

## FAST DECOUPLED POWER FLOW

Contingencies are a major concern in power system operations. For example, operating personnel need to know what power-flow changes will occur due to a particular generator outage or transmission-line outage. Contingency information, when obtained in real time, can be used to anticipate problems caused by such outages, and can be used to develop operating strategies to overcome the problems.

Fast power-flow algorithms have been developed to give power-flow solutions in seconds or less [8]. These algorithms are based on the following simplification of the Jacobian matrix. Neglecting  $\mathbf{J}_2(i)$  and  $\mathbf{J}_3(i)$ , (6.6.6) reduces to two sets of decoupled equations:

$$\mathbf{J}_1(i)\Delta\delta(i) = \Delta\mathbf{P}(i) \quad (6.9.1)$$

$$\mathbf{J}_4(i)\Delta\mathbf{V}(i) = \Delta\mathbf{Q}(i) \quad (6.9.2)$$

The computer time required to solve (6.9.1) and (6.9.2) is significantly less than that required to solve (6.6.6). Further reduction in computer time can be obtained from additional simplification of the Jacobian matrix. For example, assume  $V_k \approx V_n \approx 1.0$  per unit and  $\delta_k \approx \delta_n$ . Then  $\mathbf{J}_1$  and  $\mathbf{J}_4$  are constant matrices whose elements in Table 6.5 are the imaginary components of  $\mathbf{Y}_{\text{bus}}$ . As such,  $\mathbf{J}_1$  and  $\mathbf{J}_4$  do not have to be recalculated during successive iterations.

Simplifications similar to these enable rapid power-flow solutions. For a fixed number of iterations, the fast decoupled algorithm given by (6.9.1) and (6.9.2) is not as accurate as the exact Newton–Raphson algorithm. However, the savings in computer time is considered more important.

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## PROBLEMS

### SECTION 6.1

- 6.1 Using Gauss elimination, solve the following linear algebraic equations:

$$\begin{aligned} 5x_1 - 2x_2 - 3x_3 &= 4 \\ -5x_1 + 7x_2 - 2x_3 &= -10 \\ -3x_1 - 3x_2 + 8x_3 &= 6 \end{aligned}$$

Find the three unknowns  $x_1$ ,  $x_2$ , and  $x_3$ . Check your answers using Cramer's rule. Also check by using matrix solution of linear equations.

- 6.2 Using Gauss elimination and back substitution, solve

$$\begin{bmatrix} 10 & 3 & 0 \\ 4 & 20 & 2 \\ 5 & 2 & 14 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

- 6.3 Rework Problem 6.2 with the value of  $A_{33}$  change from 14 to 1.4.
- 6.4 Show that the Gauss elimination method, which transforms the set of  $N$  linear equations  $\mathbf{Ax} = \mathbf{y}$  to  $\mathbf{A}^{(N-1)}\mathbf{x} = \mathbf{y}^{(N-1)}$ , where  $\mathbf{A}^{(N-1)}$  is triangular, requires  $(N^3 - N)/3$  multiplications,  $N(N - 1)/2$  divisions, and  $(N^3 - N)/3$  subtractions. Assume that all the elements of  $\mathbf{A}$  and  $\mathbf{y}$  are nonzero and real. (Hint: Investigate (6.1.7). Note that during the first Gauss elimination step, each of the  $(N - 1)$  rows that are changed requires one division,  $N$  multiplications, and  $N$  subtractions.)
- 6.5 Show that, after triangularizing  $\mathbf{Ax} = \mathbf{y}$ , the back substitution method of solving  $\mathbf{A}^{(N-1)}\mathbf{x} = \mathbf{y}^{(N-1)}$  requires  $N$  divisions,  $N(N - 1)/2$  multiplications, and  $N(N - 1)/2$  subtractions. Assume that all the elements of  $\mathbf{A}^{(N-1)}$  and  $\mathbf{y}^{(N-1)}$  are nonzero and real.
- 6.6 For a digital computer with a  $25 \times 10^{-9}$ -s multiplication or division time and a  $5 \times 10^{-9}$ -s addition or subtraction time, determine how much computer time would be required to solve (6.1.1) for  $N = 100$ , using Gauss elimination and back substitution. Assume that all the elements of  $\mathbf{A}$  and  $\mathbf{y}$  are nonzero and real.
- 6.7 If 4 bytes are used to store each floating-point number in computer memory, how many kilobytes are required to store an  $N$  vector  $\mathbf{y}$  and an  $N \times N$  matrix  $\mathbf{A}$ , where  $N = 1000$ ? Assume that all the elements of  $\mathbf{y}$  and  $\mathbf{A}$  are nonzero and real.

