

Time Series Analysis

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Linear Regression

- A linear regression for a simple example of the function in the form of [$y = ax + b$]
- **Given:**
 - The observational data $(x_i, y_i); i=1,2,\dots,N$
- **Required:**

Computational Algorithm

1. Construct the following table

X_i	Y_i	$X_i Y_i$	X_i^2
X_1	Y_1	$X_1 Y_1$	X_1^2
X_2	Y_2	$X_2 Y_2$	X_2^2
\vdots	\vdots	\vdots	\vdots
\vdots	\vdots	\vdots	\vdots
\vdots	\vdots	\vdots	\vdots
X_N	Y_N	$X_N Y_N$	X_N^2
$=\Sigma X_i$	$=\Sigma Y_i$	$=\Sigma X_i Y_i$	$=\Sigma X_i^2$

2. Compute Δ from

$$\Delta = N \cdot \Sigma X_i^2 - \Sigma X_i$$

Computational Algorithm

3. Compute ***a*** and ***b*** from

$$a = \frac{1}{\Delta} [\sum Y_i \cdot \sum X_i^2 - \sum X_i \cdot \sum X_i Y_i]$$

$$b = \frac{1}{\Delta} [N \cdot \sum X_i Y_i - \sum X_i \cdot \sum Y_i]$$

4. Construct the following table

X_i	Y_i	Y_c	$Y - Y_c$	$(Y - Y_c)^2$
X_1	Y_1	$aX_1 + b$	$Y_1 - Y_{c1}$	$(Y_1 - Y_{c1})^2$
X_2	Y_2	$aX_2 + b$	$Y_2 - Y_{c2}$	$(Y_2 - Y_{c2})^2$
\vdots	\vdots	\vdots	\vdots	\vdots
\vdots	\vdots	\vdots	\vdots	\vdots
\vdots	\vdots	\vdots	\vdots	\vdots
X_N	Y_N	$aX_N + b$	$Y_N - Y_{cN}$	$(Y_N - Y_{cN})^2$
				$= \sum (Y - Y_c)^2$

Computational Algorithm

5. Compute the variance

$$\sigma^2 = \frac{\sum (Y - Y_c)^2}{N - 1}$$

6. Compute the uncertainties of *a* and *b* from

$$\delta a = \sqrt{\sigma^2 \cdot \frac{\sum X_i^2}{\Delta}}$$

$$\delta b = \sqrt{\sigma^2 \cdot \frac{N}{\Delta}}$$

Example

- Consider the following data set of 10 points and estimate the parameter a and b with uncertainties:

X_i	Y_i	X_i	Y_i
1	1	7	8
2	1	9	7
1	2	10	8
3	3	12	14
6	5	14	16