Math 373: Sheet #2

1. Let (X, τ) be topological space such that there exists a countable bases $\mathfrak{B}$ for τ. Prove there exists a countable dense subset of X. Show that $\mathbb{R}$ with Co-finite topology does not have a countable base.
2. Prove the following" Let $f: (X, \zeta) \to (Y, \eta)$ be open and onto function, and let $\mathcal{B}$ be a base for ζ. Then $\{f(B): B \in \mathcal{B}\}$ is a base for η.
3. Prove that $f: X \to Y$ is continuous function if and only if $f^{-1}(Int(B)) \subseteq Int(f^{-1}(B))$ for every $B \subseteq Y$.
4. Prove that each of the following is a topological property:
   1. Boundary,
   2. Density.
5. Let (X, ξ) be topological space. Prove that $f: X \to \mathbb{R}$ is continuous if and only if for each $a \in \mathbb{R}$, both of the sets $\{x \in X: f(x) > a\}$ and $\{x \in X: f(x) < a\}$are open sets in X.
6. Show that area is not a topological property.
7. Show that if every function $f: X \to \mathbb{R}$ is continuous, then X is a discrete space. (here $\mathbb{R}$ with usual topology)
8. Show that the identity function $I: (X, \zeta) \to (X, \eta)$ is continuous if and only if $\eta \subseteq \zeta$.