(10) The linear system

$$x_1 + 2x_2 - 2x_3 = 7$$
$$x_1 + x_2 + x_3 = 2$$
$$2x_1 + 2x_2 + x_3 = 5$$

has the sol$^{(n)}$ $(1, 2, -1)^t$

a. Show that $\rho(\underbrace{T_j}_{\text{Jacobian matrix}}) = 0$

$$x_1 = -2x_2 + 2x_3 + 7$$
$$x_2 = -x_1 - x_3 + 2$$
$$x_3 = -2x_1 - 2x_2 + 5$$

Since $x = Tx + c$

$$\Rightarrow T_j = \begin{bmatrix} 0 & -2 & 2 \\ -1 & 0 & -1 \\ -2 & -2 & 0 \end{bmatrix}$$

To find $\rho(T)$, we found eigen values for T_j

$$|T - \lambda I| = \begin{vmatrix} -\lambda & -2 & 2 \\ -1 & -\lambda & -1 \\ -2 & -2 & -\lambda \end{vmatrix} = -\lambda(\lambda^2 - 2) + 2(+\lambda - 2) + \underbrace{2(+2 - 2\lambda)}_{-4(\lambda-1)} = 0$$

$$\Leftrightarrow -\lambda^3 + 2\lambda + 2\lambda - 4 + 4 - 4\lambda = 0$$
$$\Leftrightarrow -\lambda^3 = 0$$
$$\Leftrightarrow \lambda_1 = \lambda_2 = \lambda_3 = 0$$
$$\Rightarrow \rho(T_j) = \max\{0, 0, 0\} = 0$$

b. Use the Jacobi method with $x^{(0)} = 0$ to approximate the sol$^{(n)}$ to the linear system to within 10^{-5} in l_∞ norm.

K	0	1	2	3	4
$x_1^{(k)}$	0	7	13	1	1
$x_2^{(k)}$	0	2	-10	2	2
$x_3^{(k)}$	0	5	-13	-1	-1

exact sol$^{(n)}$