Chap 1

Emergence of quantum physics

1- Blackbody radiation

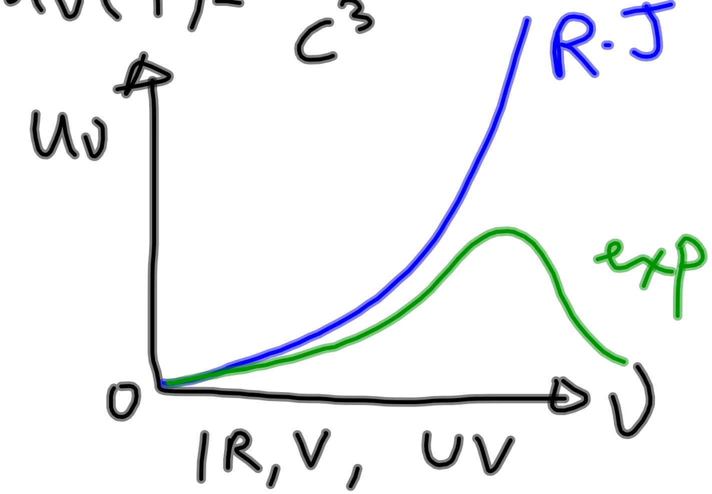
B.B 100% absorbing body

it is caracterized by its temperature T.

u_ν: density of energy for T at the frequency ν.

Rayleigh-Jeans f.

$$u_\nu(T) = \frac{8\pi}{c^3}\nu^2 kT$$



"Ultraviolet" catastrophe

exp $\searrow$ th $\nearrow$!!!

Planck in 1900:

The energy exchanged between matter and light is discrete.

$E, 2E, \dots, nE$.

0

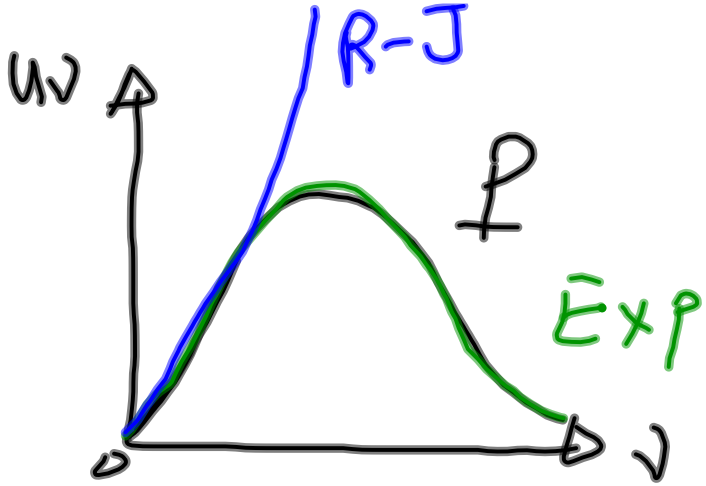
$0 < E < \infty$ R-J

$0 \quad E \quad nE$ P

$E_n = nE$

and $E = h\nu$

h: Planck ct.



Planck formula:

$$\boxed{u_\nu = \frac{8\pi h}{c^3}\,\frac{\nu^3}{e^{\frac{h\nu}{kT}} - 1}}$$

Ex: Find R-J formula from P. formula.

Sol:
$h\nu \ll kT$ so

$$e^{\frac{h\nu}{kT}} \approx 1 + \frac{h\nu}{kT} \quad \text{and}$$

$$u_\nu \approx \frac{8\pi h}{c^3}\,\frac{\nu^3}{\frac{h\nu}{kT}} = \frac{8\pi\nu^2}{c^3}\,kT$$

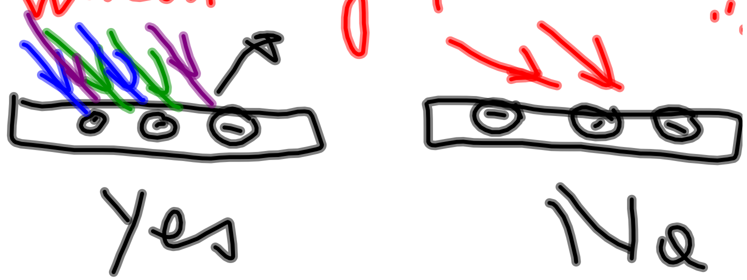
The R.J formula

2. Photoelectric effect

The effect is known from Hertz in 1887.

e^- are ejected.

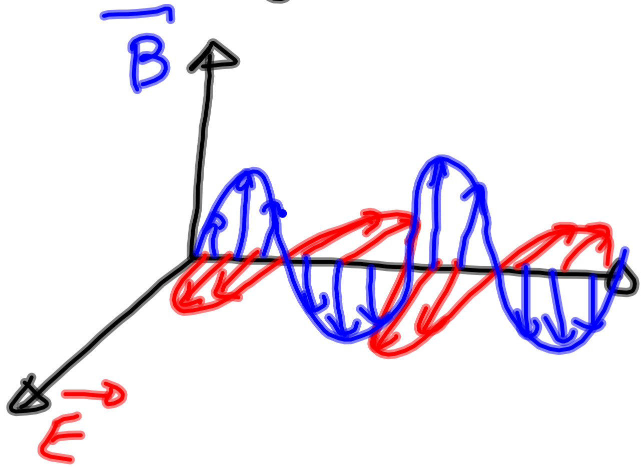
Why there is no photoelectric effect when light is red?!