### SECTION 6.2

- 6.8 Solve Problem 6.2 using the Jacobi iterative method. Start with  $x_1(0) = x_2(0) = x_3(0) = 0$ , and continue until (6.2.2) is satisfied with  $\varepsilon = 0.01$ .
- 6.9 Repeat Problem 6.8 using the Gauss-Seidel iterative method. Which method converges more rapidly?
- 6.10 Try to solve Problem 6.2 using the Jacobi and Gauss-Seidel iterative methods with the value of  $A_{33}$  changed from 14 to 0.14 and with  $x_1(0) = x_2(0) = x_3(0) = 0$ . Show that neither method converges to the unique solution.
- 6.11 Using the Jacobi method (also known as the Gauss method), solve for  $x_1$  and  $x_2$  in the system of equations

$$x_2 - 3x_1 + 1.9 = 0$$

$$x_2 - 3x_1^2 - 1.8 = 0$$

Use an initial guess  $x_1(0) = 1.0 = x_2(0)$ . Also, see what happens when you choose an uneducated initial guess  $x_1(0) = x_2(0) = 100$ .

- 6.12 Solve the following equation by the Gauss-Seidel method:

$$x_1^2 - 6x_1 + 2 = 0$$

Use the initial estimate  $x_1(0) = 1$ . Check by using the quadratic formula.

- 6.13 Take the  $z$ -transform of (6.2.6) and show that  $\mathbf{X}(z) = \mathbf{G}(z)\mathbf{Y}(z)$ , where  $\mathbf{G}(z) = (z\mathbf{U} - \mathbf{M})^{-1}\mathbf{D}^{-1}$  and  $\mathbf{U}$  is the unit matrix.

$\mathbf{G}(z)$  is the matrix transfer function of a digital filter that represents the Jacobi or Gauss-Seidel methods. The filter poles are obtained by solving  $\det(z\mathbf{U} - \mathbf{M}) = 0$ . The filter is stable if and only if all the poles have magnitudes less than 1.

- 6.14 Using the results of Problem 6.13, determine the filter poles for Examples 6.3 and 6.5. Note that in Example 6.3 both poles have magnitudes less than 1, which means the filter is stable and Jacobi converges for this example. However, in Example 6.5, one

pole has a magnitude greater than 1, which means the filter is unstable and Gauss-Seidel diverges for this example.

- 6.15 Determine the poles of the Jacobi and Gauss-Seidel digital filters for the general two-dimensional problem ( $N = 2$ ):

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

Then determine a necessary and sufficient condition for convergence of these filters when  $N = 2$ .

- 6.16 For a digital computer with a  $25 \times 10^{-9}$ -s multiplication or division time and a  $5 \times 10^{-9}$ -s addition or subtraction time, determine how much computer time per iteration would be required to solve (6.1.1) with  $N = 100$ , using Gauss-Seidel. Assume that all the elements of  $A$  and  $y$  are nonzero and real.

### SECTION 6.3

- 6.17 Use Newton-Raphson to find one solution to the polynomial equation  $f(x) = y$ , where  $y = 0$  and  $f(x) = 3x^3 + 4x^2 + 5x + 8$ . Start with  $x(0) = 1.0$  and continue until (6.2.2) is satisfied with  $\varepsilon = 0.001$ .
- 6.18 Use Newton-Raphson to find one solution to the polynomial equation  $f(x) = y$ , where  $y = 0$  and  $f(x) = x^4 + 12x^3 + 54x^2 + 108x + 81$ . Start with  $x(0) = -1$  and continue until (6.2.2) is satisfied with  $\varepsilon = 0.001$ .
- 6.19 Use Newton-Raphson to find a solution to

$$\begin{bmatrix} e^{x_1 x_2} \\ \cos(x_1 + x_2) \end{bmatrix} = \begin{bmatrix} 1.2 \\ 0.5 \end{bmatrix}$$

where  $x_1$  and  $x_2$  are in radians. (a) Start with  $x_1(0) = 1.0$  and  $x_2(0) = 0.5$  and continue until (6.2.2) is satisfied with  $\varepsilon = 0.005$ . (b) Show that Newton-Raphson diverges for this example if  $x_1(0) = 1.0$  and  $x_2(0) = 2.0$ .

- 6.20 Solve the following equations by the Newton-Raphson method:

$$x_1^2 - 4x_2 - 4 = 0$$

$$2x_1 - x_2 - 2 = 0$$

Start with an initial guess  $x_1(0) = 1.0$  and  $x_2(0) = -1.0$ .

- 6.21 The following nonlinear equation contains terms that are often found in power-flow equations:

$$y \sin y + 4 = 0$$

Find a solution by using the Newton-Raphson method with an initial guess  $y(0) = 4$  radians.

