Time Series Analysis

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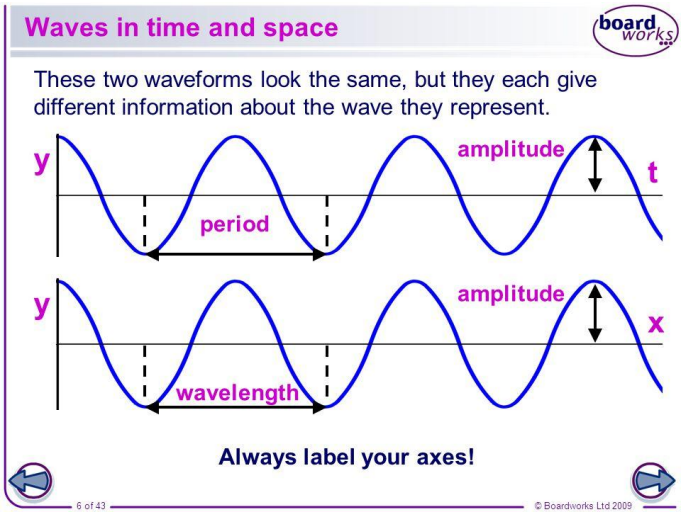
Course Goal

- Provide a basic knowledge of time series analysis
- Understand basic time series concepts and terminology
- Apply the time series analysis to analysis and solve geophysical signal problems

SIGNAL

- **What is Signal??**

A “signal” describes how some physical quantity varies over time and/or space.



- **Signal:**

  A signal is defined as a function of one or more variables which conveys information on the nature of a physical phenomenon. The value of the function can be a real valued scalar quantity, a complex valued quantity, or perhaps a vector.

- System:

  A system is defined as an entity that manipulates one or more signals to accomplish a function, thereby yielding new signals.

- **Continuos-Time Signal:**

  A signal x(t) is said to be a continuous time signal if it is defined for all time t.

- Discrete-Time Signal:

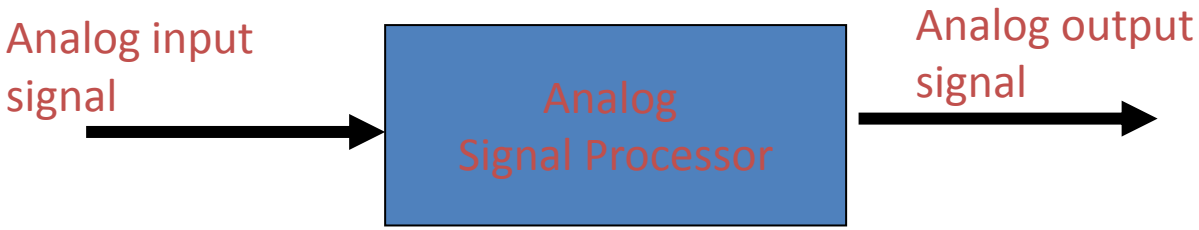
  A discrete time signal x[nT] has values specified only at discrete points in time.

- Signal Processing:

  Manipulating a signal to change its characteristics or extract information from it. **SIGNAL PROCESSING** is the analysis, interpretation and manipulation of like sound, images, time-varying measurement values and sensor data.

Basic Elements of Signal Processing System

- Types of signal processing:
  1. Analog signal processing
  2. Digital signal processing



