

Math 244 – Quiz 1

Name: _____ ID: _____ Marks: _____ (5)

Question 1 [1 marks]

Determine whether the following is **True** or **False**

(a) All homogeneous linear systems are consistent. [True]

(b) If $A = \begin{bmatrix} 1 & 2 & 6 \\ 3 & 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 2 \\ 1 & 3 \\ 0 & 3 \end{bmatrix}$, then $(AB)_{22} = 21$ [True]

Question 2 [4 marks]

Solve the following linear system using **Gaussian elimination** or **Gauss-Jordan elimination**.

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 8 \\ -x_1 - 2x_2 + 3x_3 &= 1 \\ 3x_1 - 7x_2 + 4x_3 &= 10 \end{aligned}$$

Solution:

The augmented matrix is: $\begin{bmatrix} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{bmatrix}$

We use Gauss-Jordan elimination..

$$\begin{aligned} &\begin{bmatrix} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{bmatrix} \xrightarrow{R_1+R_2} \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 3 & -7 & 4 & 10 \end{bmatrix} \xrightarrow{-3R_1+R_3} \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{bmatrix} \\ &\xrightarrow{-R_2} \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & -10 & -2 & -14 \end{bmatrix} \xrightarrow{10R_2+R_3} \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & -52 & -104 \end{bmatrix} \xrightarrow{\frac{R_3}{-52}} \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 2 \end{bmatrix} \quad (R.E.F.) \\ &\xrightarrow{5R_3+R_2} \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \xrightarrow{-2R_3+R_1} \begin{bmatrix} 1 & 1 & 0 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \xrightarrow{-R_2+R_1} \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} \quad (R.R.E.F.) \end{aligned}$$

The solution is

$$x_1 = 3, \quad x_2 = 1, \quad x_3 = 2.$$

To use the Gaussian elimination method we stop at (R.E.F.) and do a back substitution:

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 8 \\ x_2 - 5x_3 &= -9 \\ x_3 &= 2 \\ \implies x_2 &= -9 + 5(2) = -9 + 10 = 1 \implies x_1 = -1 - 2(2) + 8 = 3 \end{aligned}$$