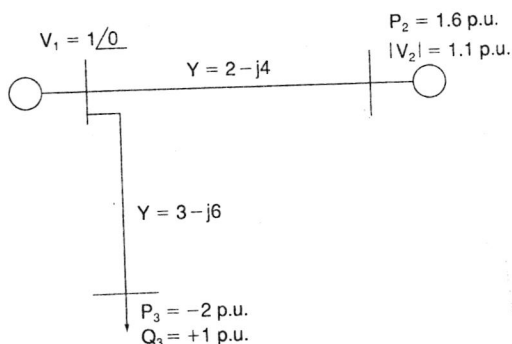
### SECTION 6.4

- 6.22 Consider the simplified electric power system shown in Figure 6.5 for which the power-flow solution can be obtained without resorting to iterative techniques. (a) Compute the elements of the bus admittance matrix  $Y_{bus}$ . (b) Calculate the phase angle  $\delta_2$  by using the real power equation at bus 2 (voltage-controlled bus). (c) Deter-

mine  $|V_3|$  and  $\delta_3$  by using both the real and reactive power equations at bus 3 (load bus). (d) Find the real power generated at bus 1 (swing bus). (e) Evaluate the total real power losses in the system.

**FIGURE 6.5**

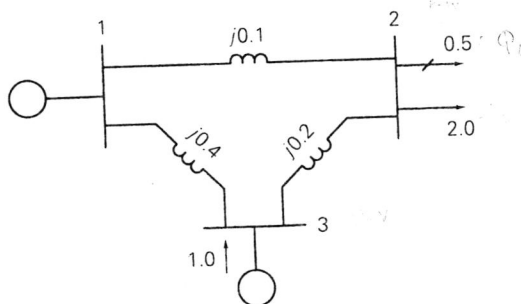
Problem 6.22



- 6.23** Compute the elements of the third row of  $Y_{bus}$  for the power system in Example 6.9.
- 6.24** In Example 6.9, double the impedance of the line from bus 2 to bus 5. Determine the new values for the second row of  $Y_{bus}$ . Verify your result using PowerWorld Simulator case Example 6.9.
- 6.25** Figure 6.6 shows a single-line diagram of a three-bus power system. Power-flow input data are given in Tables 6.9 and 6.10. (a) Determine the  $3 \times 3$  per-unit bus admittance matrix  $Y_{bus}$ . (b) For each bus  $k = 1, 2, 3$  determine which of the variables  $V_k$ ,  $\delta_k$ ,  $P_k$ , and  $Q_k$  are input data and which are unknowns.

**FIGURE 6.6**

Single-line diagram for Problem 6.25 (per unit impedances and per unit real and reactive powers are shown)

**TABLE 6.9**

Bus input data for Problem 6.25

Bus	Type	V per Unit	$\delta$ Degrees	$P_G$ per Unit	$Q_G$ per Unit	$P_L$ per Unit	$Q_L$ per Unit	$Q_{Gmin}$ per Unit	$Q_{Gmax}$ per Unit
1	Swing	1.0	0	—	—	0	0	—	—
2	Load	—	—	0	0	2.0	0.5	—	—
3	Constant voltage	1.0	—	1.0	—	0	0	-5.0	+5.0

TABLE 6.10

 Line input data for  
Problem 6.25

Line	Bus-to-Bus	R' per Unit	X' per Unit	G' per Unit	B' per Unit	Maximum MVA per Unit
1	1-2	0	0.1	0	0	3.0
2	2-3	0	0.2	0	0	3.0
3	1-3	0	0.4	0	0	3.0

(Note: There are no transformers)

## SECTION 6.5

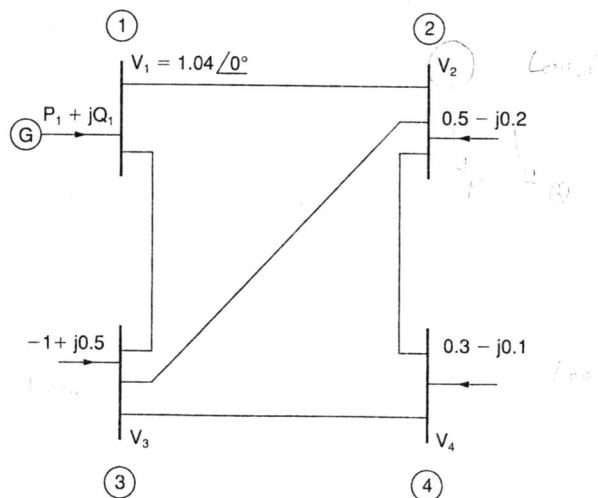
- 6.26 For the power system in Example 6.9, use Gauss-Seidel to calculate  $V_3(1)$ , the phasor voltage at bus 3 after the first iteration. Note that bus 3 is a voltage-controlled bus.
- 6.27 For the power system given in Problem 6.25, use Gauss-Seidel to compute  $V_2(1)$  and  $V_3(1)$ , the phasor voltages at bus 2 and 3 after the first iteration. Use zero initial phase angles and 1.0 per-unit initial bus voltage magnitudes.
- 6.28 The bus admittance matrix for the power system shown in Figure 6.7 is given by

$$Y_{\text{bus}} = \begin{bmatrix} 3 - j9 & -2 + j6 & -1 + j3 & 0 \\ -2 + j6 & 3.666 - j11 & -0.666 + j2 & -1 + j3 \\ -1 + j3 & -0.666 + j2 & 3.666 - j11 & -2 + j6 \\ 0 & -1 + j3 & -2 + j6 & 3 - j9 \end{bmatrix} \text{ per unit}$$

With the complex powers on load buses 2, 3, and 4 as shown in Figure 6.7, determine the value for  $V_2$  that is produced by the first and second iterations of the Gauss-Seidel procedure. Choose the initial guess  $V_2(0) = V_3(0) = V_4(0) = 1.0 \angle 0^\circ$  per unit.