Analog Signal Processing

Digital Signal Processing

Analog and Discrete Signals

- Analog signals → continuous signals

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} F(k) e^{-ikx} dx$$

$$F(k) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{ikx} dx$$

- Discrete signals → sampled signals

$$F_k = \frac{1}{N} \sum_{j=0}^{N-1} f_j e^{-2\pi ikj/N}, k = 0,1,\ldots, N-1$$

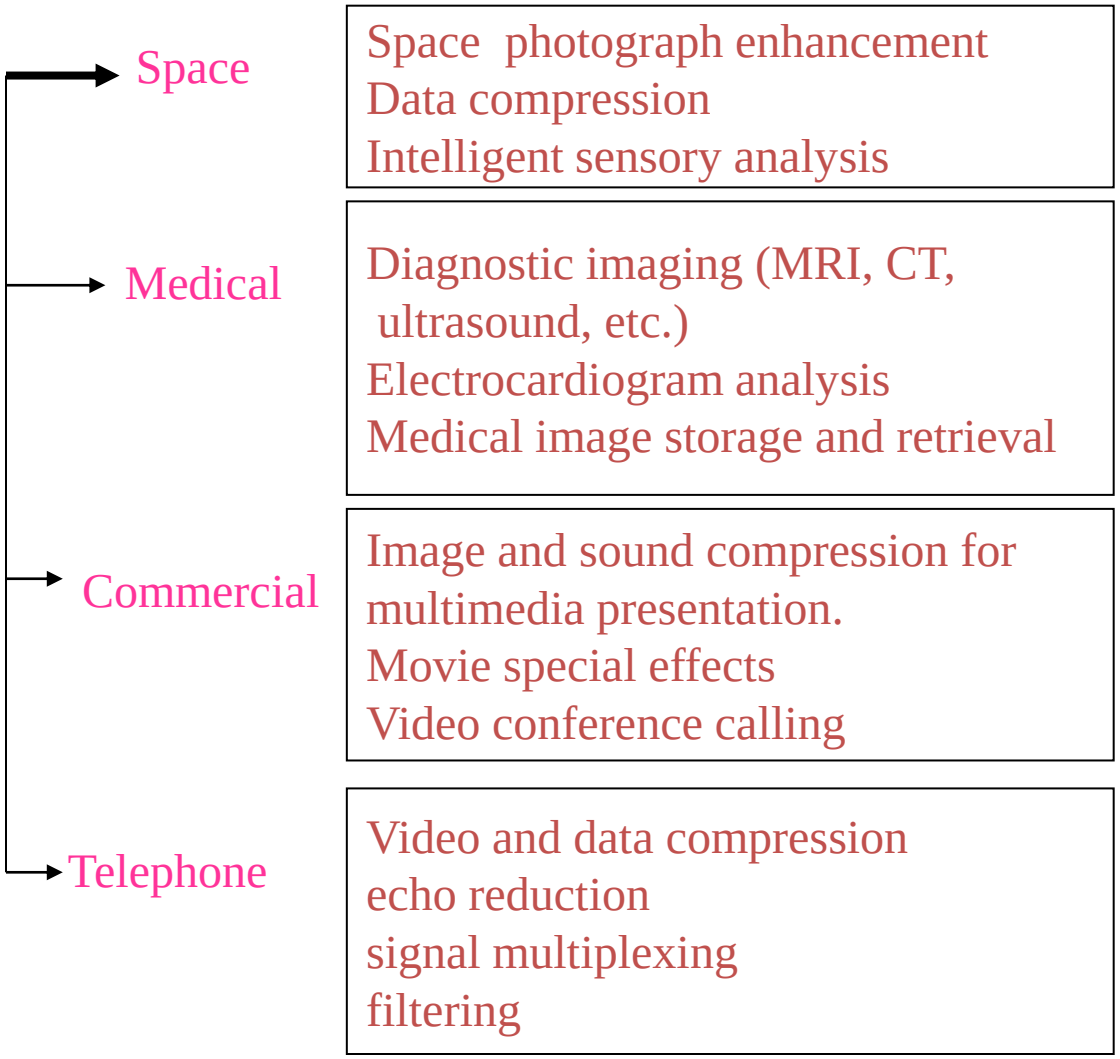
$$f_k = \sum_{j=0}^{N-1} F_j e^{2\pi ikj/N}, k = 0,1,\ldots, N-1$$

Whatever we do on the computer with data will be based on the discrete Fourier transform

