c- Show that $\rho(T_g) = 2$

$\downarrow$ gauss matrix

$$x_1^{(k)} = \left[-2x_2^{(k-1)} + 2x_3^{(k-1)} + 7\right]$$
$$x_2^{(k)} = -x_1^{(k)} - x_3^{(k-1)} + 2$$
$$x_3^{(k)} = -2x_1^{(k)} - 2x_2^{(k)} + 5$$

So, we have

$$x_1^{(k)} = -2x_2^{(k-1)} + 2x_3^{(k-1)} + 7$$
$$x_2^{(k)} = -2x_2^{(k-1)} - 3x_3^{(k-1)} - 5$$
$$x_3^{(k)} = -2(-2x_2^{(k-1)} + 2x_3^{(k-1)} \cancel{+7}) - 2(2x_2^{(k-1)} - 3x_3^{(k-1)} + 2) + 5$$
$$= \cancel{4x_2^{(k-1)}} - 4x_3^{(k-1)} - 4x_2^{(k-1)} + 6x_3^{(k-1)} - 4 + 5$$
$$= 2x_3^{(k-1)} + 1$$

$$\longrightarrow T_g = \begin{bmatrix} 0 & -2 & 2 \\ 0 & 2 & -3 \\ 0 & 0 & 2 \end{bmatrix}$$

$$|T_g - \lambda I| = 0 \iff \begin{vmatrix} -\lambda & -2 & 2 \\ 0 & 2-\lambda & -3 \\ 0 & 0 & 2-\lambda \end{vmatrix} = 0 \iff -\lambda(2-\lambda)(2-\lambda) = 0$$

$$\iff \lambda_1 = 0, \quad \lambda_2 = 2 = \lambda_3$$

So, $\rho(T_g) = \max\{0, 2, 2\} = 2$

d: Show that the Gauss-seidal method applied as in part (b) fails to give a good approximation in 25 iterations.