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| **Name:** |  |
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| Question | Marks |
| Q1(a) |  |
| Q1(b) |  |
| Q1(c) |  |
| Q1(d) |  |
| Q1(e) |  |
| Q1(f) |  |
| Q1(g) |  |
| Q1(h) |  |
| Q2(a) |  |
| Q2(b) |  |
| Q2(c) |  |
| Q3(a) |  |
| Q3(b) |  |
| Q3(c) |  |
| Q4(a) |  |
| Q4(b) |  |
| Q4(c) |  |
| Total out of 45 |  |

Question 1: (marks=3x8=24)

Prove or disprove the following:

1. Let (X, d) be a metric space and F ⊆ X be a finite subset. Then F is closed in X.
2. Let Y be a subspace of X, if A is closed in Y, and Y is closed in X, then A is closed in X.
3. If A ⊆ X and B ⊆ Y. Then = × in X × Y.
4. The image of a Hausdorff space under a continuous map is Hausdorff.
5. Every subspace of a metric space which is closed and bounded is compact
6. Let be a map from a metric space to a metric space . If is the discrete metric then is continuous
7. The intersection of two open balls in a metric space is an open ball.
8. Any two discrete topological spaces are homeomorphic

Question 2: (marks=2+3+2=7)

1. When do we say that a topological space is - space? Give an example of a - space and another not - space.
2. A topological space is a -space if and only if every finite subset of is closed.
3. Show that any finite subset of a - space doesn’t have a limit point.

Question 3: (marks=4+3+2=)

1. Let be a metric space, and be injective function from a non-empty set . Show that the function given by is a metric for .

1. Let have the topology . Let be the sequence in , where. Is it a convergent sequence? What is the limit if it exists?
2. Prove that a sequentially compactness is a topological property.

Question 4: (marks=2+3+2=7)

1. Define what do you mean by a complete metric space, and give an example of a complete metric and a non-complete metric.
2. Prove that every compact metric space is complete.
3. Describe Cauchy sequences in the Discrete Metric on . Is this Discrete Metric complete?

Question 5:

1. Let be a metrizable space. Prove that is limit point compact space if and only if is sequentially compact.
2. If is not a metrizable space, then prove that the statement in I is not true.

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