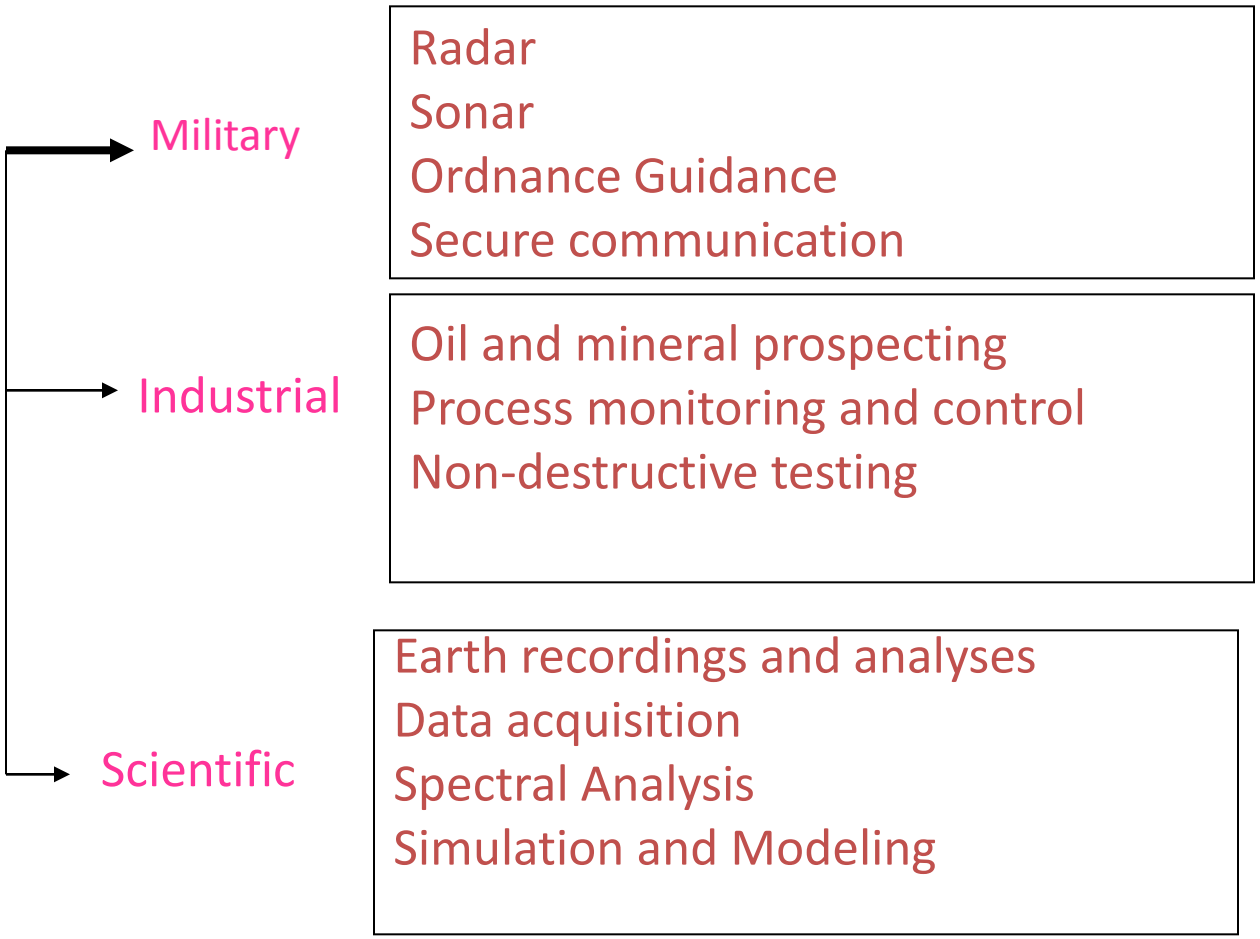
Advantages of Digital Over Analogue Signal Processing

- A digital programmable system allows flexibility in reconfiguring the DSP operations simply by changing the program. Reconfiguration of an analogue system usually implies a redesign of hardware, testing and verification that it operates properly.
- DSP provides better control of accuracy requirements.
- Digital signals are easily stored on magnetic media (tape or disk).
- The DSP allows for the implementation of more sophisticated signal processing algorithms.
- In some cases a digital implementation of the signal processing system is cheaper than its analogue counterpart.

DSP Applications



DSP Applications



Classification Signals

- Deterministic Signals

  A deterministic signal behaves in a fixed known way with respect to time. Thus, it can be modeled by a known function of time t for continuous time signals, or a known function of a sampler number (n), and sampling spacing (Δt) for discrete time signals.