FIGURE 6.7

Problem 6.28



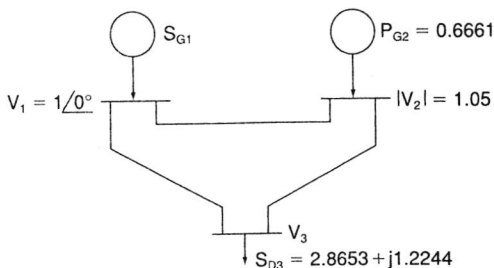
- PW** 6.29 Using PowerWorld Simulator, determine the maximum mismatch after the first, second, and third iterations for the Example 6.10 case using Gauss–Seidel. How many iterations does it take to converge to a mismatch less than 0.5 MVA?
- PW** 6.30 Repeat Problem 6.29, except first decrease the load at bus 2 to 400 MW and 140 Mvar.
- PW** 6.31 Open the PowerWorld Simulator Problem 6\_31 case. This case is similar to the Example 6.10 case, except that the maximum number of iterations has been increased, and the case has been set to automatically initialize from a flat start solution (i.e., one with all the voltage angles equal to zero and the voltage magnitudes at load buses equal to 1.0 per unit). Increase the load at bus 2 in 10-MW steps, keeping the load power factor constant. For each load increase, how many iterations are required to converge using Gauss–Seidel? What is the maximum load level before the iterations diverge?

## SECTION 6.6

- 6.32 For the power system in Example 6.9, calculate  $\Delta P_4(0)$  in Step 1 and  $J_{144}(0)$  in Step 2 of the first Newton–Raphson iteration. Assume zero initial phase angles and 1.0 per-unit initial voltage magnitudes (except  $V_3 = 1.05$ ).
- 6.33 For the power system given in Problem 6.25, use (6.6.2) and (6.6.3) to write the three power-flow equations to be solved by the Newton–Raphson method. Also, identify the three unknown variables to be solved. Do not solve the equations.
- 6.34 For the power system given in Problem 6.25, use Newton–Raphson to compute  $V_2(1)$  and  $V_3(1)$ , the phasor voltages at bus 2 and 3 after the first iteration, as follows. (a) Step 1: use (6.6.2) and (6.6.3) to compute  $\Delta \mathbf{y}(0)$ . (b) Step 2: compute the  $3 \times 3$  Jacobian matrix  $\mathbf{J}(0)$  using the equations in Table 6.5. (c) Step 3: use Gauss elimination and back substitution to solve (6.6.6). (d) Step 4: compute  $\mathbf{x}(1)$  in (6.6.7). Also, use (6.5.3) to compute  $Q_{G3}$  and verify that it is within the limits shown in Table 6.12. In Steps 1 and 2, use zero initial phase angles and 1.0 per-unit initial bus voltage magnitudes.
- 6.35 For the transmission system shown in Figure 6.8, all shunt elements are capacitors with an admittance  $y_C = j0.01$  per unit, and all series elements are inductors with an impedance  $z_L = j0.1$  per unit. Determine  $\delta_2$ ,  $|V_3|$ ,  $\delta_3$ ,  $P_{G1}$ ,  $Q_{G1}$ , and  $Q_{G2}$  for the system.

FIGURE 6.8

Problem 6.35



- 6.36** Load PowerWorld Simulator case Example 6.11; this case is set to perform a single iteration of the Newton–Raphson power flow each time **Single Solution** is selected. Verify that initially the Jacobian element  $J_{22}$  is 29.76. Then, give and verify the value of this element after each of the next three iterations (until the case converges).

