Solve the recurrence relation

$$T(n) = 4T(\tfrac{n}{3}) + cn$$

$$= 4\left[4T(n/3^2) + c(n/3)\right] + cn$$

$$= 4^2 T(n/3^2) + 4c(n/3) + cn$$

$$= 4^2\left[4T(n/3^3) + c(n/3^2)\right] + 4c(n/3) + cn$$

$$= 4^3 T(n/3^3) + 4^2 c(n/3^2) + 4c(n/3) + cn$$

$\vdots$

$$= 4^k T(n/3^k) + \sum_{i=0}^{k-1} 4^i c(n/3^i)$$

$$= cn \sum_{i=0}^{k-1} \left(\tfrac{4}{3}\right)^i$$

$$\leq cn \left[\frac{(4/3)^k - 1}{4/3 - 1}\right]$$

$$\leq 3cn\,(4/3)^k$$

Stop at $\frac{n}{3^k} = 1$

$\Rightarrow k = \log_3 n$

$$T(n) \leq 4^{\log_3 n} T(1) + 3cn\,(4/3)^{\log_3 n}$$

$$= n^{\log_3 4} T(1) + 3c \cdot n^{\log_3 4}$$

$$= O(n^{\log_3 4}) \quad = O(n^{1.26})$$

$$T(n) = 2T\left(\frac{n}{2}\right) + \sqrt{n}$$

$$= 2\left[2T(n/2^2) + \sqrt{\frac{n}{2}}\right] + \sqrt{n}$$

$$= 2^2 T(n/2^2) + 2\sqrt{\frac{n}{2}} + \sqrt{n}$$

$$= 2^2\left[2T(n/2^3) + \sqrt{\frac{n}{2^2}}\right] + 2\sqrt{\frac{n}{2}} + \sqrt{n}$$

$$= 2^3 T(n/2^3) + 2^2\sqrt{\frac{n}{2^2}} + 2\sqrt{\frac{n}{2}} + \sqrt{n}$$

$$\vdots$$

$$= 2^k T(n/2^k) + \sqrt{n}\sum_{i=0}^{k-1} 2^i \sqrt{\frac{1}{2^i}}$$

$$= \sqrt{n}\sum_{i=0}^{k-1} \sqrt{2^i}$$

$$n = 2^k \qquad \leq \sqrt{n}\left[1 + \sqrt{2} + \sqrt{2^2} + \sqrt{2^3} + \cdots + \sqrt{2^{k-1}}\right]$$

$$\Rightarrow k = \log n \qquad \leq \sqrt{n} \cdot k \cdot 2^{\frac{k-1}{2}}$$

$$T(n) \leq 2^{\log n} T(1) + k\sqrt{n} \cdot \sqrt{2^{k-1}}$$

$$= nT(1) + \log n \cdot \sqrt{n} \cdot \sqrt{2^{(\log n) - 1}}$$

$$\leq nT(1) + \sqrt{n} \cdot \log n \cdot \sqrt{n}$$

$$= \Theta(n \log n)$$

Solve

$$T(n) = T(n-1) + c$$

$$= T(n-2) + c + c$$

$$= T(n-3) + c + c + c$$

$$\vdots$$

$$= T(n-k) + kc$$

stop when $n-k=1 \Rightarrow k = n-1$

$$\vdots$$

$$T(n) = T(1) + (n-1)c$$

$$= O(n)$$