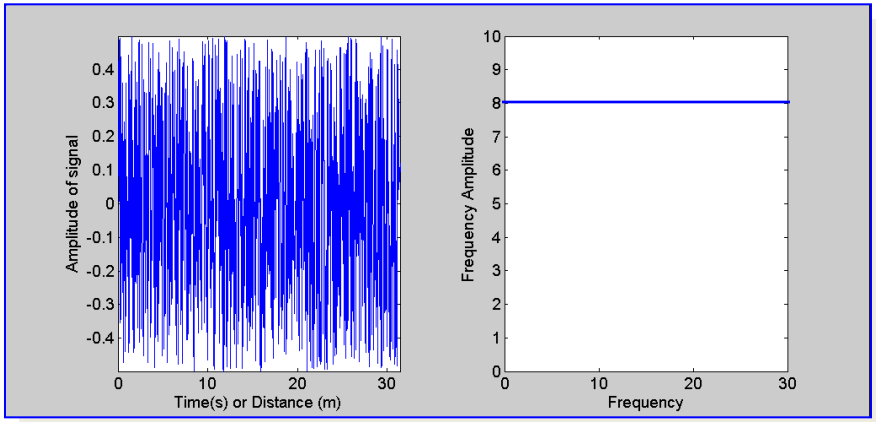
- Random or Stochastic Signals:

  In many practical situations, there are signals that either cannot be described to any reasonable degree of accuracy by explicit mathematical formulas, or such a description is too complicated to be of any practical use. The lack of such a relationship implies that such signals evolve in time in an unpredictable manner. We refer to these signals as random.

Basic Vocabulary

- **Random signals:**

  Difficult to define when they commence and finish. They may contain all frequencies. A spectrum with constant contribution of all frequencies is called a white spectrum.



- **Transient Signals:**

  Limited in time (or space) in comparison with a harmonic wave form that is infinite

Periodic Signals

Periodic Signals

Concept of **periodic signal**

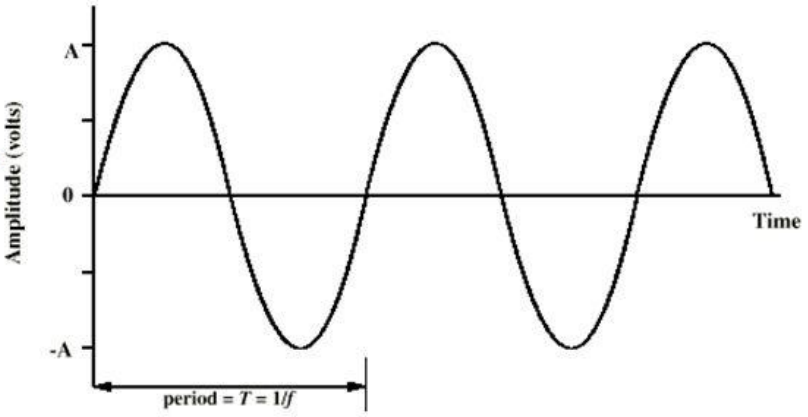
- The same signal pattern repeats over time.
- Otherwise, a signal is aperiodic.

Sine Wave: represented by three parameters,

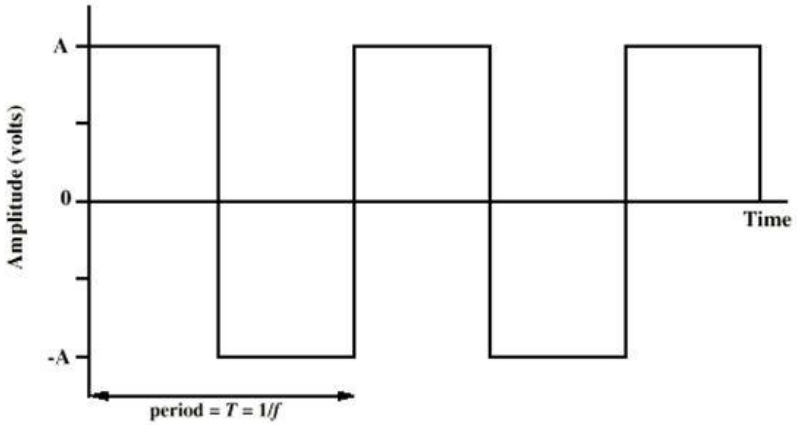
$$s(t)=A\sin(2\pi ft+\phi)$$

- Peak Amplitude (A)
  - maximum strength of signal
  - measured in volts
- Frequency (f)
  - Rate of change of signal
  - Hertz (Hz) or cycles per second
  - Period = time for one repetition (T)
  - T = 1/f
- Phase (ϕ)
  - Relative position in time within a single period of a signal

Figure (a) displays the value of a signal at a given point in space as a function of time.



