Time Series Analysis

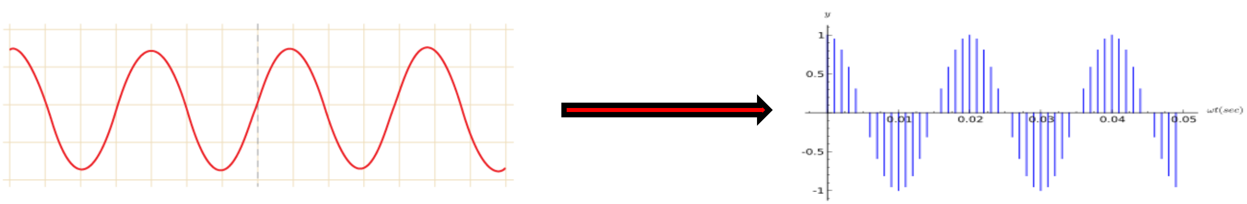
AK Abdelfattah

Digitization/Sampling

- What happens when digitize a signal (ground motion, temperature, etc.) in space and/or time?
- What are the consequence of a particular sampling rate on the information content?
- How can we treat (process, transform) the observed signals to extract relevant information?

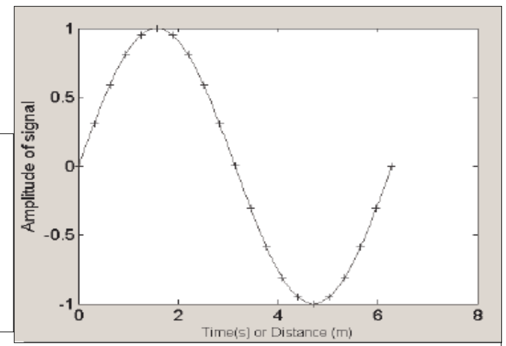
Digitization/Sampling

- This is the conversion of a continuous time signal into a discrete time signal obtained by taking “samples” of the continuous time signal at discrete time instants. Thus, if *x(t)* is the input to the sampler, the output is *x(n.Δt)*, where *Δt* is called the *Sampling interval*.



Wavelength/Period

- The most important concepts in processing observations are spatial and temporal frequencies.



T period
f frequency
ω angular frequency

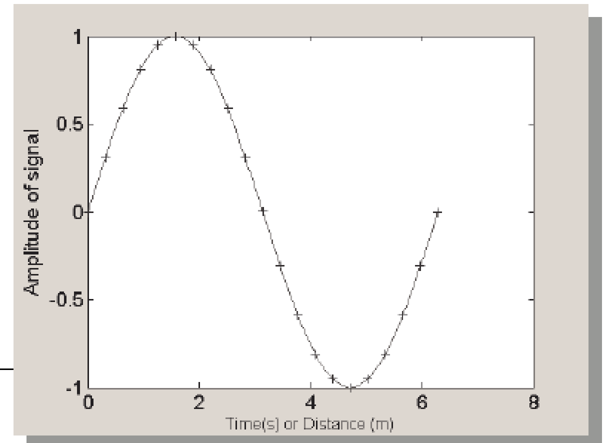
temporal frequencies

$T = 1/f$
$\omega = 2\pi f$

Harmonic oscillation in time:
$f(t) = A \sin(\omega t) = A \sin(2\pi f t) = A \sin((2\pi/T)\, t)$
A motion amplitude

Wavelength/Period

... and the space analogue ...