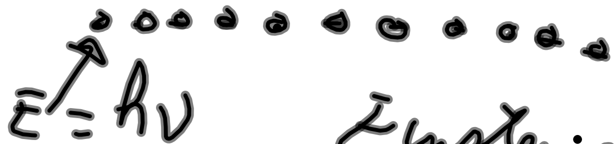
Einstein in 1905:

light = photons

energy of a photon $= h\nu$

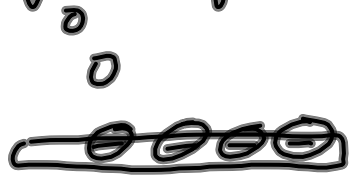


Maxwell model of light



Einstein model

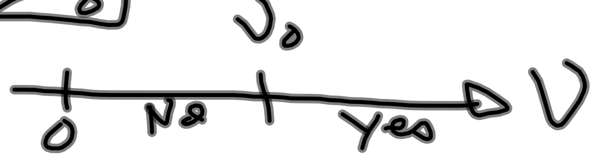
Explanation of the effect:

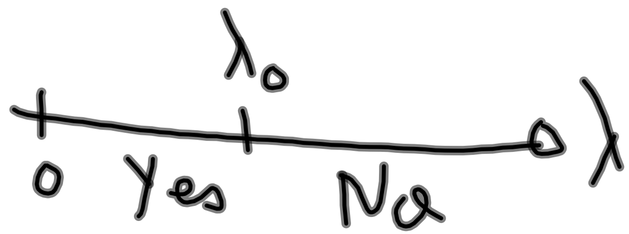


$$h\nu = W_{ext} + KE$$

So $h\nu_0 = W_{ext} = \frac{hc}{\lambda_0}$

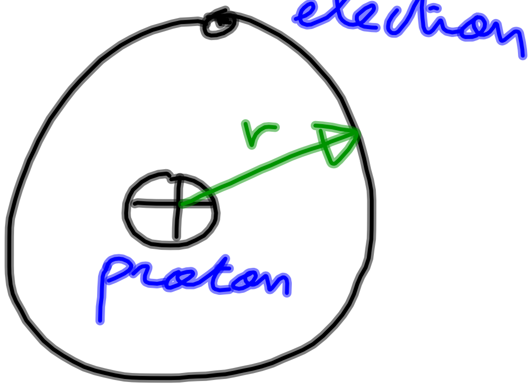
$$\boxed{\nu \geq \nu_0}$$





3- Bohr model of the hydrogen atom

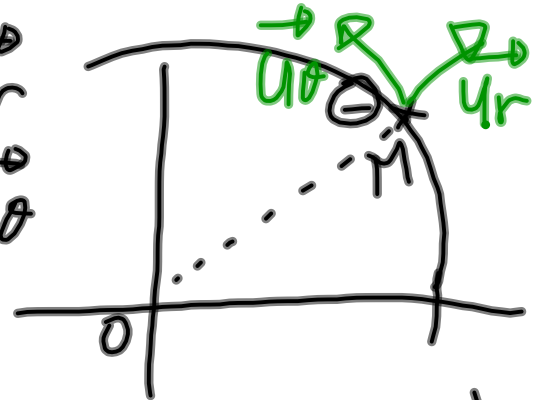
(1913)



circular orbit of radius r.

$$\vec{OM} = r\,\vec{u}_r$$

$$\vec{v} = r\omega\,\vec{u}_\theta$$



ω: angular velocity $\left(\omega = \frac{v}{r}\right)$

$$\vec{a} = -r\omega^2\,\vec{u}_r$$

$$\vec{F} = m\vec{a} = \frac{1}{4\pi\varepsilon_0}\,\frac{-e^2}{r^2}\,\vec{u}$$

$$\boxed{m r \omega^2 = \frac{1}{4\pi\varepsilon_0}\,\frac{e^2}{r^2}}$$

$$KE = \frac{1}{2} m v^2 = \frac{1}{2} m r^2 \omega^2$$

$$\boxed{KE = \frac{1}{8\pi\varepsilon_0}\frac{e^2}{r}}$$

$$PE = -\int \vec{F}\cdot d\vec{r}$$

$$= -\int \frac{e^2}{4\pi\varepsilon r^2}\, dr$$

$$\boxed{PE = -\frac{e^2}{4\pi\varepsilon_0 r}}$$

$$E = KE + PE$$

$$\boxed{E = -\frac{1}{8\pi\varepsilon_0}\frac{e^2}{r}}$$

Duality

wave-corpuscle

(De Broglie

$m\vec{v}$

$$\lambda = \frac{h}{mv}$$

$$\boxed{2\pi r = n\lambda}$$

n: principal quantum number

$n \in \mathbb{N}^*$
$(n = 1, 2, 3, \ldots)$

$$2\pi r = n\frac{h}{mv}$$

$$mvr = n\frac{h}{2\pi} = n\hbar$$

$$\boxed{L = mvr = n\hbar}$$

Bohr quantificati

L: angular momentum

$$(mvr)^2 = n^2 \hbar^2$$

$$(mr^2\omega)^2 = n^2 \hbar^2$$

$$m^2 r^4 \omega^2 = n^2 \hbar^2$$

and we have:

$$m r^2 \omega^2 = \frac{1}{4\pi\varepsilon_0} \frac{e^2}{r}$$

So:

$$m r^{\cancel{2}} = \frac{n^2 \hbar^2 4\pi\varepsilon_0 \cancel{r}}{e^2}$$

$$\boxed{r = n^2 \frac{4\pi\varepsilon_0 \hbar^2}{m e^2}}$$

$$\boxed{r_n = n^2 r_1}$$

with:

$$\boxed{r_1 = \frac{4\pi\varepsilon_0 \hbar^2}{m e^2} = 0.529 \text{ Å}}$$

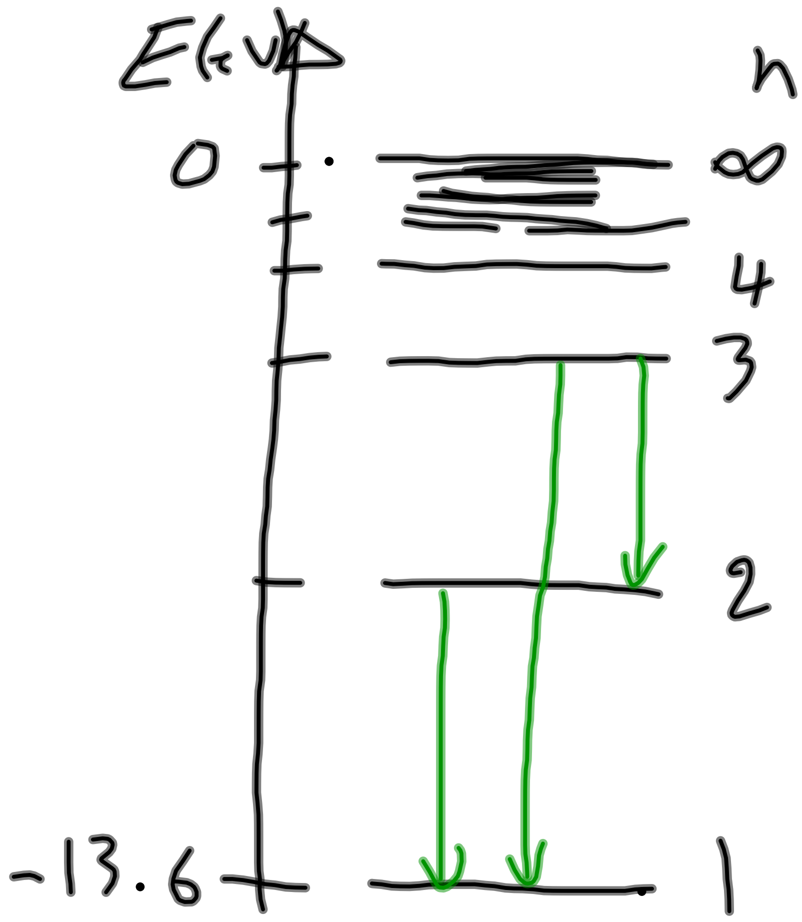
Bohr radius

and the energy:

$$E = -\frac{1}{8\pi\varepsilon_0}\,\frac{e^2}{n^2 \frac{4\pi\varepsilon \hbar^2}{me^2}}$$

$$E_1 = -\frac{me^4}{32\pi^2\varepsilon_0^2\hbar^2} = -13.6\ \text{eV}$$

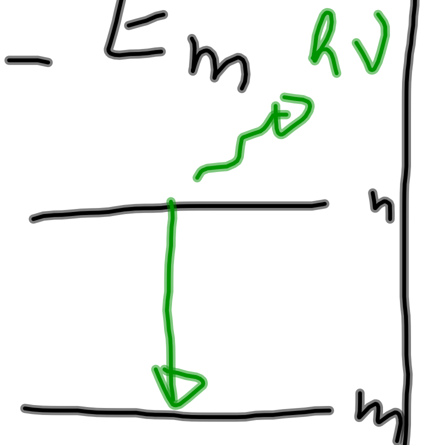
$$E_n = \frac{E_1}{n^2}$$



Energy diagram of the hydrogen

Rydberg formula

$$h\nu = E_n - E_m$$



$$\frac{hc}{\lambda} = -E_1\left(\frac{1}{m^2}-\frac{1}{n^2}\right)$$

$$\frac{1}{\lambda} = \left(-\frac{E_1}{hc}\right)\left(\frac{1}{m^2}-\frac{1}{n^2}\right)$$

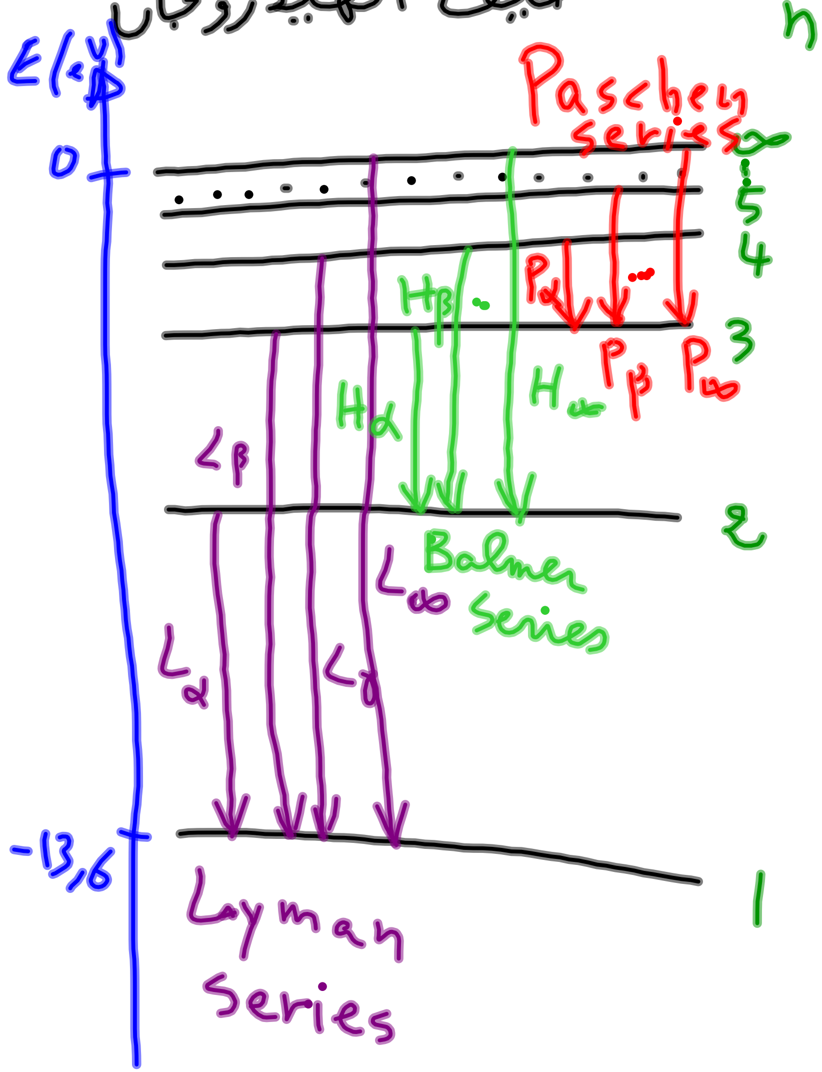
$$\boxed{\frac{1}{\lambda} = R\left(\frac{1}{m^2}-\frac{1}{n^2}\right)}$$