## SECTION 6.7

- PW 6.37** Use PowerWorld Simulator to determine the Mvar rating of the shunt capacitor bank in the Example 6.14 case that increases  $V_2$  to 1.00 per unit. Also determine the effect of this capacitor bank on line loadings and total real power losses (total real and reactive losses are shown on the **Case Information, Case Summary** dialog). To vary the capacitor's nominal Mvar rating, right-click on the capacitor symbol to view the **Switched Shunt** dialog and then change the Nominal Mvar field.
- PW 6.38** Use PowerWorld Simulator to modify the Example 6.9 case by inserting a second line between bus 2 and bus 4. Give the new line a circuit identifier of "2" to distinguish it from the existing line. The line parameters of the added line should be identical to those of the existing line 2–4. Determine the new line's effect on  $V_2$ , the line loadings, and on the total real power losses.
- PW 6.39** Using PowerWorld Simulator with the Example 6.9 case, change the generator 3 voltage setpoint between 1.00 and 1.08 per unit in 0.005 per-unit steps. Show the variation in the reactive power output of generator 3,  $V_2$ , and total real power losses.
- PW 6.40** Open PowerWorld Simulator case Problem 6\_40. This case is identical to Example 6.9 except that the transformer between buses 1 and 5 is now a tap-changing transformer with a tap range between 0.9 and 1.1 and a tap step size of 0.00625. The tap is on the high side of the transformer. As the tap is varied between 0.975 and 1.1, show the variation in the reactive power output of generator 1,  $V_5$ ,  $V_2$ , and the total real power losses.
- PW 6.41** Open PowerWorld Simulator case Problem 6\_41. This case is identical to the case from the previous problem except that a second transformer has been installed between buses 1 and 5. This new transformer is identical to the first transformer except that its tap ratio is fixed at 1.0. As the tap on the first transformer is varied between 0.9 and 1.1, show the variation in the reactive power output of generator 1, the real and reactive power flow through the two transformers (measured on the bus 1 side), and the total real power losses.
- PW 6.42** Open PowerWorld Simulator case Example 6\_13. As in Example 6.13, remove the TIM69 to HANA69 line. Determine the Mvar rating of the shunt capacitor bank at Bus HANA69 necessary to correct the HANA69 voltage back to 0.950 per unit. Use <ctrl> up arrow to zoom the one-line to better see the one-line values.
- PW 6.43** Open PowerWorld Simulator case Example 6\_13. Plot the variation in the total system real power losses as the generation at bus BLT138 is varied in 20-MW blocks between 0 MW and 400 MW. What value of BLT138 generation minimizes system losses?
- PW 6.44** Repeat Problem 6.43, except first remove the 138-kV line from BLT138 to BOB138.
- PW 6.45** Open PowerWorld Simulator case Example 6\_13. Sequentially open (remove from service) each of the case's three 345-kV transmission lines and the six 345/138-kV transformers (always closing the previous device). Record the impact each outage has on system losses. Which device has the largest impact on system losses?

## SECTION 6.8

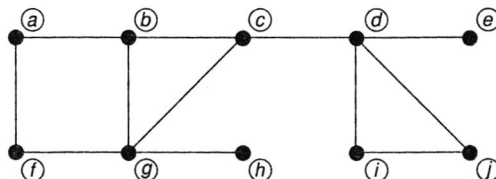
- 6.46** Using the compact storage technique described in Section 6.8, determine the vectors **DIAG**, **OFFDIAG**, **COL**, and **ROW** for the following matrix:

$$S = \begin{bmatrix} 17 & -9.1 & 0 & 0 & -2.1 & -7.1 \\ -9.1 & 25 & -8.1 & -1.1 & -6.1 & 0 \\ 0 & -8.1 & 9 & 0 & 0 & 0 \\ 0 & -1.1 & 0 & 2 & 0 & 0 \\ -2.1 & -6.1 & 0 & 0 & 14 & -5.1 \\ -7.1 & 0 & 0 & 0 & -5.1 & 15 \end{bmatrix}$$

- 6.47** If 4 bytes of computer storage are used for each floating-point number and 2 bytes for each integer, determine the total number of bytes required to store the **S** matrix in Problem 6.46 (a) with compact storage and (b) without compact storage.
- 6.48** Reorder the rows of the matrix given in Problem 6.46 such that after one Gauss elimination step,  $S_{\text{reordered}}^{(1)}$  has the same number of zeros in columns 2–6 as the original matrix.
- 6.49** For the triangular factorization of the corresponding  $Y_{\text{bus}}$ , number the nodes of the graph shown in Figure 6.9 in an optimal order.

FIGURE 6.9

Problem 6.49



- 6.50** For a small power system, the numerical values for initialization of the power-flow mismatch equations is given by

$$\begin{array}{c} \begin{array}{cc} \textcircled{2} & \textcircled{3} \\ \textcircled{2} & \begin{bmatrix} 45.443 & 0 & -26.365 & | & 8.882 & 0 \\ 0 & 41.269 & -15.421 & | & 0 & 8.133 \\ -26.365 & -15.421 & 41.786 & | & -5.273 & -3.084 \\ \hline -9.089 & 0 & 5.273 & | & 44.229 & 0 \\ 0 & -8.254 & 3.084 & | & 0 & 40.459 \end{bmatrix} \end{array} \end{array} \begin{array}{c} \textcircled{4} \\ \textcircled{2} \\ \textcircled{3} \end{array} \begin{array}{c} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta\delta_4 \\ \hline \Delta V_2/V_2 \\ \Delta V_3/V_3 \end{bmatrix} \\ \text{corrections} \end{array}$$

Jacobian

$$= \begin{array}{c} \begin{bmatrix} -1.597 \\ -1.940 \\ 2.213 \\ \hline -0.447 \\ -0.835 \end{bmatrix} \\ \text{mismatches} \end{array}$$



Order the rows of the Numerical Jacobian so as to minimize fill-in elements when the lower- and upper-triangular factors are calculated. Numerically evaluate the triangular factors and, from the mismatch equations, compute the voltage corrections in the first iteration of the Newton–Raphson method.

