	7	8	9	10	11	12	Total
Answer													

Question I:

Choose the correct answer, then fill in the table above:

(1) The compound proposition $p \leftrightarrow p$, for any proposition p, is always a

(a) Tautology (b) Contradiction (c) Contingency (d) None of the previous

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(2) The existential quantifier "$\exists\, x \in \mathbb{N}: 3x = 1$ " is

(a) True (b) False (c) None of the previous

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(3) The argument $\dfrac{q \wedge \neg p}{\therefore \neg p}$ is

(a) Valid (b) Not valid (c) None of the previous

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(4) If $A\ and\ B$ are subsets of the universal set U, such that $\bar{A} \cap B = B$, then

(a) $\bar{A} \subset B$ (b) $B \subset \bar{A}$ (c) $A = B$ (d) None of the previous

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(5) If $R = \{(x, y) | x \geq y\}$, is a relation on $\mathbb{Z}$, then the symmetric closure of R is

(a) $\{(x, y) | x < y\}$ (b) $\{(x, y) | x \leq y\}$ (c) $\{(x, y) | x = y\}$ (d) None of the previous

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(6) Let $U = \{1, 2, 3\}$ be a nonempty set and let $R = \{(A, B) | A \subseteq B, A, B \in P(U)\}$ be the inclusion partially ordered relation. Then $\{1,2\}\ and\ \{1\}$ are

(a) Comparable (b) Incomparable (c) None of the previous

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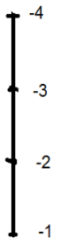
(7) The Hasse graph 

represents the poset on $\{-4,-3,-2,-1\}$ given by:

(a)$\{(x,y)|x\ divides\ y\}$ (b) $\{(x,y)|x \leq y\})$ (c) $\{(x,y)|x \geq y\}$ (d) None of the previous

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(8) There exists a graph with vertices of degrees

(a) 1,2,4,6 (b) 1,2,3,6 (c) 1,2,3,5 (d) None of the previous

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(9) Let $G = (V,E)$ be an undirected graph with 3 vertices each of degree 4, then G has

(a) 6 edges (b) 12 edges (c) 7 edges (d) None of the previous

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(10) The graph K_3 is

(a) Not connected (b) Bipartite (c) Not bipartite (d) None of the previous

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(11) A tree with 90 vertices has

(a) 90 edges (b) 89 edges (c) 91 edges (d) None of the previous

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(12) The dual of the Boolean expression $(x+1)+\bar{y}.0+\bar{x}.1$ is

(a) $(x.0).(\bar{y}+1)(\bar{x}+0)$ (b) $(\bar{x}.0).(y+1)(x+0)$

(c) $(x.1).(\bar{y}+0).(\bar{x}+1)$ (d) None of the previous

Question II:

A. Without using truth tables prove the following

$$p \rightarrow (q \wedge p) \equiv \neg p \vee q.$$

B. Prove that " n^2 is odd if and only if $1 - n$ is even", where n is an integer.

C. Use Mathematical Induction to prove that

$$\frac{1}{(1)(2)} + \frac{1}{(2)(3)} + --- + \frac{1}{(n-1)(n)} = \frac{n-1}{n}, \qquad n \geq 2.$$

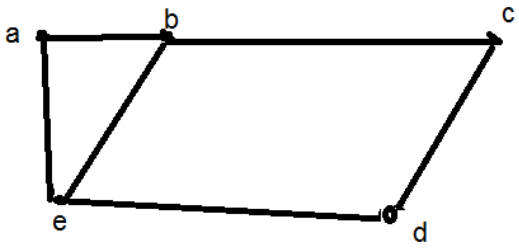
Question III:

A. Prove that the relation R defined by:

$$xRy \Leftrightarrow x \equiv y\ (mod 3)$$

is an equivalence relation on the set $\mathbb{Z}$. Then write a partition of $\mathbb{Z}$ corresponding to this relation.

B. Answer the following questions about the following graph G:



G

(i) Is the graph connected? Justify your answer.

(ii) Find $deg(e)$. Find a path from $a\ to\ c$. What is its length?

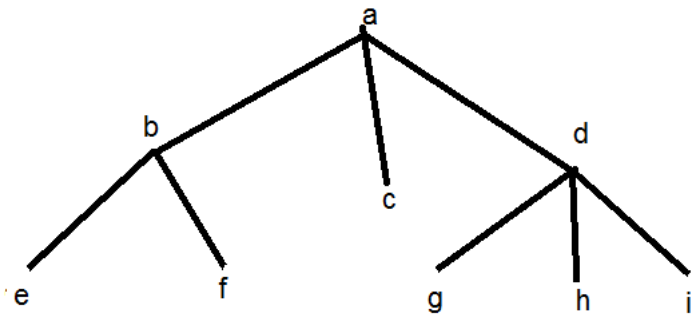
(iii) Are the graphs G and W_4 isomorphic? Justify your answer.

(iv) Draw the subgraph of G induced by the vertices a, b, e.

(v) Is G a planar? If so, how many regions does it have? Justify your answer.

Question IV:

A. Answer the following questions about the tree below:



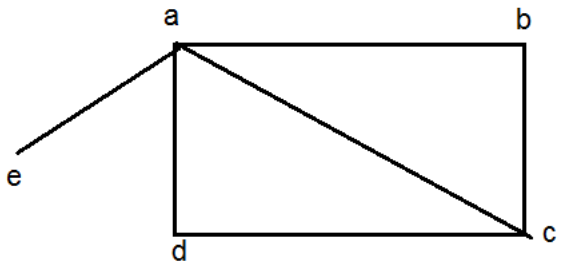
(i) Which vertex is the root? Which vertices are internal? Which vertices are leaves?

(ii) Which vertex is the parent of f? Which vertices are siblings of c?

(iii) Is the tree a full-3ary tree? Justify your answer.

(iv) Find the level of each vertex in the tree. What is the height of the tree? Is the tree balanced? Justify your answer.

B. Draw a spanning tree of the graph below:



C. Draw a binary search tree for the words: **plate, knife, spoon, table, glass, napkin** (using alphabetical order).

Question V:

A. Find a Boolean expression for the function that has the values in the following table:

x	y	z	$F(x,y,z)$
1	1	1	0
1	1	0	1
1	0	1	0
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	1
0	0	0	0

B. Find the **sum of product** expansion for the following Boolean function:

$$f(x,y,z) = x\bar{z} + yz.$$

Find **the value** of $f(1,0,1)$.

C. Construct a circuit from inverters, AND gates, and OR gates to produce the output

$$xy + \bar{x}y.$$

D. Use K-maps to minimize the sum of product expansion

$$xyz + xy\bar{z} + \bar{x}y\bar{z} + \bar{x}\bar{y}z.$$

Good Luck ☺