King Saud University
Department of Mathematics

244
Second Midterm, December 2016

NAME:

Group Number/Instructor name:

ID:

- Duration of the exam: 90 minutes

Question	Grade
I	
II	
III	
IV	
Total	

Question	1	2	3	4	5
Answer					

I) Choose the correct answer (write it on the table above):

1) If $u = (-3, 4, x)$ is a vector in $\mathbb{R}^3$ such that $\|u\| = 6$ then $x =$

(a) $\pm\sqrt{2}$	(b) 1	(c) $\pm\sqrt{11}$	(d) None

2) The unit vector that has the same direction as $v = (2, -1, 0, -2)$ is

(a) $3v$	(b) $\frac{1}{3}v$	(c) $\|v\|$	(d) None

3) (1) If A is invertible matrix, then $\det\left((kA)^{-1}A^T\right) =$

(a) 1	(b) k	(c) $\frac{1}{k}$	(d) None

4) If $\|u + v\| = 5$ and $\|u - v\| = 1$, then the dot product $u \cdot v$ equals

(a) 6	(b) 4	(c) 1	(d) None

5) The distance between e_1 and $(1, 4, -2, \sqrt{3})$ in the Euclidean space $\mathbb{R}^4$ is

(a) 5	(b) $\sqrt{17}$	(c) 6	(d) None

II) a) Determine whether the following is **True** or **False**.

(1) If $\mathbf{u}$ is a column matrix and $\mathbf{v}$ is a column matrix then $\mathbf{u} \cdot \mathbf{v} = \mathbf{u}\mathbf{v}^{\mathbf{T}}$. ( )

(2) If A is invertible matrix, then $\mathbf{adj}(A)$ must be invertible matrix. ( )

(3) The minor $M_{i,i+1}$ is the same as the cofactor $C_{i,i+1}$. ( )

(4) For every $n \times n$ matrix A, it is true that $\det(A^2) = (\det(A))^2$. ( )

(5) If $\mathbf{u} \cdot \mathbf{v} = 0$, then either $\mathbf{u} = \mathbf{0}$ or $\mathbf{v} = \mathbf{0}$. . ( )

(6) The vector $\mathbf{0}$ is orthogonal to any vector $\mathbf{u}$ in $\mathbb{R}^m$. ( )

b) If u and v are vectors in R^n and $(u - 2v) \cdot (3u) + (6v - u) \cdot u = 0$ then find the value of u.

III) a) Let $V = \{(x, 4) \in R^2, x \neq 0\}$ with the following addition and scalar multiplication on $\mathbf{u} = (x, 4) \in V$ and $\mathbf{v} = (y, 4) \in V$

$$\mathbf{u} + \mathbf{v} = (xy, 4)$$
$$k\mathbf{u} = (kx, 4)$$

1. Find the object $\mathbf{0} \in V$ such that $\mathbf{u} + \mathbf{0} = u$ for all $\mathbf{u} \in V$.
2. If $\mathbf{u} \in V$. Find the object $-\mathbf{u} \in V$ such that $-\mathbf{u} + \mathbf{u} = \mathbf{0}$.
3. Show that V is not a vector space.

b) If $A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & 0 & 5 \\ 1 & 7 & 2 \end{bmatrix}$. Find A^{-1} using its adjoint.

IV) a) Compute the determinant of

$$A = \begin{bmatrix} 3 & 3 & 0 & 5 \\ 2 & 2 & 0 & -2 \\ 4 & 1 & -3 & 0 \\ 2 & 10 & 3 & 2 \end{bmatrix}$$

b) Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 1 & 2 & -2 \\ 3 & 1 & 0 \end{bmatrix}$, $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$. Solve for x_1, using **Cramer's rule**.

Scrap paper. This page will not be graded.