OPER 441: Modeling and Simulation
Exercises Sheet #6

Question 1:

Consider the triangular distribution:

$$F(x) = \begin{cases} 0 & x < a \\ \dfrac{(x-a)^2}{(b-a)(c-a)} & a \le x \le c \\ 1 - \dfrac{(b-x)^2}{(b-a)(b-c)} & c < x \le b \\ 1 & b < x \end{cases}$$

a) Use a spreadsheet to generate 1000 observations of the triangular distribution with $a = 2$, $c = 5$, $b = 10$.

b) Use your favorite statistical software to make a histogram of 1000 observations from your implementation of the triangular distribution with
$a = 2$, $c = 5$, $b = 10$.

Question 2:

A firm is trying to decide whether or not to invest in two proposals A and B that have the net cash flows shown in the following table, where $N(\mu, \sigma)$ represents that the cash flow value comes from a normal distribution with the provided mean and standard deviation.

End of Year	0	1	2	3	4
A	$N(-250, 10)$	$N(75, 10)$	$N(75, 10)$	$N(175, 20)$	$N(150, 40)$
B	$N(-250, 5)$	$N(150, 10)$	$N(150, 10)$	$N(75, 20)$	$N(75, 30)$

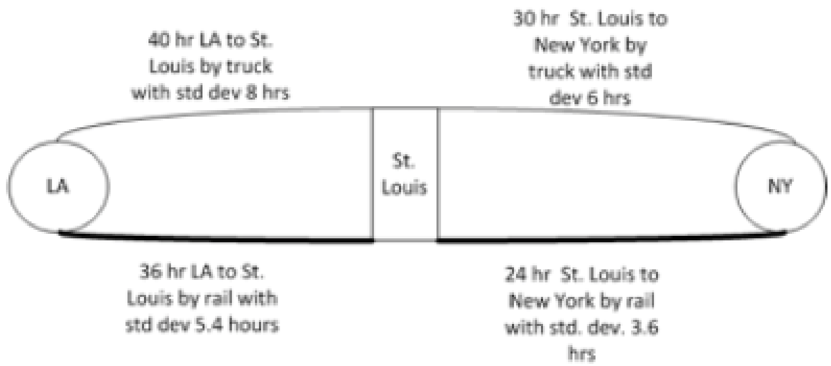
The interest rate has been varying recently and the firm is unsure of the rate for performing the analysis. To be safe, they have decided that the interest rate should be modeled as a beta random variable over the range from 2 to 7 percent with $\alpha_1 = 4.0$ and $\alpha_2 = 1.2$. Given all the uncertain elements in the situation, they have decided to perform a simulation analysis in order to assess the situation.

a) Compare the expected present worth of the two alternatives. Estimate the probability that alternative A has a higher present worth than alternative B.

b) Determine the number of samples needed to be 95% confidence that you have estimated the $P\{PW(A) > PW(B)\}$ to within $\pm$ 0.10.

Question 3:

Shipments can be transported by rail or trucks between New York and Los Angeles. Both modes of transport go through the city of St. Louis. The mean travel time and standard deviations between the major cities for each mode of transportation are shown in the following figure.



Assume that the travel times (in either direction) are lognormally distributed as shown in the figure. For example, the time from NY to St. Louis (or St. Louis to NY) by truck is 30 hours with a standard deviation of 6 hours. In addition, assume that the transfer time in hours in St. Louis is triangularly distributed with parameters (8, 10, 12) for trucks (truck to truck). The transfer time in hours involving rail is triangularly distributed with parameters (13, 15, 17) for rail (rail to rail, rail to truck, truck to rail). We are interested in determining the shortest total shipment time combination from NY to LA. Develop a spreadsheet simulation for this problem.

a) How many shipment combinations are there?
b) Write a spreadsheet expression for the total shipment time of the truck only combination.
c) We are interested in estimating the average shipment time for each shipment combination and the probability that the shipment combination will be able to deliver the shipment within 85 hours.
d) Estimate the probability that a shipping combination will be the shortest.
e) Determine the sample size necessary to estimate the mean shipment time for the truck only combination to within 0.5 hours with 95% confidence

Question 4:

The times to failure for an automated production process have been found to be randomly distributed according to a Rayleigh distribution:

$$f(x) = \begin{cases} 2\beta^{-2} x e^{(-(x/\beta)^2)} & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

Setup a spreadsheet to generate 5 random numbers from your algorithm with = 2.

