Math 316 **Name:** ____________________
Spring 2016
First Mid-Term Exam
22/2/2016
Time Limit: 90 minutes Student Number ____________________

This exam contains 6 pages (including this cover page) and 5 questions.
Total of points is 20.

Grade Table (for teacher use only)

Question:	1	2	3	4	5	Total
Points:	4	3	5	4	4	20
Score:						

(a)	(b)	(c)	(d)

1. (4 points) **Choose the correct answer. Write your answer in the previous table.**

   (a) The dimension of the vector space C over $\mathbb{R}$ is

   A. 0 B. 2 C. 1 D. None of the previous

   (b) The space of all polynomials defined on an interval I, $P(I)$,

   A. is finite dimensional vector space
   B. is an infinite dimensional vector space
   C. it is not even a vector space
   D. None of the previous.

   (c) The orthogonal projection of the vector $v = (1, -1, 1)$ in the direction of the vector $u = (1, 1, 1)$ is

   A. $u_1 = (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$
   B. $u_2 = u_1 = (\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$
   C. $u_3 = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$
   D. $u_4 = (\frac{-1}{3}, \frac{1}{3}, \frac{1}{3})$

(d) If $V = C(0,1)$ associated with the inner product $< f, g >= \int_0^1 f(x)g(x)\,dx$. Let $f = x$ and $g = ax + b$. If g is orthogonal to f then a, b satisfy the relation

A. $a + b = 0$ B. $\frac{a}{2} + \frac{b}{3} = 0$ C. $\frac{a}{3} + \frac{b}{2} = 0$ D. None of the previous

2. (3 points) **Determine whether the following is True or False. Correct the sentence in the box if it is not correct.**

(a) **True False.** Any orthogonal set in any inner product space is linearly independent.

(b) **True False.** The Gram-Schmidt process allows us to construct an orthogonal set from any finite set of vectors in the inner product space.

(c) **True False.** In any inner product space every Cauchy sequence is convergent.

(d) **True False.** The function $f(x) = x^{\alpha} \in \mathcal{L}^2(0,1)$ if $\alpha > -1$.

3. (5 points) Let $x = (2, -1, 3)$, $y = (4, 5, 7)$ be in the Euclidean inner product space $\mathbb{R}^3$. Compute

   (a) $< x, x >$ and $\|x\|$.

   (b) $< y, y >$ and $\|y\|$.

   (c) $< x + y, x + y >$ and $\|x + y\|$.

   (d) Verify both of the Cauchy-Schwartz inequality and the triangle inequality for the vectors x and y.

4. (4 points) Let $f_n(x) = x(1-x)^n$, $0 \leq x \leq 1$.

   (a) Calculate the point wise limit f and its domain.

   (b) Is the convergence uniform? Justify your answer.

(c) Determine the convergence in $\mathcal{L}^2$.

5. (4 points) (a) Study the uniform convergence of the series $\sum \frac{\sin kx}{k^{2/3}}$, $x \in [-\pi, \pi]$.

(b) Study the uniform convergence in $\mathcal{L}^2$.