(a) Sine wave



(b) Square wave

Basic Vocabulary

- **Non-Periodic Signal:**

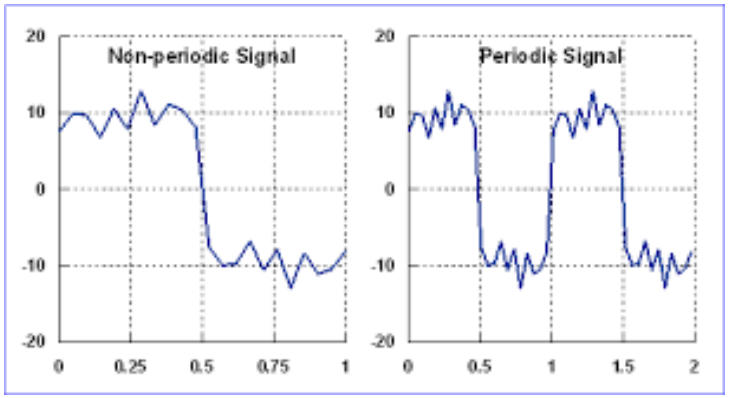
  is the signal that defined over a finite interval of space or time and vanishes outside that interval.

- **Periodic Signal:**

  is the signal that repeats itself indefinitely with time or space. Mathematically, this is stated as: $f(x) = f(x \pm T)$, T is called the period of the signal. We will have more about period later. $T=2\pi m/a$

  For combined function such as $f(t)=\cos(a_1(t+T))+\cos(a_2(t+T))$, $T=2\pi m/a_1=2\pi n/a_2$, then $a_1/a_2=m/n$, m and n are integer numbers

  These relations are very useful to examine the periodicity of any function

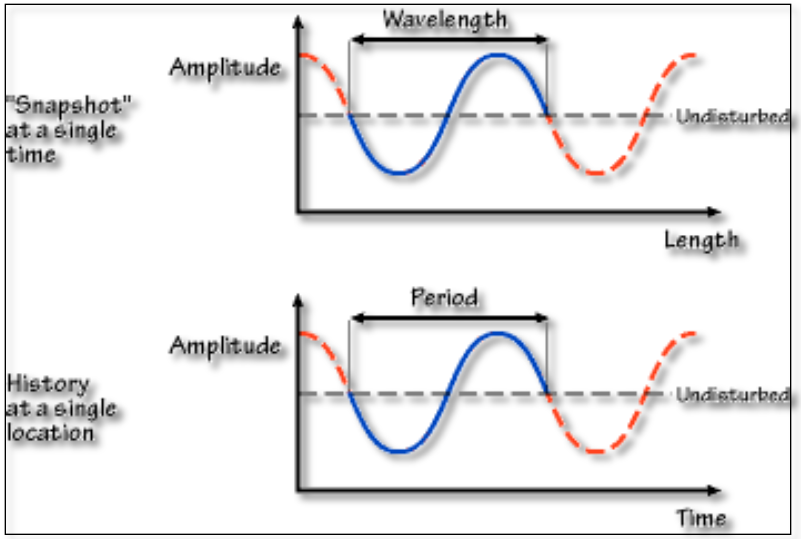


Basic Vocabulary

- **Period and Wave Length (T &λ):**

If the signal is a function of time (ms, sec, hours, etc.) then the signal is characterized by **PERIOD**, which is the interval of time between tow identically located points on the signal.

On the other hand, if the signal is measured against distance (cm, m, km, etc.), then the signals is identified by **WAVE LENGTH in distance units.**