Question 5:

A firm produces Y-Box gaming stations for the consumer market. Their profit function is:

Profit = (unit price - unit cost) × (quantity sold) − fixed costs

Suppose that the unit price is $200 per gaming station, and that the other variables have the following probability distributions:

Unit Cost	80	90	100	110
Probability	0.20	0.40	0.30	0.10

Quantity Sold	1000	2000	3000
Probability	0.10	0.60	0.30

Fixed Cost	50000	65000	80000
Probability	0.40	0.30	0.30

Use a spreadsheet to generate 1000 observations of the profit).

a) Make a histogram of your observations using your favorite statistical analysis package.
b) Estimate the mean profit from your sample and compute a 99% confidence interval for the mean profit.
c) Estimate the probability that the profit will be positive.

Question 6:

T. Wilson operates a sports magazine stand before each game. He can buy each magazine for 33 cents and can sell each magazine for 50 cents. Magazines not sold at the end of the game are sold for scrap for 5 cents each. Magazines can only be purchased in bundles of 10. Thus, he can buy 10, 20, and so on magazines prior to the game to stock his stand. The lost revenue for not meeting demand is 17 cents for each magazine demanded that could not be provided. Mr. Wilson's profit is as follows:

$$\text{Profit} = (\text{revenue from sales}) - (\text{cost of magazines}) - (\text{lost profit from excess demand}) + (\text{salvage value from sale of scrap magazines})$$

Not all game days are the same in terms of potential demand. The type of day depends on a number of factors including the current standings, the opponent, and whether or not there are other special events planned for the game day weekend. There are three types of game days demand: high, medium, low. The type of day has a probability distribution associated with it.

Type of Day	High	Medium	Low
Probability	0.35	0.45	0.20

The amount of demand for magazines then depends on the type of day according to the following distributions:

	High		Medium		Low	
Demand	PMF	CDF	PMF	CDF	PMF	CDF
40	0.03	0.03	0.1	0.1	0.44	0.44
50	0.05	0.08	0.18	0.28	0.22	0.66
60	0.15	0.23	0.4	0.68	0.16	0.82
70	0.2	0.43	0.2	0.88	0.12	0.94
80	0.35	0.78	0.08	0.96	0.06	1.0
90	0.15	0.93	0.04	1.0		
100	0.07	1.0				

Let Q be the number of units of magazines purchased (quantity on hand) to setup the stand. Let D represent the demand for the game day. If $D > Q$, Mr. Wilson sells only Q and will have lost sales of D-Q. If $D < Q$, Mr. Wilson sells only D and will have scrap of Q-D. Assume that he has determined that $Q = 50$.

Make sure that you can estimate the average profit and the probability that the profit is greater than zero for Mr. Wilson. Develop a spreadsheet model to estimate the average profit with 95% confidence to within plus or minus $0.5.

Initial sample size of 100 yields, s = 13.4. Thus, sample size needed is:

$$n \geq \left(\frac{z_{\alpha/2}s}{E}\right)^2 = \left(\frac{1.96 \times 13.4}{0.5}\right)^2 = 2759.19 = 2760$$

Question 8:

The time for an automated storage and retrieval system in a warehouse to locate a part consists of three movements. Let X be the time to travel to the correct aisle. Let Y be the time to travel to the correct location along the aisle. And let Z be the time to travel up to the correct location on the shelves. Assume that the distributions of X, Y, and Z are as follows:

- $X \sim$ lognormal with mean 20 and standard deviation 10 seconds
- $Y \sim$ uniform with minimum 10 and maximum 15 seconds
- $Z \sim$ uniform with minimum of 5 and a maximum of 10 seconds

Develop a spreadsheet that can estimate the average total time that it takes to locate a part and can estimate the probability that the time to locate a part exceeds 60 seconds. Base your analysis on 1000 observations.