## CASE STUDY QUESTIONS

- A. Is voltage collapse more likely to occur in a power system serving an urban load, an industrial load, a residential-commercial load, or a mix of different load types?
- B. The voltage collapse phenomenon is studied via power-flow programs. What other studies can be performed with power-flow programs?

## DESIGN PROJECT I: SYSTEM PLANNING

Time given: 3 weeks

Approximate time required: 15 hours

As a planning engineer for Metropolis Light and Power (MLP) your job is to ensure that the transmission system is adequate for any base case or first-contingency loading situation. Traditionally, MLP has served the load in the metropolitan Metropolis region, along with several nearby rural areas. However, your job has definitely gotten tougher over the last several years. Because of the uncertainty associated with restructuring, management has been reluctant to spend money to strengthen the system. Also, new generation additions are no longer centrally determined by MLP, but rather left to the discretion of numerous independent generation companies.

During this time, though, the load has continued to grow. A robust economy in the Metropolis area has been helping to stress the transmission system. The stresses of the load growth are now most apparent on the western portion of the grid. Anchored by a successful new industrial park with associated commercial and residential development, load growth at the HANA69, AMANS69, and HISKY69 substations has exceeded 7% per year.

Anticipated future peak load conditions for the MLP system are shown in Figure 6.10 as DesignCase1. Unfortunately, there are several contingency violations. Your job is to identify the problems and then make recommendations for the best transmission system additions to strengthen the western portion of the system so that there will be no violations for either the base case loading or during any single transmission element contingency. Secure system operations requires that no lines or transformers be loaded higher than 100% of their ratings and that all bus voltage magnitudes are between 0.95 and 1.06 per unit. The table that follows summarizes various rights-of-way that can be used for the installation of new lines. Design costs are also provided.

8. In power systems with generation areas that can be temporarily separated from load areas, braking resistors can improve stability. When separation occurs, the braking resistor is inserted into the generation area for a second or two, preventing or slowing acceleration in the generation area. Shelton et al. [8] describe a 3-GW-s braking resistor.

## PROBLEMS

### SECTION 13.1

- 13.1 A three-phase, 60-Hz, 400-MVA, 13.8-kV, 4-pole steam turbine-generating unit has an  $H$  constant of 5.0 p.u.-s. Determine: (a)  $\omega_{\text{syn}}$  and  $\omega_{m\text{syn}}$ ; (b) the kinetic energy in joules stored in the rotating masses at synchronous speed; (c) the mechanical angular acceleration  $\alpha_m$  and electrical angular acceleration  $\alpha$  if the unit is operating at synchronous speed with an accelerating power of 400 MW.
- 13.2 Calculate  $J$  in  $\text{kg m}^2$  for the generating unit given in Problem 13.1.
- 13.3 Generator manufacturers often use the term  $WR^2$ , which is the weight in pounds of all the rotating parts of a generating unit (including the prime mover) multiplied by the square of the radius of gyration in feet.  $WR^2/32.2$  is then the total moment of inertia of the rotating parts in  $\text{slug-ft}^2$ . (a) Determine a formula for the stored kinetic energy in  $\text{ft-lb}$  of a generating unit in terms of  $WR^2$  and rotor angular velocity  $\omega_m$ . (b) Show that

$$H = \frac{2.31 \times 10^{-4} WR^2 (\text{rpm})^2}{S_{\text{rated}}} \quad \text{per unit-seconds}$$

where  $S_{\text{rated}}$  is the voltampere rating of the generator and rpm is the synchronous speed in r/min. Note that  $1 \text{ ft-lb} = 746/550 = 1.356 \text{ joules}$ . (c) Evaluate  $H$  for a three-phase generating unit rated 800 MVA, 3600 r/min, with  $WR^2 = 4,000,000 \text{ lb-ft}^2$ .

- 13.4 The generating unit in Problem 13.1 is initially operating at  $p_{mp.u.} = p_{ep.u.} = 0.7$  per unit,  $\omega = \omega_{\text{syn}}$ , and  $\delta = 12^\circ$  when a fault reduces the generator electrical power output by 70%. Determine the power angle  $\delta$  five cycles after the fault commences. Assume that the accelerating power remains constant during the fault. Also assume that  $\omega_{p.u.}(t) = 1.0$  in the swing equation.
- 13.5 Repeat Problem 13.4 for a bolted three-phase fault at the generator terminals that reduces the electrical power output to zero. Compare the power angle with that determined in Problem 13.4.
- 13.6 A third generating unit rated 400 MVA, 15 kV, 0.90 power factor, 16 poles, with  $H_3 = 3.5$  p.u.-s is added to the power plant in Example 13.2. Assuming all three units swing together, determine an equivalent swing equation for the three units.