R: Rydberg constan

$(R = 109713\ \mathrm{cm}^{-1})$

Hydrogen spectra

طيف الهيدروجان



Ex: calculate $\lambda(H\alpha)$ in Å

$$\frac{1}{\lambda(H\alpha)} = R\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$$

$$= R\left(\frac{1}{4} - \frac{1}{9}\right)$$

$$\frac{1}{\lambda(H\alpha)} = \frac{5R}{36} \Rightarrow \boxed{\lambda(H\alpha) = \frac{36}{5R}}$$

$$R = 109737.3\ cm^{-1}$$

$$\lambda(H\alpha) = 6.561\ 10^{-5}\ cm$$

$$= 6561\ Å$$

H-W:
Calculate

1°) $\lambda(L_\beta)$ in Å

2°) $\nu(H_\gamma)$ in Hz

3°) $\lambda(P_\infty)$ in nm

Sol:

1°) $\frac{1}{\lambda(L_\beta)} = R\left(1-\frac{1}{9}\right) = \frac{8R}{9}$

$\lambda(L_\beta) = \frac{9}{8R} = 1025$ Å

2°) $\frac{1}{\lambda(H_\gamma)} = R\left(\frac{1}{4}-\frac{1}{25}\right) = \frac{21R}{100}$

So $\lambda(H_\gamma) = \frac{100}{21R} = 4339$ Å

$\left(R = 109737.3\ cm^{-1}\right)$

$\nu = \frac{c}{\lambda} = \frac{3\times10^{8}}{4339\times10^{-10}}$

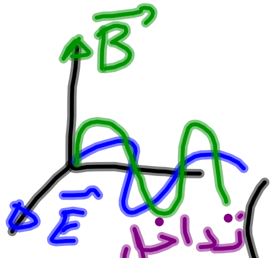
$= 6.914\times10^{14}$ Hz

3°) $\frac{1}{\lambda(P_\infty)} = \frac{R}{9} \Rightarrow \lambda(P_\infty) = \frac{9}{R} = 820.1$ n

(chap 2)

Wave particle, duality and SCHRÖDINGER equation

1- Duality of light:



e-m wave (interference- diffraction) تداخل حيود

o o o o photons

$E = h\nu$

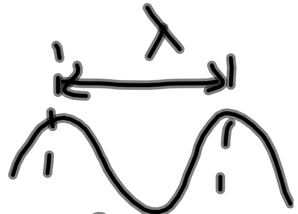
(photoelectric effect Compton effect)

2- Duality of matter

matter as corpuscules (atoms...)

matter as wave

$m\vec{v}$ we associate



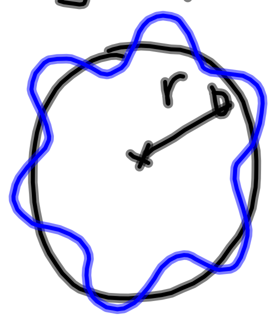
$\left(\hbar = \frac{h}{2\pi}\right)$

$$\lambda = \frac{h}{mv}$$

hyp. of L. De Broglie

Ex: for H:

$2\pi r = n\lambda$



So, the quant. of Bohr

$L = mvr = n\hbar$.

$l = mv$: ـل خم الزاوي $= n\hbar$

3. Schrödinger equation

we write a one dimension wave:

$$\boxed{y(x,t) = A\sin(kx - wt)}$$

where $k = \frac{2\pi}{\lambda}$ and $w = 2\pi\nu$

in Q.M: $E = h\nu$,

$\lambda = \frac{h}{p}$ and $y \xrightarrow{Q.M} \psi$

$$\boxed{\psi(x,t) = A\sin\left[\frac{1}{\hbar}(px - Et)\right]}$$

which equation verify ψ?

First the equation for y

$$y' = \frac{dy}{dt} = -wA\cos(kx - wt)$$

$$y'' = \frac{d^2y}{dt^2} = -w^2 A\sin(kx - wt)$$

$$= -w^2 y \quad \text{or}$$

$$\boxed{y'' + w^2 y = 0}$$

This is the equation of motion for the wave y

S̈ eq:

$$\psi(x,t) = A e^{\frac{i}{\hbar}(px - Et)}$$

$$\frac{\partial \psi}{\partial x} = \frac{ip}{\hbar}\psi$$

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{p^2}{\hbar^2}\psi$$

we have

$E = \frac{p^2}{2m} + V$ so $p^2 = 2m(E-V)$

and

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{2m}{\hbar^2}(E-V)\psi$$

or:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\psi = 0$$

in 3D:

$$\boxed{\Delta \psi + \frac{2m}{\hbar^2}(E-V)\psi = 0}$$

This is the S̈ eq.

$$\Delta \psi = \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} + \frac{\partial^2 \psi}{\partial z^2}$$

The Laplacian of ψ

we can re-write the S̈ eq as:

$$E\Psi = \underbrace{\left(-\frac{\hbar^2}{2m}\Delta + V\right)}_{\substack{H \\ \text{Hamiltonian}}}\Psi$$

The S̈ eq become:

$$\boxed{H\Psi = E\Psi}$$

(H is an operator)

