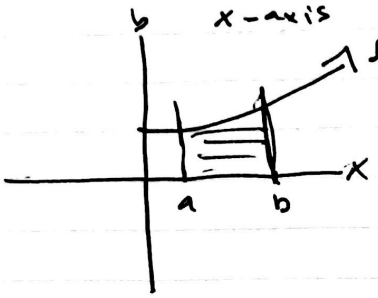
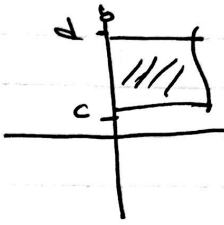
Area underneath the curve

x-axis



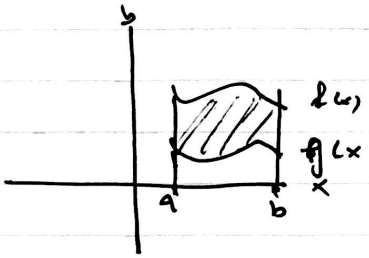
$$A = \int_a^b f(x)\,dx$$

y-axis

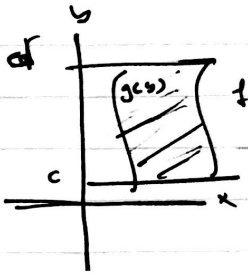


$$A = \int_c^d f(y)\,dy$$

Area between two curves.



$$A = \int_a^b f(x) - g(x)\,dx$$



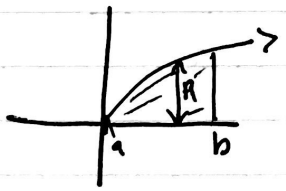
$$A = \int_c^d f(y) - g(y)\,dy$$

---

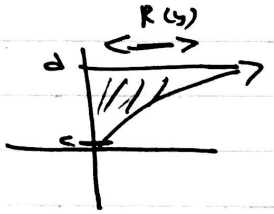
disk and washer method.

* disk

x-axis



$$V = \pi \int_a^b R(x)^2\,dx$$

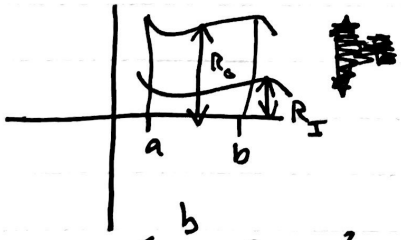


$$V = \pi \int_c^d [R(y)]^2\,dy$$

---

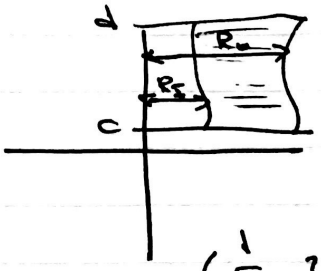
* washer

x-axis



$$V = \pi \int_a^b (R_o^2 - R_I^2)\,dx$$

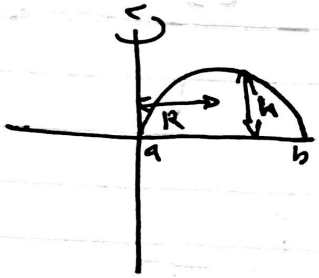
y-axis



$$V = \pi \int_c^1 [R_o^2 - R_I^2]\,dy$$

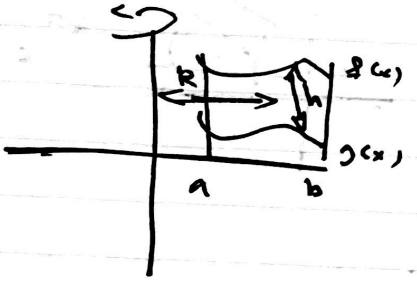
★ Shell Method.

