King Saud University
Department of Mathematics

Final Exam

2^{nd} **semester 1437 H**

Course Title: Math 425 (Partial Differential Equations)

Date: 24 May 2016; (8-11) am

(. . . . . .) Name . . . . ID . . . .

Question	Grade
Q1	
Q2	
Q3	
Q4	
Total	

Question 1

I. Solve Cauchy problem:

$$x^2 u_{xx} - 2x u_{xy} + u_{yy} + u_y = 0;$$
$$u(1, y) = y^2, \; u_x(1, y) = e^y$$

II. Find the general solution of

(1) $x^2p + y^2q = 0.$

(2) $5p + 4q + z = x^3 + 2e^{3y}.$

Question 2

I. Solve the Heat equation:

$$u_t = 9u_{xx}$$

subject to:

$$u_x(0,t) = u_x(\pi,t) = 0, \quad t \geq 0$$
$$u(x,0) = \cos 4x, \quad 0 \leq x \leq \pi.$$

II. Prove that <u>if</u> the following boundary value problem

$$\begin{aligned} \Delta u = f \quad &\text{in } \Omega \\ u = g \quad &\text{on } \partial\Omega \end{aligned}$$

has a solution then it is unique.

Question 3

I. Find the potential at all point of space outside of a sphere of radius $R = 3$, where $u(3, \theta, \phi) = \cos^3(\theta)$.

(Hint: You may need the definition of Legendre's function $P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n}(x^2 - 1)^n$)

II. Find the formal series soltuion of the initial value problem

$$\begin{aligned}
&u_{tt} - c^2 u_{xx} = xe^t \\
&u(0,t) = 0,\ u(\pi,t) = 0, \quad t \geq 0 \\
&u(x,0) = \sin x,\ u_t(x,0) = 0, \quad 0 \leq x \leq \pi
\end{aligned}$$

Question 4

Find the general solution of $\Delta u = 0$ inside a circle C with radius $r = a$ s.t. u is a single-valued and continuous on $\bar{C}$.

(Hint: You may need to use that $\Delta u = u_{rr} + \frac{1}{r}u_r + \frac{1}{r^2}u_{\theta\theta}$,

$$\int_0^{2\pi} \cos n\theta \cos m\theta d\theta = \int_0^{2\pi} \sin n\theta \sin m\theta d\theta = \begin{cases} 0, & n \neq m \\ \pi, & n = m \end{cases}$$

and $\int_0^{2\pi} \cos n\theta \sin m\theta d\theta = 0, \quad \forall\ n, m$)