DERIVATIVE FORMULAS

General Rules

$\frac{d}{dx}[f(x)+g(x)] = f'(x)+g'(x)$

$\frac{d}{dx}[f(x)-g(x)] = f'(x)-g'(x)$

$\frac{d}{dx}[cf(x)] = c\,f'(x)$

$\frac{d}{dx}[f(g(x))] = f'(g(x))\,g'(x)$

$\frac{d}{dx}[f(x)\,g(x)] = f'(x)\,g(x)+f(x)\,g'(x)$

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)\,g(x)-f(x)g'(x)}{[g(x)]^2}$

Power Rules

$\frac{d}{dx}(x^n) = nx^{n-1}$

$\frac{d}{dx}(c) = 0$

$\frac{d}{dx}(cx) = c$

$\frac{d}{dx}(\sqrt{x}) = \frac{1}{2\sqrt{x}}$

Exponential

$\frac{d}{dx}[e^x] = e^x$

$\frac{d}{dx}[a^x] = a^x \ln a$

$\frac{d}{dx}\left[e^{u(x)}\right] = e^{u(x)}\,u'(x)$

$\frac{d}{dx}[e^{rx}] = r\,e^{rx}$

Trigonometric

$\frac{d}{dx}(\sin x) = \cos x$

$\frac{d}{dx}(\cos x) = -\sin x$

$\frac{d}{dx}(\tan x) = \sec^2 x$

$\frac{d}{dx}(\cot x) = -\csc^2 x$

$\frac{d}{dx}(\sec x) = \sec x\,\tan x$

$\frac{d}{dx}(\csc x) = -\csc x\,\cot x$

Inverse Trigonometric

$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$

$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$

$\frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$

$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\,\sqrt{x^2-1}}$

$\frac{d}{dx}(\csc^{-1} x) = -\frac{1}{|x|\,\sqrt{x^2-1}}$

Hyperbolic

$\frac{d}{dx}(\sinh x) = \cosh x$

$\frac{d}{dx}(\cosh x) = \sinh x$

$\frac{d}{dx}(\tanh x) = \mathrm{sech}^2 x$

$\frac{d}{dx}(\coth x) = -\mathrm{csch}^2 x$

$\frac{d}{dx}(\mathrm{sech}\, x) = -\mathrm{sech}\, x\,\tanh x$

$\frac{d}{dx}(\mathrm{csch}\, x) = -\mathrm{csch}\, x\,\coth x$

Inverse Hyperbolic

$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$

$\frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$

$\frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$

$\frac{d}{dx}(\coth^{-1} x) = \frac{1}{1-x^2}$

$\frac{d}{dx}(\mathrm{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$

$\frac{d}{dx}(\mathrm{csch}^{-1} x) = -\frac{1}{|x|\,\sqrt{x^2+1}}$

other $\frac{d}{dx}(\ln u) = \frac{1}{u}$, $\sinh u = \frac{e^u + e^{-u}}{2}$, $\cosh u = \frac{e^{u} - e^{-u}}{2}$

2.4 Important Formulas for Integration

BRIEF TABLE OF INTEGRALS

1. $\int u^n\,du = \dfrac{u^{n+1}}{n+1} + C,\ n \neq -1$

2. $\int \dfrac{1}{u}\,du = \ln|u| + C$

3. $\int e^u\,du = e^u + C$

4. $\int a^u\,du = \dfrac{1}{\ln a}a^u + C$

5. $\int \sin u\,du = -\cos u + C$

6. $\int \cos u\,du = \sin u + C$

7. $\int \sec^2 u\,du = \tan u + C$

8. $\int \csc^2 u\,du = -\cot u + C$

9. $\int \sec u \tan u\,du = \sec u + C$

10. $\int \csc u \cot u\,du = -\csc u + C$

11. $\int \tan u\,du = -\ln|\cos u| + C$

12. $\int \cot u\,du = \ln|\sin u| + C$

13. $\int \sec u\,du = \ln|\sec u + \tan u| + C$

14. $\int \csc u\,du = \ln|\csc u - \cot u| + C$

15. $\int u \sin u\,du = \sin u - u\cos u + C$

16. $\int u \cos u\,du = \cos u + u\sin u + C$

17. $\int \sin^2 u\,du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$

18. $\int \cos^2 u\,du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$

19. $\int \tan^2 u\,du = \tan u - u + C$

20. $\int \cot^2 u\,du = -\cot u - u + C$

21. $\int \sin^3 u\,du = -\frac{1}{3}\left(2 + \sin^2 u\right)\cos u + C$

22. $\int \cos^3 u\,du = \frac{1}{3}\left(2 + \cos^2 u\right)\sin u + C$

23. $\int \tan^3 u\,du = \frac{1}{2}\tan^2 u + \ln|\cos u| + C$

24. $\int \cot^3 u\,du = -\frac{1}{2}\cot^2 u - \ln|\sin u| + C$

25. $\int \sec^3 u\,du = \frac{1}{2}\sec u \tan u + \frac{1}{2}\ln|\sec u + \tan u| + C$