Ex: $H = \begin{pmatrix} 1 & 2 \\ 8 & 1 \end{pmatrix}$

find Ψ and E of the system

Sol:

$$H\Psi = E\Psi$$

$$\begin{pmatrix} 1 & 2 \\ 8 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = E\begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{cases} a + 2b = Ea \\ 8a + b = Eb \end{cases}$$

or

$$\begin{cases} (1-E)a + 2b = 0 \\ 8a + (1-E)b = 0 \end{cases}$$

we must have

$$\begin{vmatrix} 1-E & 2 \\ 8 & 1-E \end{vmatrix} = 0$$

so $(1-E)^2 = 16$

$$\begin{cases} 1-E_1 = 4 \\ 1-E_2 = -4 \end{cases}$$

$E_1 = -3$ and $E_2 = 5$

$H\Psi_1 = E_1 \Psi_1$

$$\begin{pmatrix} 1 & 2 \\ 8 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = -3\begin{pmatrix} a \\ b \end{pmatrix}$$

$a + 2b = -3a$

$4a + 2b = 0$ or $2a + b = 0$

$$\Psi_1 = \alpha\begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

a normalized vector is

$$\psi_1 = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 \\ -2 \end{pmatrix}$$

For ψ_2, we have

$$\begin{pmatrix} 1 & 2 \\ 8 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = 5\begin{pmatrix} a \\ b \end{pmatrix}$$

$$a + 2b = 5a$$

$$b = 2a$$

$$\psi_2 = \alpha\begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

normalized ψ_2:

$$\psi_2 = \frac{1}{\sqrt{5}}\begin{pmatrix} 1 \\ 2 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} \end{pmatrix}$$

Ex: $H = \hbar\omega H_0$

with $H_0 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$

Sol: $H|\psi\rangle = E|\psi\rangle$

or $H_0|\psi\rangle = e|\psi\rangle$

with: $E = \hbar\omega e$

so

$$\begin{vmatrix} 1-e & 1 \\ 1 & 1-e \end{vmatrix} = 0$$

$(1-e)^2 = 1$

$1-e = 1$ or $1-e = -1$

$e_0 = 0$ or $e_1 = 2$

and

$$\boxed{\begin{aligned} E_0 &= \hbar\omega e_0 = 0 \\ E_1 &= \hbar\omega e_1 = 2\hbar\omega \end{aligned}}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = e\begin{pmatrix} a \\ b \end{pmatrix}$$

0 or 2

$|\psi_0\rangle$ $|\psi_1\rangle$

Chap3
operators in Q.M
Postulates of Q.M

P_1: $|\Psi\rangle$ state function

$|\Psi\rangle \in H$ (Hilbert space)
ket Ψ.

The dual is the bra Ψ. ($\langle\Psi|$)

$\langle\Psi|\Psi'\rangle \in \mathbb{R}$

P_2:

$P\,dV = |\Psi|^2 dV$
= probability in dV

So $|\Psi|^2$ = density of probability

Condition of normalization

$$\int |\Psi|^2 dV = 1$$

$$\int \Psi^*(x,y,z)\,\Psi(x,y,z)\,dV = 1$$

in 1D:

$$\int |\Psi(x)|^2 dx = \int_{-\infty}^{+\infty} |\Psi(x)|^2 dx = 1$$

$\ddot{S}$ eq:

$$\Psi(x,t) = A e^{\frac{i}{\hbar}(px - Et)}$$

$$\frac{\partial \Psi}{\partial x} = \frac{ip}{\hbar}\Psi$$

$$\frac{\partial^2 \Psi}{\partial x^2} = -\frac{p^2}{\hbar^2}\Psi$$

we have

$E = \frac{p^2}{2m} + V$ so $p^2 = 2m(E-V)$

and

$$\frac{\partial^2 \Psi}{\partial x^2} = -\frac{2m}{\hbar^2}(E-V)\Psi$$

or:

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{2m}{\hbar^2}(E-V)\Psi = 0$$

in 3D:

$$\boxed{\Delta \Psi + \frac{2m}{\hbar^2}(E-V)\Psi = 0}$$

This is the $\ddot{S}$ eq.

$$\Delta \Psi = \frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2}$$

The Laplacian of Ψ

we can re-write the S̈ eq as:

$$E\Psi = \underbrace{\left(-\frac{\hbar^2}{2m}\Delta + V\right)}_{H}\Psi$$

Hamiltonian

The S̈ eq become:

$$\boxed{H\Psi = E\Psi}$$

(H is an operator)

Ex: $H = \begin{pmatrix} 1 & 2 \\ 8 & 1 \end{pmatrix}$

find Ψ and E of the system

Sol:

$$H\Psi = E\Psi$$

$$\begin{pmatrix} 1 & 2 \\ 8 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = E\begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{cases} a + 2b = Ea \\ 8a + b = Eb \end{cases}$$

or

$$\begin{cases}(1-E)a+2b=0\\ 8a+(1-E)b=0\end{cases}$$

we must have

$$\begin{vmatrix}1-E & 2\\ 8 & 1-E\end{vmatrix}=0$$

so $(1-E)^2=16$

$$\begin{cases}1-E_1=4\\ 1-E_2=-4\end{cases}$$

$E_1=-3$ and $E_2=5$

$H\Psi_1=E_1\Psi_1$

$$\begin{pmatrix}1 & 2\\ 8 & 1\end{pmatrix}\begin{pmatrix}a\\ b\end{pmatrix}=-3\begin{pmatrix}a\\ b\end{pmatrix}$$

$a+2b=-3a$

$4a+2b=0$ or $2a+b=0$

$$\Psi_1=\alpha\begin{pmatrix}1\\ -2\end{pmatrix}$$

a normalized vector is

$$\psi_1 = \frac{1}{\sqrt{5}}\begin{pmatrix}1\\-2\end{pmatrix}$$

For ψ_2, we have

$$\begin{pmatrix}1 & 2\\8 & 1\end{pmatrix}\begin{pmatrix}a\\b\end{pmatrix} = 5\begin{pmatrix}a\\b\end{pmatrix}$$

$$a + 2b = 5a$$

$$b = 2a$$

$$\psi_2 = \alpha\begin{pmatrix}1\\2\end{pmatrix}$$

normalized ψ_2:

$$\psi_2 = \frac{1}{\sqrt{5}}\begin{pmatrix}1\\2\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{5}}\\ \frac{2}{\sqrt{5}}\end{pmatrix}$$

Chap3
operators in Q.M
Postulates of Q.M

P_1: $|\Psi\rangle$ state function

$|\Psi\rangle \in H$ (Hilbert space)

ket Ψ.

