Department of Mathematics

College of Sciences

King Saud University

Math 373

Final Exam

First semester, 1430 H

Time: 3 Hour

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| **Q1.** Let be an infinite set, and  a nonempty finite set. Let   1. Prove that is a topology on . 2. Find a base for different than itself. 3. Is  a Hausdorff space? 4. Is  a compact space? 5. Is  a metrizable space? 6. If ,  and , find and   **Q2. (a)** Define the following:   1. Cauchy sequences in metric spaces. 2. Complete metric space. 3. Uniform continuity in metric spaces. 4. Compact space.   **(b)** Prove or disprove the following statements:   1. A subspace of a complete metric space is complete. 2. For an open ball in a metric space the boundary 3. Every closed subset of a compact topological space is compact. 4. Completeness in metric spaces is a topological property.   **Q3.** Let  and be any topological spaces and let be a function. Show that:  **(a)** is closed iff ,  **(b)** is continuous iff  **(c)** If is compact and is continuous, then is compact.  **Q4**. **(a)** Prove that every metric space is Hausdorff.  **(b)** If is a finite metric space, then show that the metric topology is the discrete topology.  **(c)** If the space  is metrizable and  show that is metrizable.  **(d)** Characterize the convergent sequences in discrete topological spaces.  **(e)** Let be a metric space. Prove that if  is a convergent sequence in , then the set is a bounded subset of .  **Bonus Question:**  Let be a continuous onto function from a compact space to a Hausdorff space . Let be an arbitrary function, where is a topological space. Show that if is continuous, then is continuous. |

Good Luck