Math 246 **Name:** ____________________
Fall 2015
Mid 2
23/6/1436
Time Limit: 90 Minutes Teaching Assistant ____________________

This exam contains 7 pages (including this cover page) and 6 questions.
Total of points is 24.

Rest of introduction. Rest of introduction. Rest of introduction. Rest of introduction. Rest of introduction. Rest of introduction. Rest of introduction. Rest of introduction.

Grade Table (for teacher use only)

Question	Points	Score
1	6	
2	3	
3	4	
4	3	
5	4	
6	4	
Total:	24	

(a)	(b)	(c)	(d)	(e)	(f)

1. (6 points) **Choose the correct answer. Write your answer in the previous table.**

(a) The value of a that makes $A = \begin{bmatrix} -3 & a^2 \\ 4 & 0 \end{bmatrix}$ symmetric is

A. 2 B. -2 C. both A and B D. -3

(b) If $A = \begin{bmatrix} 2-x & 5 & x^2 \\ 0 & x+3 & x-1 \\ 0 & 0 & x \end{bmatrix}$ is invertible matrix , then $x =$

A. 0 B. 5 C. -3 D. 2

(c) If $A^{-2} = \begin{bmatrix} 16 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then

A. $A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ B. $A = \begin{bmatrix} \frac{1}{4} & 0 & 0 \\ 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 \end{bmatrix}$ C. $A = \begin{bmatrix} 256 & 0 & 0 \\ 0 & 81 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

(d) For $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 7 & 6 \\ 1 & 0 & 8 \end{bmatrix}$ the cofactor C_{23} is

A. 2 B. -2 C. 3 D. 0

(e) If A is 4×4 matrix with $\det A = 3$, then $\det(2A(A^T)^{-1}) =$

A. 2.3^8 B. 16 C. 16.3^8 D. none.

(f) The coordinate vector of $w = (1, 0)$ relatives to the basis $\{u_1 = (1, -1),\ u_2(1, 1)\}$ of $\mathbb{R}^2$ is

A. $(\frac{1}{2}, -\frac{1}{2})$ B. $(\frac{1}{2}, \frac{2}{3})$ C. $(\frac{1}{2}, \frac{1}{2})$ D. $(1, 0)$.

---

2. (3 points) **Determine whether the following is True or False.**

(a) If A is a square matrix and the linear system has multiple solutions for x, then $\det A = 0$ ( )

(b) The system

$$\begin{aligned} x_1 - 3x_2 &= b_1 \\ 4x_1 - 12x_2 &= b_2 \end{aligned}$$

is consistent for all values of b_1 and b_2. ( )

(c) The vectors $\{(1, 0, 0), (0, 1, 0), (0, 0, 0)\}$ are linearly dependent ( ).

3. (4 points) Use Crammer's rule to solve the system

$$\begin{aligned} x_1 - 2x_2 + x_3 &= 2 \\ 2x_1 - x_2 \phantom{{}+x_3} &= 2 \\ 4x_1 \phantom{{}-2x_2} - 3x_3 &= 1 \end{aligned}$$

4. (3 points) Determine whether the set of all points

$$V = \{(x, y, z) \in \mathbb{R}^3 \ : \ xyz = 0\}$$

with the standard addition and scalar multiplications on $\mathbb{R}^3$ is a subspace of $\mathbb{R}^3$ or not.

5. (4 points) Prove one of the following statements

   (a) Let V be a vector space and W is a nonempty subset of V. Assume that the following conditions hold

      1 If u and v are vectors in W, then $u + v \in W$.

      2 If k is a scalar and u is a vector in W, then $ku \in W$. Prove that W is a subspace of V.

   (b) Prove that A set S with two or more vectors is linearly dependent if and only if at least one vector is expressible as a linear combinations of the other vectors in S.

The following question is a **bonus** question.

6. (4 points) [Bonus question] Find an elementary matrix E that satisfies $EB = A$ where

$$B = \begin{bmatrix} 2 & 6 & -8 \\ 0 & 5 & 3 \\ 0 & -5 & 25 \end{bmatrix}, \qquad A = \begin{bmatrix} 2 & 6 & -8 \\ 0 & 5 & 3 \\ 4 & 7 & 9 \end{bmatrix}.$$