The dual is the bra Ψ. ($\langle\Psi|$)

$\langle\Psi|\Psi'\rangle \in \mathbb{R}$

P_2:

$P\,dV = |\Psi|^2 dV$

= probability in dV

So $|\Psi|^2$ = density of probability

Condition of normalization

$$\int |\Psi|^2 dV = 1$$

$$\int \Psi^*(x,y,z)\Psi(x,y,z)\,dV = 1$$

in 1D:

$$\int |\Psi(x)|^2 dx = \int_{-\infty}^{+\infty} |\Psi(x)|^2 dx = 1$$

Ex: Suppose that the particle is in $[0, a]$

Calculate the normalization constante A.

$$\Psi(x) = A \sin(kx)$$

Sol:

$$\int_0^a \sin^2(kx)\,dx = \frac{1}{A^2}$$

P_3:

$\mathcal{A}$: observable

A: operator

$$A|\Psi_n\rangle = a_n|\Psi_n\rangle$$

a_n: eigenvalues
(The only values that can be observed)

Ex:
Total energy $\mathcal{E}$

the corresponding operator is the Hamiltonian H.

E_n: eigenvalues

we write:

$$H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

Observable		Operator
Name	Symbol	
Position	x, y, z	X, Y, Z
Momentum	p_x, p_y, p_z	$P_x = \frac{\hbar}{i}\frac{\partial}{\partial x}$; $P_y = \frac{\hbar}{i}\frac{\partial}{\partial y}$; $P_z = \frac{\hbar}{i}\frac{\partial}{\partial z}$
Kinetic energy	$KE = T$	$-\frac{\hbar^2}{2m}\Delta$
Potential energy	$V(r)$	V
Total energy	E	$H = -\frac{\hbar^2}{2m}\Delta + V$

P4: The time-dependent S̈ equation is:

$$H|\psi\rangle = i\hbar \frac{\partial}{\partial t}|\psi\rangle$$

Chap 4

ONE DIMENSIONAL POTENTIALS

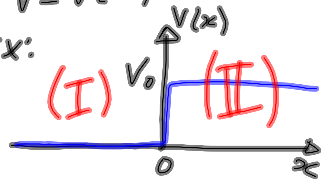
$$V = V(x)$$

Chap 4

ONE DIMENTIONAL POTENTIALS

$V = V(x)$

Ex:



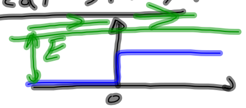
The potential "Step" خطوة

$V(x<0) = 0$ (region I)

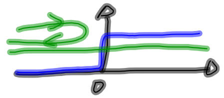
$V(x>0) = V_0$ (region II)

Classical study:

- $E > V_0$



- $E < V_0$



$$v_{I} = \sqrt{\frac{2E}{m}}$$

$$v_{II} = \sqrt{\frac{2(E-V_0)}{m}}$$

(I) → (II) deceleration تباطؤ

$v \downarrow$

Quantum study

1. Region I $(V(x<0)=0)$

$\ddot{S}$ eq:

$$\psi'' + k^2\psi = 0$$

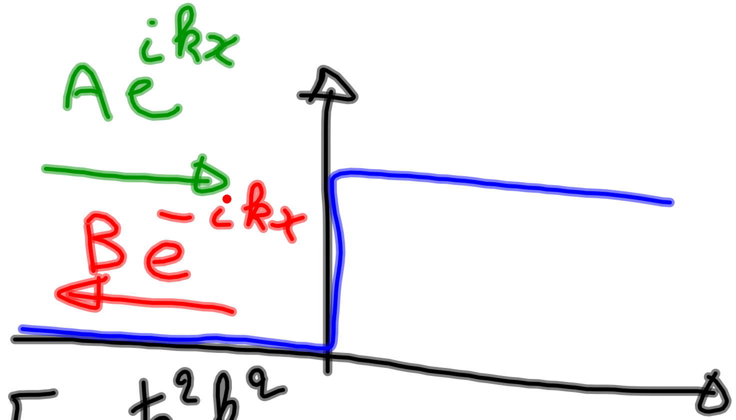
where $k^2 = \frac{2mE}{\hbar^2}$

The solution is:

$$\psi_I(x) = Ae^{ikx} + Be^{-ikx}$$

Ae^{ikx} represent an incident wave ساقطة

Be^{-ikx} represent a reflected wave منعكسة



$$E = \frac{\hbar^2 k^2}{2m} = \frac{1}{2} m v_I^2$$

So $\boxed{v_I = \frac{\hbar k}{m}}$

2) Region II (Case $E > V_0$)

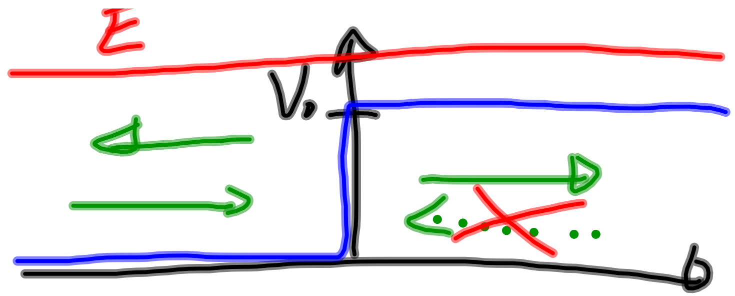
$$\psi'' + \frac{2m}{\hbar^2}(E - V_0)\psi = 0$$

$$q^2 = \frac{2m}{\hbar^2}(E - V_0)$$

So: $\psi'' + q^2\psi = 0$

The solution is:

$$\psi_{II}(x>0) = Ce^{iqx} + \cancel{De^{-iqx}}$$



D = 0 for physical reasons (No wave coming from ∞)