### SECTION 13.2

- 13.7 The synchronous generator in Figure 13.4 delivers 0.9 per-unit real power at 1.08 per-unit terminal voltage. Determine: (a) the reactive power output of the generator; (b) the generator internal voltage; and (c) an equation for the electrical power delivered by the generator versus power angle  $\delta$ .

- 13.8 The generator in Figure 13.4 is initially operating in the steady-state condition given in Problem 13.7 when a three-phase-to-ground bolted short circuit occurs at bus 3. Determine an equation for the electrical power delivered by the generator versus power angle  $\delta$  during the fault.

### SECTION 13.3

- 13.9 The generator in Figure 13.4 is initially operating in the steady-state condition given in Example 13.3 when circuit breaker B12 inadvertently opens. Use the equal-area criterion to calculate the maximum value of the generator power angle  $\delta$ . Assume  $\omega_{p.u.}(t) = 1.0$  in the swing equation.
- 13.10 The generator in Figure 13.4 is initially operating in the steady-state condition given in Example 13.3 when a temporary three-phase-to-ground short circuit occurs at point F. Three cycles later, circuit breakers B13 and B22 permanently open to clear the fault. Use the equal-area criterion to determine the maximum value of the power angle  $\delta$ .
- 13.11 If breakers B13 and B22 in Problem 13.10 open later than 3 cycles after the fault commences, determine the critical clearing time.
- 13.12 Rework Problem 13.10 if circuit breakers B13 and B22 open after 3 cycles and then reclose when the power angle reaches  $35^\circ$ . Assume that the temporary fault has already self-extinguished when the breakers reclose.
- 13.13 The generator in Figure 13.4 is initially operating in the steady-state condition given in Problem 13.7 when circuit breaker B12 inadvertently opens. Use the equal-area criterion to calculate the maximum value of the generator power angle  $\delta$ . Assume  $\omega_{p.u.}(t) = 1.0$  in the swing equation.
- 13.14 The generator in Figure 13.4 is initially operating in the steady-state condition given in Problem 13.7 when a temporary three-phase-to-ground short circuit occurs at point F. Three cycles later, circuit breakers B13 and B22 permanently open to clear the fault. Use the equal-area criterion to calculate the maximum value of the generator power angle  $\delta$ . Assume  $\omega_{p.u.}(t) = 1.0$  in the swing equation.
- 13.15 If breakers B13 and B22 in Problem 13.14 open later than three cycles after the fault commences, determine the critical clearing time.

### SECTION 13.4

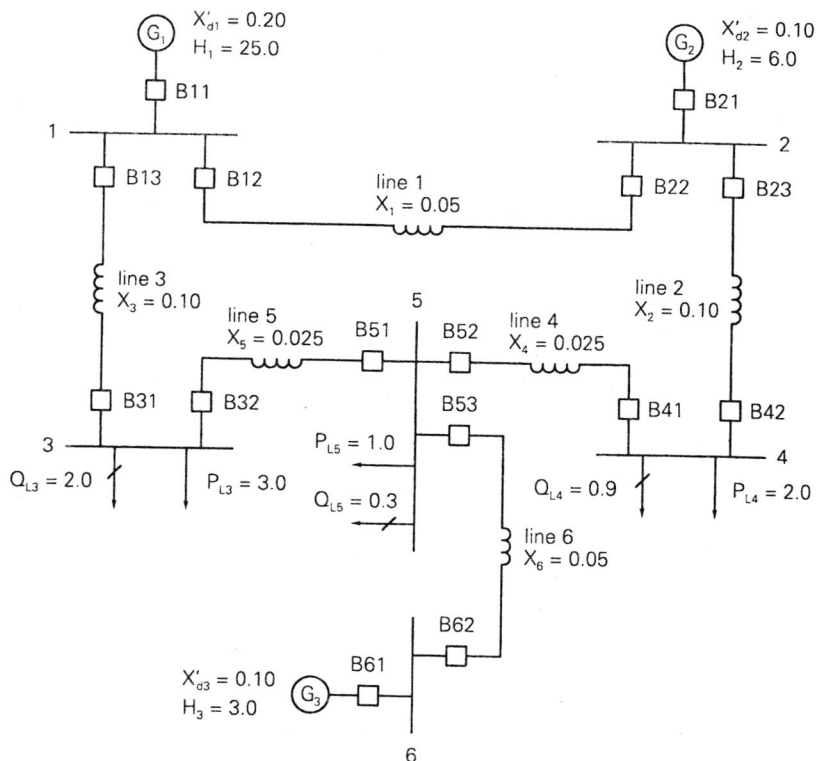
- 13.16 Verify the maximum power angle determined in Problem 13.9 by applying the modified Euler's method to numerically integrate the swing equation. Write and run a computer program.
- 13.17 Investigate the effect of generating-unit damping torque on the maximum power angle in Problem 13.16. Damping, which is caused by friction and windage, can be represented by subtracting from the per-unit accelerating power  $p_{ap.u.}(t)$  [used in (13.4.8) and (13.4.12)] the term  $B\omega_{p.u.}(t)$ , where  $B$  is a per-unit damping coefficient. Compare the maximum power angle using  $B = 0.01$  per unit with that computed in Problem 13.16. Discuss the effect of generating-unit damping torques on stability.
- 13.18 Verify the critical clearing time determined in Problem 13.11 by applying the modified Euler's method. Write and run a computer program.
- 13.19 In Problem 13.12, assume that the circuit breakers open at  $t = 3$  cycles and then reclose at  $t = 24$  cycles (instead of when  $\delta$  reaches  $35^\circ$ ). Determine the maximum power angle by applying the modified Euler method. Write and run a computer program.