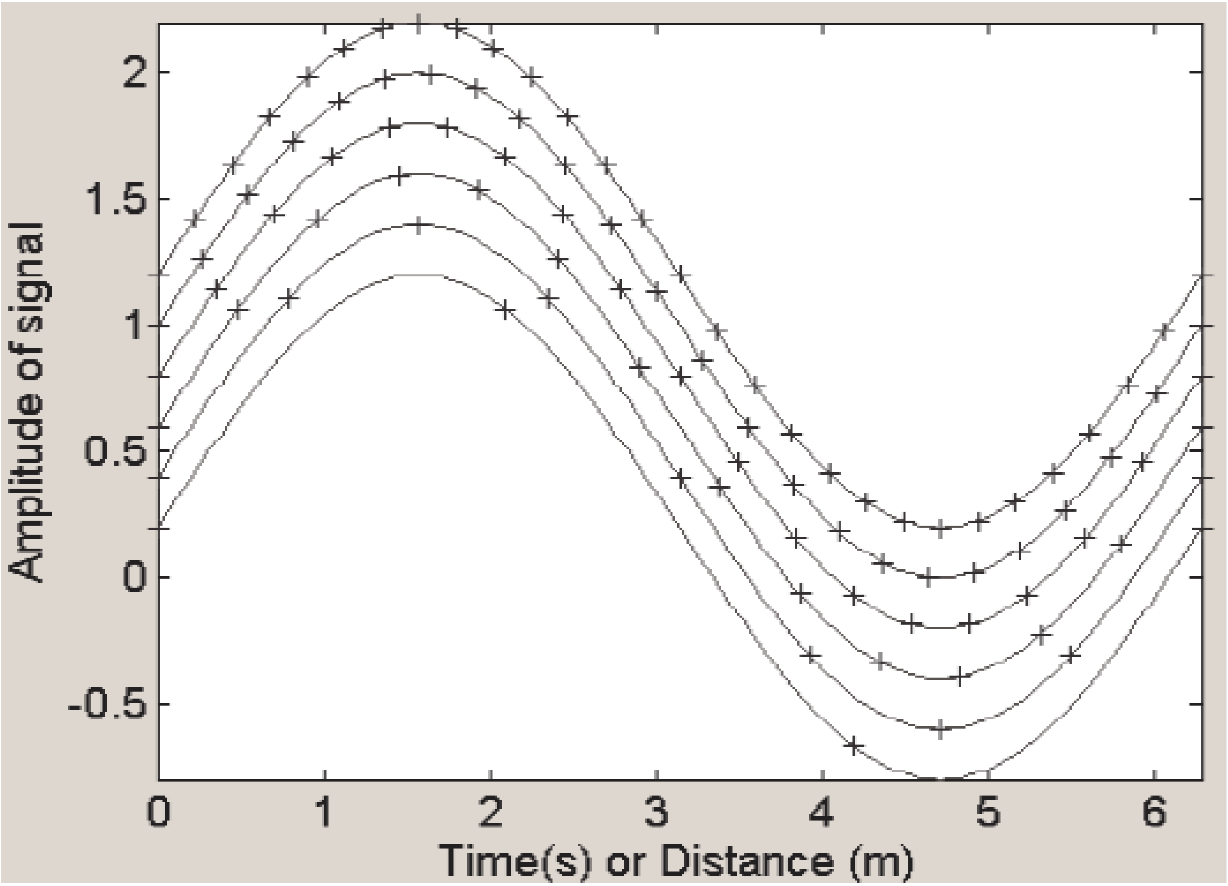
λ wavelength
k spatial wavenumber

$k=2\pi/\lambda$

spatial frequencies

Harmonic oscillation in time:
$f(x) = A \sin(kx) = A \sin((2\pi/\lambda)\, x)$
A motion amplitude

Sampling rate



- Sampling frequency, sampling rate is the number of sampling points per unit distance or unit time.

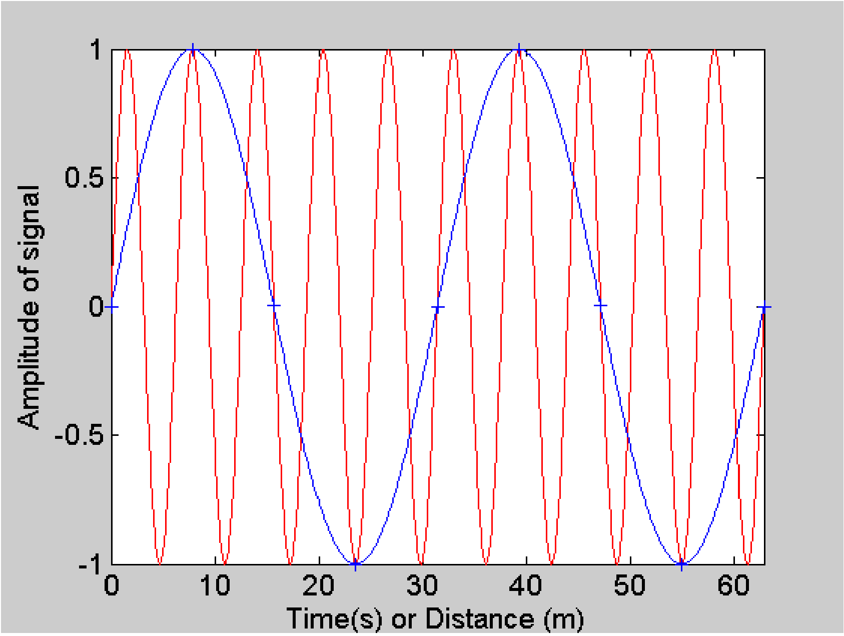
Nyquist Frequency

- The frequency half of the sampling rate dt is called the Nyquist frequency $f_N=1/(2dt)$. The distortion of a physical signal higher than the Nyquist frequency is called aliasing.

- Aliasing must be avoided by ensuring Nyquist is above the maximum frequency in the signal, or high-pass/ anti-aliasing filtering.

- The frequency of physical signal is > $\mathbf{f_N}$ is sampled wit (+) leading to the erroneous blue oscillation.

- How can we avoid aliasing



Resolution and Dynamic Range

- What about the amplitude response of our records? What are the limits of the analogue sensor and digital data-logger?. (example: a seismographic station consists of an analogue seismometer, and analogue-to-digital (AD) converter, which usually includes data storage / communication device and called a data-logger)
- **Resolution :** smallest signal that can be resolved.
- **Dynamic range :** ratio between the largest amplitude (A_{max}) and smallest amplitude (A_{min}) of the signals that can be accurately recorded.

bits	dynamic range ($2^{\#bits-1}$)	DR_{dB} (#bits-1).6	Orders of magnitude
8	256/2	42	2
12	4,096/2	66	3
16	65,536/2	90	4.5
20	1,048,576/2	114	6
24	16,777,216/2	138	7

Dynamic range

It is described the precision of the sampling of our physical signal in amplitude?

The unit is Decibel (dB) and is defined as the ratio of two power values (and power is proportional to amplitude square)

In terms of amplitudes

Dynamic range = $20 \log_{10}(A_{max}/A_{min})$ dB

Example: with 1024 units of amplitude ($A_{min}=1$, $A_{max}=1024$)

$$20 \log_{10}(1024/1) \text{ dB} \approx 60 \text{ dB}$$

Signal and Noise

- Almost all signals contain noise.
- The signal-to-noise ratio has an important concept to consider in all geophysical measurements.
- Can you give examples of noise in the various methods?