$$KE = E - V_0 = \frac{q^2\hbar^2}{2m} = \frac{1}{2}mv^2$$

$$\boxed{v_{II} = \frac{\hbar q}{m}}$$

Physically, we can put

$$\psi(x<0) = e^{ikx} + Re^{-ikx}$$

$$\psi(x>0) = Te^{iqx}$$

ψ, cont in 0: $1+R=T$

ψ' cont in 0:

$$ik(1-R) = iqT$$

So:

$$1+R=T$$

$$1-R=\frac{q}{k}T$$

$$2 = \left(1+\frac{q}{k}\right)T$$

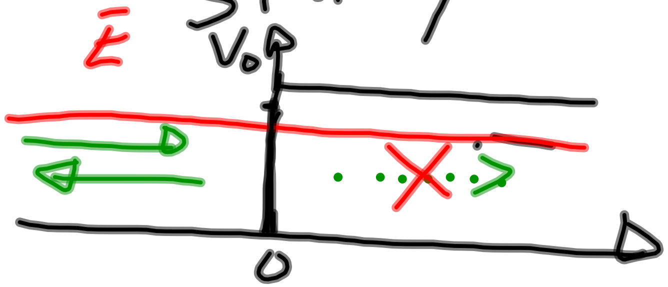
$$1 = \frac{k+q}{2k}T$$

$$\boxed{T = \frac{2k}{k+q}}$$

$$R = \frac{k-q}{k+q}$$

Study of the case where $\underline{\underline{E<V_0}}$

1) Classical study



No transmission

All the wave will be reflected.

2) Quantum study

* Region I: $\psi'' + k^2\psi = 0$

$$\psi(x<0) = e^{ikx} + Re^{-ikx}$$

* Region II:

$$\psi'' + \frac{2m}{\hbar^2}(E-V_0)\psi = 0$$

$$\psi'' - \alpha^2\psi = 0 \quad \text{where}$$

$$\alpha^2 = \frac{2m(V_0-E)}{\hbar^2}$$

$$\psi_{II}(x>0) = \cancel{Ce^{\alpha x}} + De^{-\alpha x}$$

For mathematical reasons $C=0$:

$$\lim_{x\to\infty}\Psi(x)=0$$

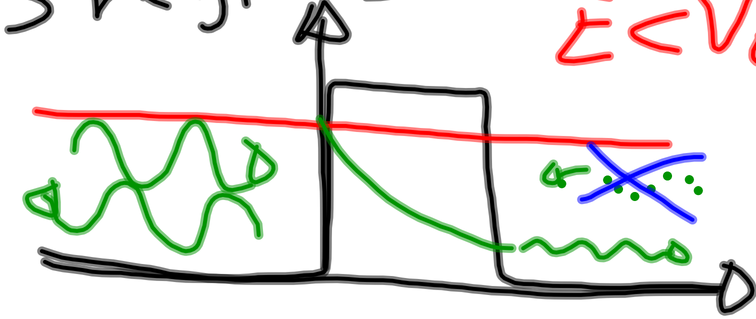
So $C=0$. and

$$\Psi_{II}(x>0)=De^{-\alpha x}$$

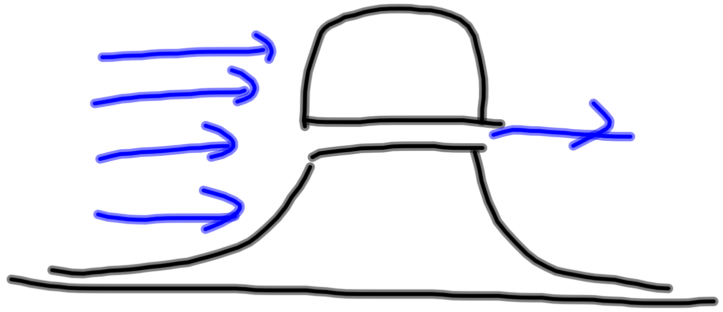
Evanecenting wave

Suppose that we have 3 regions

$E<V_0$

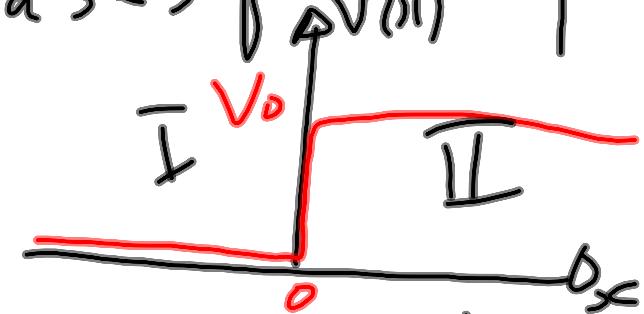


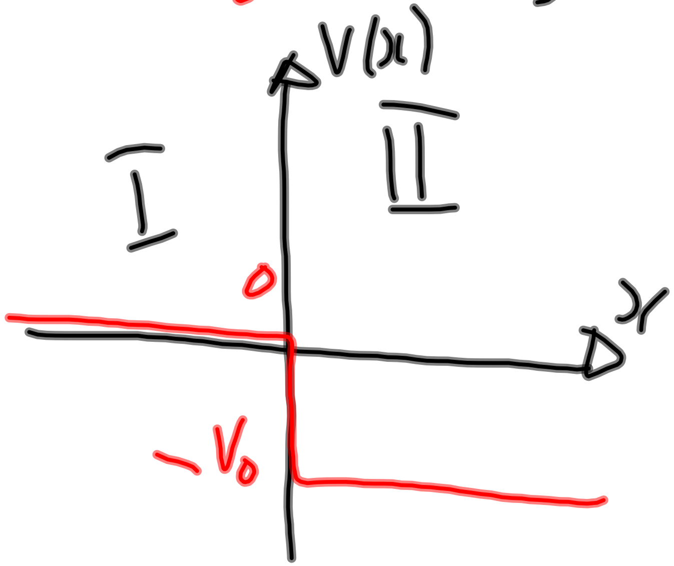
we have transmission even when $E<V_0$.
Classecly there is NO transmission.
But with Q.M. there is a probability for transmission.
This is the:
Tunneling effect



