Final Exam

Question 1: (10 points)

(a) Find the solution of the following LP problem by inspection (without using the simplex algorithm).

$$\begin{aligned} \max z &= x_1 \\ \text{s.t.} \quad 5x_1 + x_2 &= 10 \\ 6x_1 + x_3 &= 3 \\ 3x_1 + x_4 &= 9 \\ x_1, x_2, x_3, x_4 &\geq 0. \end{aligned}$$

(b) Use the two phase method to find the optimal solution to the following LP:

$$\begin{aligned} \max z &= 4x_1 + 3x_2 \\ \text{s.t.} \quad 2x_1 + x_2 &\geq 3 \\ 4x_1 + x_2 &\leq 5 \\ x_1, x_2 &\geq 0. \end{aligned}$$

Question 2: (6 points)

If the following LP:

$$\begin{aligned} \max z &= 2x_1 + x_2 \\ \text{s.t.} \quad x_1 + x_2 &\leq 6 \\ 4x_1 + x_2 &\leq 12 \\ x_1, x_2 &\geq 0 \end{aligned}$$

has the optimal solution $z^* = 8$ at $x_1^* = 2, x_2^* = 4$, and $BV = \{x_1, x_2\}$.

(a) If $c_1 = 2$ is changed to $c_1 = 2+\Delta$, find the values of Δ such that the optimal solution remains optimal, and find the value of the new solution x_B^* and z^*.

(b) What is the solution if $\Delta = 1$, and if $\Delta = 3$?

Question 3: (6 points)

(a) Solve the following problems by finding the optimal solution and the range of λ for which the optimal solution remains optimal.

$$\begin{aligned} \max z =& (2 - 2\lambda)x_1 + (3 + 5\lambda)x_2 \\ \text{s.t.} \quad & x_1 + x_2 \leq 6 \\ & x_1 + 2x_2 \leq 8 \\ & x_1, x_2 \geq 0. \end{aligned}$$

(b) Use the optimal tableau to find the solution if $\lambda = 1$.

Question 4: (7 points)

Use branch and bound to solve the following integer problem.

$$\begin{aligned} \max z = & 3x_1 + 5x_2 \\ \text{s.t.} \quad & 3x_1 + 3x_2 \leq 13 \\ & 3x_1 + 6x_2 \leq 16 \\ & x_1, x_2 \text{ are nonnegative integers.} \end{aligned}$$

Question 5: (9 points)

You are given the following constraints

$$\begin{aligned} x_1 + x_2 &\geq 6 \\ 2x_1 + x_2 &\geq 8 \\ x_1 + x_2 &\leq 5 \\ x_1, x_2 &\geq 0 \end{aligned}$$

(a) Can we solve any problem with these constraints? why?
(b) If constraint one falls short, there is 2SR penalty for each unit. If constraint two falls short, there is 1SR penalty for each unit.
(1) Formulate the goal programming problem and solve it.
(2) Explain what the solution of this problem represents.

Question 6: (7 points)

Consider the following LP:

$$\begin{aligned} \max z = & c_1x_1 + c_2x_2 \\ \text{s.t.} \quad & a_1x_1 + a_2x_2 \leq b_1 \\ & a_3x_1 + a_4x_2 \leq b_2 \\ & x_1, x_2 \geq 0, \end{aligned}$$

you are given that the optimal tableau for this LP is

$$\begin{aligned} z \qquad\quad + \; 2s_1 + s_2 &= 1 \\ x_2 + 3s_1 + 2s_2 &= 2 \\ x_1 \quad + 4s_1 + 3s_2 &= 3 \end{aligned}$$

Determine a_i, b_j, c_j for $i = 1, 2, 3, 4$, $j = 1, 2$.

Question 7: (5 points)

Consider the maximization problem with the optimal tableau:

$$\begin{aligned} z + 2x_1 + x_2 &= 10 \\ 3x_1 + 2x_2 + s_1 &= 3 \\ 4x_1 + 3x_2 + s_2 &= 5 \end{aligned}$$

Determine the second best bfs to this LP.

Good Luck

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