King Saud University

College of Sciences

Department of Mathematics

First Examination	Math 244	Semester I, (1441),	Duration: 1hr. 30 mn

Calculators are not allowed

Question 1

1. Consider the matrices A and B such that $A=\begin{pmatrix} 1 & 1 & 2 \\ -1 & 0 & 1 \\ 2 & 1 & 2 \end{pmatrix}$ and $AB = A + 2I_3$.

   Find the matrices A^{-1} and B.

2. Consider the matrices $C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$ and $D = \begin{pmatrix} -2 & 1 & -1 & 2 \\ 1 & 2 & 1 & 3 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & -1 & 0 \end{pmatrix}$.

   If E is a 4×4 matrix such that $EC^2 + ED = 2I_4$, then find E^{-1}.

3. Find the set of solutions of the linear system with the augmented matrix

   $$[A : B] = \left[\begin{array}{cccc|c} 1 & 0 & -2 & 1 & 4 \\ -1 & 0 & 2 & 0 & -2 \\ 0 & 0 & 1 & -1 & 1 \\ 0 & 0 & 0 & 1 & 2 \end{array}\right].$$

Question 2

1. a) Find the matrix $\operatorname{adj}(A)$ if $A = \begin{pmatrix} 1 & 2 & -3 \\ 3 & -1 & 2 \\ -2 & 4 & -2 \end{pmatrix}$.

   b) Find $\operatorname{adj}(A).A$.

2. Find the conditions on a, b such that the following linear system is consistent

$$\begin{cases} x + 2y + z + 3t = a \\ 2x + y + 3z + 2t = b \\ -x + 7y - 4z + 9t = 1 \end{cases}$$

Question 3

1. a) Show that the set $E = \{(x, y, z, w) \in \mathbb{R}^4;\ x - y + z = 0,\ 2x + y - z = 0\}$ is a subspace of $\mathbb{R}^4$.

   b) Find a basis of the subspace E.

2. Let $v_1 = (1, -1, 2, 0, 3), v_2 = (-1, 2, 0, 1, 2), v_3 = (1, 1, -1, 1, 1)$.

   a) Show that the set of vectors $\{v_1, v_2, v_3\}$ is linearly independent.

   b) Find $a, b, c \in \mathbb{R}$ such that $(-2, 3, 5, 2, 11) = av_1 + bv_2 + cv_3$.

Solution of the first Examination Math 244 Semester I 1441

Question 1

1. $A^{-1} = \begin{pmatrix} -1 & 0 & 1 \\ 4 & -2 & -3 \\ -1 & 1 & 1 \end{pmatrix}$, $B = I + 2A^{-1} = \begin{pmatrix} -1 & 0 & 2 \\ 8 & -3 & -6 \\ -2 & 2 & 3 \end{pmatrix}$.

2. $C^2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$, $E^{-1} = \frac{1}{2}(C^2 + D) = \frac{1}{2}\begin{pmatrix} -1 & 1 & -1 & 2 \\ 1 & 6 & 1 & 3 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & -1 & 1 \end{pmatrix}$.

3. Gauss-Jordan $\left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 8 \\ 0 & 0 & 1 & 0 & 3 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$
   $x = 8, y \in \mathbb{R}, z = 3, t = 2$. $S = \{(8, y, 3, 2);\ y \in \mathbb{R}\}$.

Question 2

1. a) $\operatorname{adj}(A) = \begin{pmatrix} -6 & -8 & 1 \\ 2 & -8 & -11 \\ 10 & -8 & -7 \end{pmatrix}$.

   b) $\operatorname{adj}(A).A = |A| I_3 = -32 I_3$.

2. $-5a + 3b + 1 = 0$.

Question 3

1. a) $E = \{(x, y, z, w) \in \mathbb{R}^4;\ x - y + z = 0,\ 2x + y - z = 0\} = \{X = (x, y, z, w) \in \mathbb{R}^4;\ AX = 0\}$, where $A = \begin{pmatrix} 1 & -1 & 1 & 0 \\ 2 & 1 & -1 & 0 \end{pmatrix}$ is a subspace of $\mathbb{R}^4$.

   b) $\begin{cases} x - y + z &= 0 \\ 2x + y - z &= 0 \end{cases} \iff x = 0, y = z$. $E = \{(0, y, y, w) = y(0,1,1,0) + w(0,0,0,1);\ y, w \in \mathbb{R}\}$. Then $B = \{(0,1,1,0), (0,0,0,1)\}$ is a basis of E.

2. a) $xv_1 + yv_2 + zv_3 = (0,0,0,0,0) \iff x = y = z = 0$.

   b) $a = 2, b = 3, c = -1$.