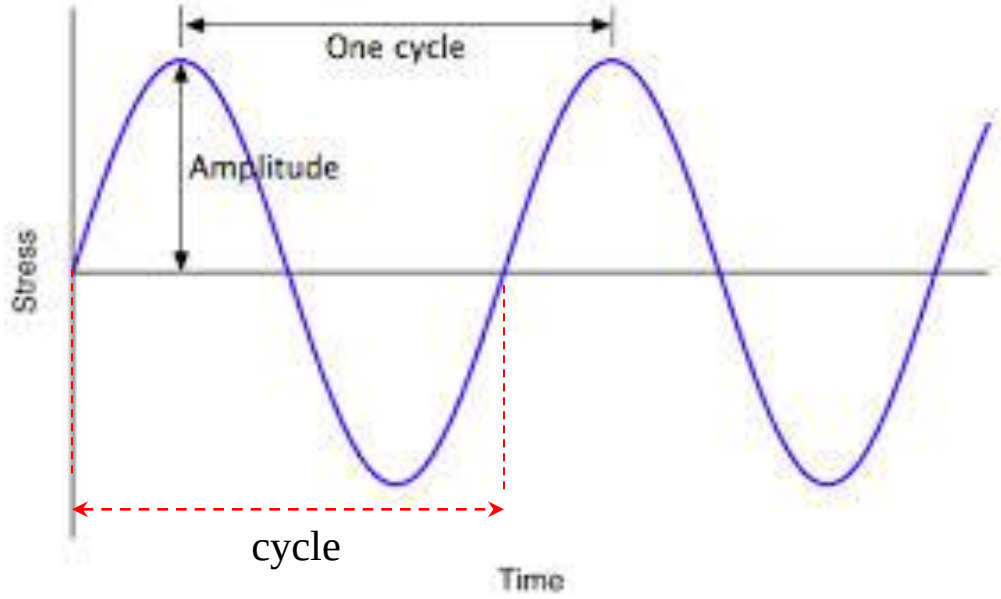
$$v = f\lambda$$

v : speed $(m.s^{-1})$
λ : wavelength (m)
f : frequency $(Hz$ or $s^{-1})$

Basic Vocabulary

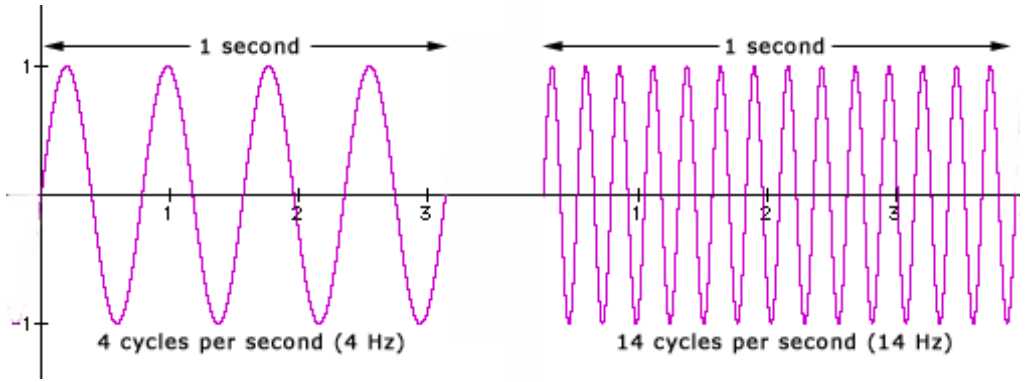
- **Cycle:**

  Two identically located points on a periodic signal define one cycle of the signal. Note a cycle is neither a primary physical units nor does it have one. It is only a descriptive term.

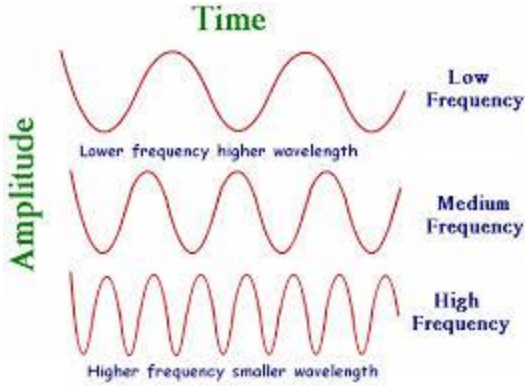


Basic Vocabulary

- **Frequency:**
  Is the number of repetitions cycles of a periodic signal per unit time or distance. Comparing the two figures (a and b) to the right, you will notice that in the first the signal makes one cycle per unit measure, whereas in the second the signal executes two cycles in the same length of time/space unit. Hence, the former signal (a) has a frequency of 4 cycles/sec and the latter (b) has a frequency of 14 cycles/sec. Figure c shows the characteristics of frequency contents.



(a) (b)



(c)

Basic Vocabulary

- **Phase:**

  The phase of a signal refers to a sinusoidal function such as the following:

$$x(t) = A \cdot \cos(2\pi f t + \varphi)$$
$$y(t) = A \cdot \sin(2\pi f t + \varphi) = A \cdot \cos\left(2\pi f t + \varphi - \tfrac{\pi}{2}\right)$$

  where *A*, *f*, and φ are constant parameters called the *amplitude*, *frequency*, and *phase* of the sinusoid.

Basic Vocabulary

- **Phase shift:**

  is any change that occurs in the phase of one quantity, or in the phase difference between two or more quantities