$$f(t) \longrightarrow F(p)$$

$$f'(t) \longrightarrow pF(p) - f(0)$$

$$f''(t) \longrightarrow p\left(pF(p) - f(0)\right) - f'(0)$$

$$y \longrightarrow Y(p)$$

$$y' \longrightarrow pY(p) + 3$$

$$y'' \longrightarrow p^2Y(p) + 3p - 5$$

$$y'' - 3y' + 2y = 4e^{2t} \text{ with } y(0) = -3$$
$$\text{and } y'(0) = 5$$

$$y = y(t) \xrightarrow{L.T} Y(p)$$

$$y' \xrightarrow{L.T} pY(p) + 3$$

$$y'' \longrightarrow p^2 Y(p) + 3p - 5$$

$$e^{\lambda t} \xrightarrow{L.T} \frac{1}{p-\lambda}$$

$$4e^{2t} \longrightarrow \frac{4}{p-2}$$

$$\left(p^2 Y(p) + 3p - 5\right) - 3\left(pY(p) + 3\right) + 2Y(p) = \frac{4}{p-2}$$

$$\underbrace{\left(p^2 - 3p + 2\right)}_{(p-2)(p-1)} Y(p) = \frac{4}{p-2} - 3p + 5 + 9$$

$$(p-2)(p-1)Y(p) = 14 + \frac{4}{p-2} - 3p$$

$$= \frac{14(p-2)+4-3p(p-2)}{p-2}$$

$$Y(p) = \frac{-3p^2+20p-24}{(p-2)^2(p-1)} = \frac{a}{p-1} + \frac{b}{p-2} + \frac{c}{(p-2)^2}$$

$$= \frac{a(p-2)^2 + b(p-2)(p-1) + c(p-1)}{(p-2)^2(p-1)}$$

$$= \frac{a(p^2-4p+4) + b(p^2-3p+2) + cp - c}{(p-2)^2(p-1)}$$

$$= \frac{\overbrace{(a+b)}^{-3}p^2 + \overbrace{(-4a-3b+c)}^{20}p + \overbrace{4a+2b-c}^{-24}}{(p-2)^2(p-1)}$$

$$\begin{cases} a+b=-3 \\ -4a-3b+c=20 \\ 4a+2b-c=-24 \end{cases}$$

$+b=+4$

$a=-3-b=-3-4=-7$

$c=4a+2b+24$

$=-28+8+24=4$

So:

$$Y(p) = -\frac{7}{p-1} + \frac{4}{p-2} + \frac{4}{(p-2)^2}$$

we know that:

$$-7e^{t} \to -\frac{7}{p-1} \,;\, 4e^{2t} \to \frac{4}{p-2}$$

$$4te^{2t} \to \frac{4}{(p-2)^2}$$

So

$$\boxed{y(t) = -7e^{t} + 4\left(e^{2t} + te^{2t}\right)}$$

let's verify the sol:

$$y(t) = -7e^{t} + 4e^{2t} + 4te^{2t}$$

$$y'(t) = -7e^{t} + 8e^{2t} + 4e^{2t} + 8te^{2t}$$

$$= -7e^{t} + 12e^{2t} + 8te^{2t}$$

$$y''(t) = -7e^{t} + 32e^{2t} + 16te^{2t}$$

So:

$$\cancel{-7e^{t}} + 32e^{2t} + \cancel{16te^{2t}} + \cancel{21e^{t}} - 36e^{2t}$$

$$\cancel{-24te^{2t}} \cancel{-14e^{t}} + 8e^{2t} + \cancel{8te^{2t}}$$

$$= 4e^{2t}$$