

7.4

1. Compute the condition numbers of the following matrices relative to $\|\cdot\|_\infty$.

a. $\begin{bmatrix} \frac{1}{2} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{2} \end{bmatrix} = A$

$$\|A\|_\infty = \max \left\{ \frac{1}{2} + \frac{1}{3}, \frac{1}{3} + \frac{1}{2} \right\} = \frac{5}{6}$$

$$A^{-1} = \frac{1}{\det A} \begin{bmatrix} \frac{1}{2} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} 18 & -24 \\ -24 & 36 \end{bmatrix}$$

$$\Rightarrow \|A^{-1}\|_\infty = \max \{ 24+18, 24+36 \} = 60$$

$$\therefore \kappa(A) = \left(\frac{5}{6}\right)(60) = \|A\|_\infty \|A^{-1}\|_\infty = 50$$

* * * * *

b. $\begin{bmatrix} 3.4 & 1.6 \\ 6.8 & 2.9 \end{bmatrix} = A$

$$\|A\|_\infty = \max \{ 3.4+1.6, 6.8+2.9 \} = 9.7$$

$$A^{-1} = \frac{100}{43} \begin{bmatrix} 2.9 & -1.6 \\ -6.8 & 3.4 \end{bmatrix} = \begin{bmatrix} \frac{290}{43} & -\frac{160}{43} \\ -\frac{680}{43} & \frac{340}{43} \end{bmatrix}$$

$$\|A^{-1}\|_\infty = \max \left\{ \frac{450}{43}, \frac{1070}{43} \right\} = \frac{1070}{43} = 24.9$$

$$\Rightarrow \kappa(A) = \|A\|_\infty \|A^{-1}\|_\infty = 241.373$$

2. The following systems A_nb have x as the actual solⁿ and \tilde{x} as an approx. solⁿ. Using the result in Ex. 1 to compute $\|x - \tilde{x}\|_\infty$ and $\kappa(A) \frac{\|b - A\tilde{x}\|_\infty}{\|A\tilde{x}\|_\infty}$

a. $\frac{1}{2}x_1 + \frac{1}{3}x_2 = \frac{1}{63}$

$$\frac{1}{3}x_1 + \frac{1}{4}x_2 = \frac{1}{168}$$

$$x = \left(\frac{1}{4}, \frac{1}{6} \right)^t$$

$$\tilde{x} = (0.142, -0.166)$$

$$x - \tilde{x} = (8.57 \times 10^{-4}, 0.333)$$

$$\Rightarrow \|x - \tilde{x}\|_\infty = 0.333$$