## SECTION 13.5

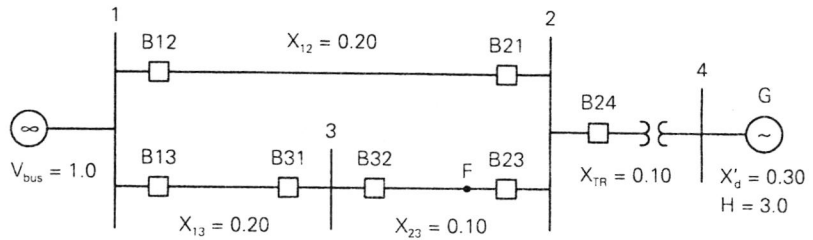
- 13.20** Consider the six-bus power system shown in Figure 13.13, where all data are given in per-unit on a common system base. All resistances as well as transmission-line capacitances are neglected. (a) Determine the  $6 \times 6$  per-unit bus admittance matrix  $Y_{\text{bus}}$  suitable for a power-flow computer program. (b) Determine the per-unit admittance matrices  $Y_{11}$ ,  $Y_{12}$ , and  $Y_{22}$  given in (13.5.5), which are suitable for a transient stability study.

**FIGURE 13.13**

Single-line diagram of a six-bus power system (per-unit values are shown)



- 13.21** Modify the matrices  $Y_{11}$ ,  $Y_{12}$ , and  $Y_{22}$  determined in Problem 13.20 for (a) the case when circuit breakers B12 and B22 open to remove line 1–2; and (b) the case when the load  $P_{L4} + jQ_{L4}$  is removed.
- 13.22** Figure 13.14 shows a single-line diagram of the same circuit given in Example 13.3, except that bus numbers 1 and 2 are interchanged and a new bus 4 is defined for the generator. All reactances are given in per-unit on a common system base. The infinite bus voltage is  $1.0 \angle 0^\circ$  per unit, and the generator initially delivers 1.0 per unit real power at 1.05 per unit terminal voltage, as in Problem 13.7. Using PowerWorld Simulator, create the bus input data with bus 1 as the swing bus, buses 2 and 3 as load buses, and bus 4 as a constant voltage magnitude bus. Also create the line input and

**FIGURE 13.14**Circuit for Problem  
13.22

transformer input data files. Then run PowerWorld Simulator, and verify the generator reactive power output determined in Problem 13.7.

## CASE STUDY QUESTIONS

- How have electric utility companies changed their power system planning practices since the 1965 Northeast blackout?
- Since 1965, has the U.S. electric power system become less prone to blackouts or more prone?
- Will there be another blackout as large in scale as the 1965 Northeast blackout?

## REFERENCES

- G. W. Stagg and A. H. El-Abiad, *Computer Methods in Power Systems* (New York: McGraw-Hill, 1968).
- O. I. Elgerd, *Electric Energy Systems Theory*, 2d ed. (New York: McGraw-Hill, 1982).
- C. A. Gross, *Power System Analysis* (New York: Wiley, 1979).
- W. D. Stevenson, Jr., *Elements of Power System Analysis*, 4th ed. (New York: McGraw-Hill, 1982).
- E. W. Kimbark, "Suppression of Ground-Fault Arcs on Single-Pole Switched EHV Lines by Shunt Reactors," *IEEE Trans PAS*, 83 (March 1964), pp. 285–290.
- K. R. McClymont et al., "Experience with High-Speed Rectifier Excitation Systems," *IEEE Trans PAS*, vol. PAS-87 (June 1986), pp. 1464–1470.
- E. W. Cushing et al., "Fast Valving as an Aid to Power System Transient Stability and Prompt Resynchronization and Rapid Reload after Full Load Rejection," *IEEE Trans PAS*, vol. PAS-90 (November/December 1971), pp. 2517–2527.
- M. L. Shelton et al., "Bonneville Power Administration 1400 MW Braking Resistor," *IEEE Trans PAS*, vol. PAS-94 (March/April 1975), pp. 602–611.
- R. McCaw, "The Great Blackout," *Power Engineering* (December 1965), pp. 36A–36C.