Matrix

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2- Size of Matrix: If a matrix A has n rows and m columns, then we say A is "n by m matrix" and we write it as "$n \times m$".

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(ii) $\begin{bmatrix} 1 & x & x^2 & e^x \\ x+1 & \sin(x) & -x & 8 \\ 2^x & 0 & 15 & (x^3+5)^{100} \end{bmatrix}$ is 3×4 matrix.

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Exercise: Can we find a matrix which is square, row and column at the same time??.

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Example: Let A be the matrix $\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$.

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$$\begin{aligned} A^2 &= AA = \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 12 & 1 \end{bmatrix} \\ A^3 &= A^2 A = \begin{bmatrix} 4 & 0 \\ 12 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 28 & 1 \end{bmatrix} \\ A^{-3} &= (A^3)^{-1} = \frac{1}{8} \begin{bmatrix} 1 & 0 \\ -28 & 8 \end{bmatrix} \end{aligned}$$

$$A^2 - 2A + I$$

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A^2 - 2A + I &= \begin{bmatrix} 4 & 0 \\ 12 & 1 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 8 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}
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\end{aligned}
$$