26. $\int \csc^3 u\,du = -\frac{1}{2}\csc u \cot u + \frac{1}{2}\ln|\csc u - \cot u| + C$

27. $\int \sin au \cos bu\,du = \dfrac{\sin(a-b)u}{2(a-b)} - \dfrac{\sin(a+b)u}{2(a+b)} + C$

28. $\int \cos au \cos bu\,du = \dfrac{\sin(a-b)u}{2(a-b)} + \dfrac{\sin(a+b)u}{2(a+b)}$

29. $\int e^{au} \sin bu\,du = \dfrac{e^{au}}{a^2+b^2}(a\sin bu - b\cos bu) + C$

30. $\int e^{au} \cos bu\,du = \dfrac{e^{au}}{a^2+b^2}(a\cos bu + b\sin bu) + C$

31. $\int \sinh u\,du = \cosh u + C$

32. $\int \cosh u\,du = \sinh u + C$

33. $\int \operatorname{sech}^2 u\,du = \tanh u + C$

34. $\int \operatorname{csch}^2 u\,du = -\coth u + C$

35. $\int \tanh u\,du = \ln(\cosh u) + C$

36. $\int \coth u\,du = \ln|\sinh u| + C$

37. $\int \ln u\,du = u\ln u - u + C$

38. $\int u \ln u\,du = \frac{1}{2}u^2 \ln u - \frac{1}{4}u^2 + C$

39. $\int \dfrac{1}{\sqrt{a^2-u^2}}\,du = \sin^{-1}\dfrac{u}{a} + C$

40. $\int \dfrac{1}{\sqrt{a^2+u^2}}\,du = \ln\left|u + \sqrt{a^2+u^2}\right| + C$

41. $\int \sqrt{a^2-u^2}\,du = \dfrac{u}{2}\sqrt{a^2-u^2} + \dfrac{a^2}{2}\sin^{-1}\dfrac{u}{a} + C$

42. $\int \sqrt{a^2+u^2}\,du = \dfrac{u}{2}\sqrt{a^2+u^2} + \dfrac{a^2}{2}\ln\left|u + \sqrt{a^2+u^2}\right| + C$

43. $\int \dfrac{1}{a^2+u^2}\,du = \dfrac{1}{a}\tan^{-1}\dfrac{u}{a} + C$

44. $\int \dfrac{1}{a^2-u^2}\,du = \dfrac{1}{2a}\ln\left|\dfrac{a+u}{a-u}\right| + C$

7.2 Important Results

1. $\int_{-\pi}^{\pi}\cos nx dx = \frac{\sin nx}{n}\Big|_{-\pi}^{\pi} = \frac{1}{n}[\sin n\pi - \sin(-n\pi)] = \frac{1}{n}[0-0] = 0$

   $\sin n\pi = 0 \qquad \sin(-n\pi) = -\sin n\pi = 0$

2. $\int_{-\pi}^{\pi}\sin nx dx = -\frac{\cos nx}{n}\Big|_{-\pi}^{\pi} = \frac{1}{n}[\cos n\pi - \cos(-n\pi)] = -\frac{1}{n}[\cos n\pi - \cos n\pi] = 0$

   $\cos(-\theta) = \cos\theta$

   $\cos n\pi = (-1)^n$

   $\cos(-n\pi) = \cos n\pi = (-1)^n$

3. $\int_{0}^{\pi}\cos nx dx = \frac{\sin nx}{nx}\Big|_{0}^{\pi} = [\sin n\pi - \sin 0] = \frac{1}{n}[0-0] = 0$

   $\sin n\pi = 0, \sin 0 = 0$

4. $\int_{0}^{\pi}\sin nx dx = -\frac{\cos nx}{n}\Big|_{0}^{\pi} = -\frac{1}{n}[\cos n\pi - \cos 0] = -\frac{1}{n}[(-1)^n - 1]$

5. $\int e^{ax}\cos bx dx = \frac{e^{ax}}{a^2+b^2}[a\cos bx + b\sin bx]$

6. $\int e^{ax}\sin bx dx = \frac{e^{ax}}{a^2+b^2}[a\sin bx - b\cos bx]$

7. $\int x\sin nx dx = -x\frac{\cos nx}{n} + \frac{\sin nx}{n^2}$

8. $\int x\cos nx dx = x\frac{\sin nx}{n} + \frac{\cos nx}{n^2}$

9. $\int x^2\sin nx dx = -x^2\frac{\cos nx}{n} + 2x\frac{\sin nx}{n^2} + 2\frac{\cos nx}{n^3}$

10. $\int x^2\cos nx dx = x^2\frac{\sin nx}{n} + 2x\frac{\cos nx}{n^2} - 2\frac{\sin nx}{n^3}$

11. $\sin a \ \sin b = \frac{1}{2}[\cos(a-b) - \cos(a+b)]$

    $\cos a \ \cos b = \frac{1}{2}[\cos(a-b) + \cos(a+b)]$

    $\sin a \ \cos b = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$

    $\cos a \ \sin b = \frac{1}{2}[\sin(a+b) - \sin(a-b)]$