King Saud University

Department of Mathematics

First semester 1434-1435 H

MATH 425 MATH 423

Final Exam

Duration: 180 Minutes

Question 1:

Q1-a) Find the general solution of

1. $xzp + yzq = xy$
2. $({D_x}^2 - {D_y}^2 - 3D_x + 3D_y)z = e^{x+2y}$
3. $xs + q = 4x + 2y + 2$

Q1-b) Classify the following equation. Reduce to its normal form, and then solve it

$$x^2 r + 2xys + y^2 t = 0$$

Question 2:

Q2-a) Find the solution of the Dirichlet problem

$$\nabla^2 u = 0, \qquad 0 < r < 1$$

$$u(1,\theta) = sin\theta \qquad 0 \le \theta \le 2\pi$$

Q2-b) Find the surface which intersects the surfaces of the system

$z(x+y) = c(3z+1)$ orthogonally and which pass through the circle $x^2 + y^2 = 1,\ z = 1$

Question 3:

Find the solution $u(r,\theta)$ of Laplace equation outside of the sphere of radius $r = 1$,

$if\ u(1,\theta) = 2cos^2\boldsymbol{\theta} - \mathbf{1}$

Question 4:

Q4-a) Find the solution of the wave equation on the real line $-\infty < x < \infty$ with initial condition

$$u(x,0) = e^x$$

$$u_t(x,0) = sinx$$

Q4-b) Discuss a solution of the wave equation $u_{xx} + u_{yy} = u_{tt}$ satisfied by

$$u(0,y,t) = u(1,y,t) = u(x,0,t) = u(x,1,t) = 0$$

And

$$u(x,y,0) = A\sin\pi x\sin 2\pi y$$

$$u_t\ (x,y,0) = 0$$

Question 5:

Q5-a) Solve initial boundary value problem

$$\frac{\partial u}{\partial t} = K\frac{\partial^2 u}{\partial x^2} \qquad 0 < x < 10$$

$$u(0,t) = 0, u(10,t) = 100, \quad t > 0$$

$$u(x,0) = 20 \quad 0 < x < 10$$

Q5-b) The heat conduction in the round rod with heat sources present is described by the PDE

$$u_t - u_{xx} = F(x,t), 0 < x < 1, t > 0$$

Subject to

$$u(0,t) = u(1,t) = 0$$

$$u(x,0) = f(x)$$

Find $u(x,t)$.

Question 6: *Please resolve either A or B*

A) Let $R \subseteq \mathbb{R}^3$ be a region and $p(x, y, z) \in R$

Let $S(p, r)$ denote the sphere with center at p and with radius r

1. Define the spherical mean $\overline{\varphi}$ of φ on $S(p, r)$
2. Prove $\overline{\varphi}\,(r) = \varphi\,(\bar{Q})$ where $\bar{Q}$ is some point on $S(p, r)$
3. Prove that if u is harmonic function then $\overline{u}\,(r) = u\,(p)$.

B) A homogeneous thermally conducting cylinder occupies the region

$$0 \le r \le a,\ \ 0 \le \theta \le 2\Pi, 0 \le z \le b.$$

The top $z = 5$ and the lateral surface $r = 3$ are held at 0^0, while the base $z = 0$ is held at 100^0.

Assume that there is no source of heat generation within the cylinder.

Show temperature in within the cylinder is

$$u(r, z) = 200 \sum_{n=1}^{N} \frac{J_0\,(\xi_n r/a) \sinh[(\xi_n/a)(z - h)]}{\xi_n sinh(-\xi_n h/a) J_1(\xi_n)}$$

Where ξ_n are positive zeros of $J_0\,(\xi)$