Question 9:

Lead-time demand may occur in an inventory system when the lead-time is other than instantaneous. The lead-time is the time from the placement of an order until the order is received. The lead-time is a random variable. During the lead-time, demand also occurs at random. Lead-time demand is thus a random variable defined as the sum of the demands during the lead-time, or LDT = $\sum_{i=1}^{T} D_i$ where i is the time period of the lead-time and T is the lead-time. The distribution of lead-time demand is determined by simulating many cycles of lead-time and the demands that occur during the lead-time to get many realizations of the random variable LDT. Notice that LDT is the *convolution* of a random number of random demands. Suppose that the daily demand for an item is given by the following probability mass function:

Daily Demand (items)	4	5	6	7	8
Probability	0.10	0.30	0.35	0.10	0.15

The lead-time is the number of days from placing an order until the firm receives the order from the supplier.

a) Assume that the lead-time is a constant 10 days. Use a spreadsheet to simulate 1000 instances of LDT. Report the summary statistics for the 1000 observations. Estimate the chance that LDT is greater than or equal to 10. Report a 95% confidence interval on your estimate. Use your favorite statistical program to develop a frequency diagram for LDT.

b) Assume that the lead-time has a shifted geometric distribution with probability parameter equal to 0.2 Use a spreadsheet to simulate 1000 instances of LDT. Report the summary statistics for the 1000 observations. Estimate the chance that LDT is greater than or equal to 10. Report a 95% confidence interval on your estimate. Use your favorite statistical program to develop a frequency diagram for LDT.

Question 10:

Setup a spreadsheet to generate 5 random numbers from the negative binomial distribution with parameters ($r = 4$, $p = 0.4$) using:

a) The convolution method
b) The number of Bernoulli trials to get 4 successes.

Question 11:

Setup a spreadsheet that will generate 30 observations from the following probability density function using the Acceptance-Rejection algorithm for generating random variates.

$$f(x) = \begin{cases} \frac{3x^2}{2} & -1 \le x \le 1 \\ 0 & \text{otherwise} \end{cases}$$

Question 12:

Describe a simulation experiment that would allow you to test the lack of memory property empirically. Implement your simulation experiment in a spreadsheet and test the lack of memory property empirically. Explain in your own words what lack of memory means.

Question 13:

Parts arrive to a machine center with three drill presses according to a Poisson distribution with mean λ. The arriving customers are assigned to one of the three drill presses randomly according to the respective probabilities p_1, p_2, and p_3 where $p_1 + p_2 + p_3 = 1$ and $p_i > 0$ for i = 1, 2, 3. What is the distribution of the inter-arrival times to each drill press? Specify the parameters of the distribution.

a) Suppose that p_1, p_2, and p_3 equal to 0.25, 0.45, and 0.3 respectively and that λ is equal to 12 per minute. Setup a spreadsheet to generate the first 3 arrival times.

This question demonstrates the splitting property of a Poisson distribution. Each machine experience a Poisson process with mean $\lambda \times p_i$. Thus, the distribution of the inter-arrival times to each drill press will be exponential with mean $1/(\lambda \times p_i)$

Because of the splitting rule for Poisson processes, the drill presses each see arrivals according to the following three Poisson processes:

$$\lambda_1 = \lambda p_1 = 12 * 0.25 = 3$$

$$\lambda_2 = \lambda p_2 = 12 * 0.45 = 5.4$$

$$\lambda_3 = \lambda p_3 = 12 * 0.3 = 3.6$$