rotated by y-axis



$$V = 2\pi \int_a^b R(x)\, h(x)\, dx$$

x ← $R(x)$; $h(x)$ → $f(x)$

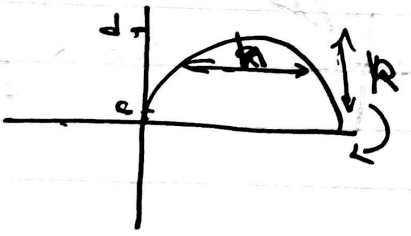


$$V = 2\pi \int_a^b R(x)\, h(x)\, dx$$

x ← $R(x)$; $h(x)$ → $[f(x) - g(x)]$

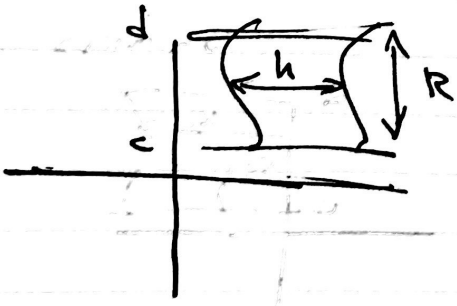
---

rotated by x-axis



$$V = 2\pi \int_c^d R(y)\, h(y)\, dy$$

y ← $R(y)$; $h(y)$ → $f(y)$

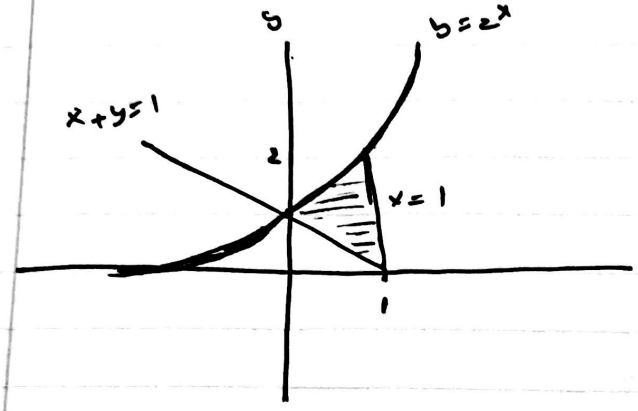


$$V = 2\pi \int_c^d R(y)\, h(y)\, dy$$

y ← $R(y)$; $h(y)$ → $[f(y) - g(y)]$

(7.5)

43) $y = 2^x$ $\quad y = 1 - x$ $\quad x = 1$

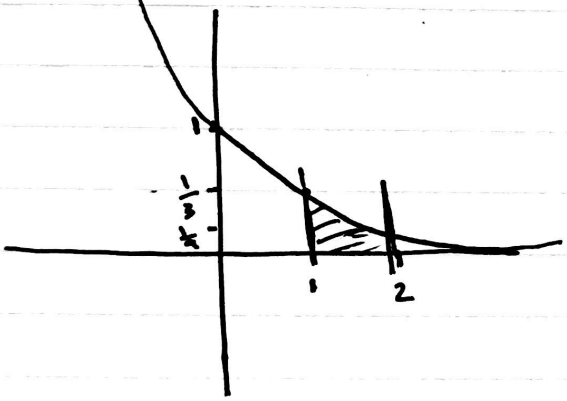


$$A = \int_0^1 2^x - (1 - x)\, dx$$

$$= \frac{1}{\ln 2} - \frac{1}{2} \approx 0.94$$

---

44)



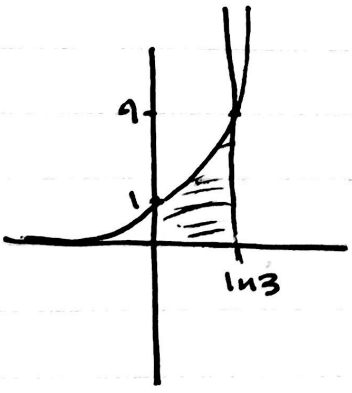
$y = 3^{-x}$ $\quad x = 1$, $x = 2$ $\quad$ x-axis.

$$V = \pi \int_1^2 (3^{-x})^2\, dx$$

$$= \frac{4\pi}{81 \ln 3}$$

(7.4)

37) $y = e^{2x}$ $\quad y = 0$, $x = 0$, $x = \ln 3$



$$A = \int_0^{\ln 3} e^{2x}\, dx = 4$$