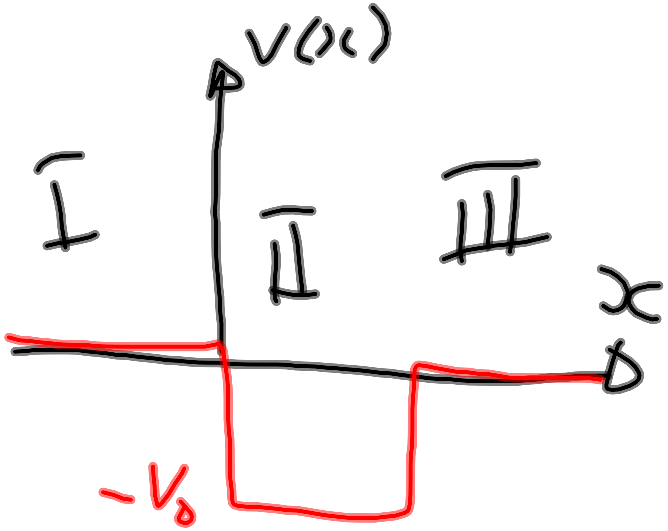
ظاهرة النفق

Some particular cases for 1D problems:



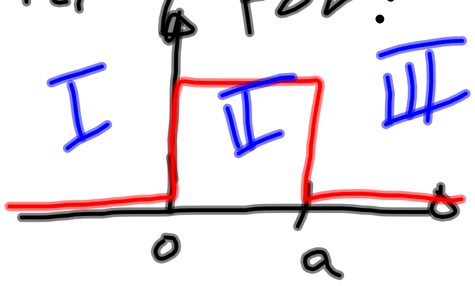


Barrier



Well

Ex: find ψ for: $(E < V_0)$



$V_I \equiv V(x<0) = 0$

$V_{II} = V(0<x<a) = V_0$

$V_{III} = V(x>a) = 0$

region I:

$$\psi'' + k^2\psi = 0$$

$$k^2 = \frac{2mE}{\hbar^2}$$

$$\psi_I(x) = A e^{ikx} + B e^{-ikx}$$

region II:

$$\psi'' - \alpha^2\psi = 0$$

$$\alpha^2 = \frac{2m(V_0 - E)}{\hbar^2}$$

$$\psi_{II}(x) = C e^{\alpha x} + D e^{-\alpha x}$$

region III: $\psi'' + k^2\psi =$

$$\psi_{III}(x) = E e^{ikx} + \cancel{F e^{-ik}}$$

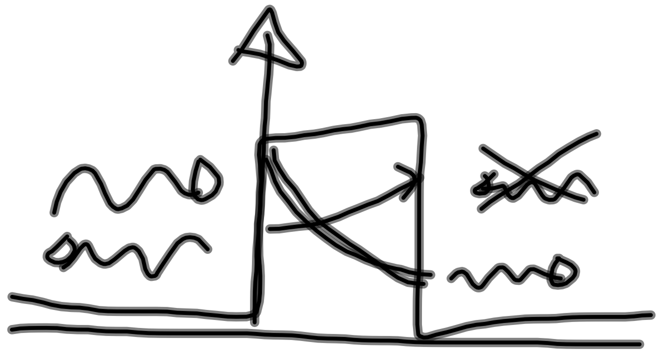
No reflection so F =

How to find A, B, C, D, E?

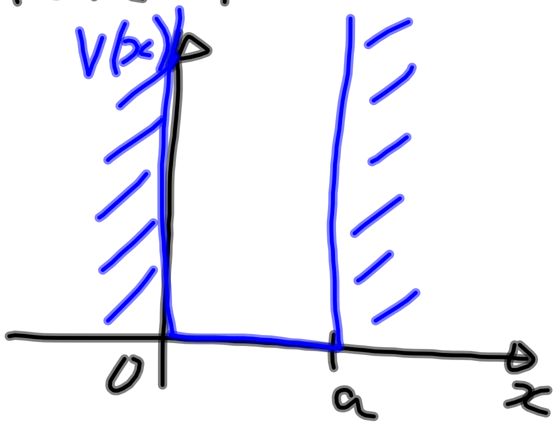
Ψ cont. in 0:

$\Psi_I(0) = \Psi_{II}(0)$

$A + B = C + D$



Particle in a box:



$V(x<0) = V(x>a) = +\infty$

$V(0<x<a) = 0$

For $x<0$ and $x>a$ the potential is infinity

So the particle can not be there.

The wave function is zero:

$\Psi(x<0) = \Psi(x>a) = 0$

$$\boxed{V=\infty \Rightarrow \Psi = 0}$$

$\underline{0 \leq x \leq a}$:

$$\Psi'' + k^2 \Psi = 0$$

with $k^2 = \frac{2mE}{\hbar^2}$

So:

$$\Psi(0 \leq x \leq a) = A \sin kx + B \cos k$$

* we have $\psi(0) = 0$ so

$$A \cancel{\sin 0} + B \underbrace{\cos 0}_{1} = \psi(0) = 0$$

$\boxed{B = 0}$ and

$$\psi(0 \leq x \leq a) = A \sin kx$$

* we have $\psi(a) = 0$ so

$$\sin ka = 0$$

$$ka = n\pi$$

$$k^2 a^2 = n^2 \pi^2$$

$$\frac{2mE}{\hbar^2} a^2 = n^2 \pi^2 \text{ so}$$

$$\boxed{E_n = \frac{n^2 \pi^2 \hbar^2}{2 m a^2}}$$

n: quantum number

$n = 1, 2, 3, \ldots$ $n \in \mathbb{N}^*$

calculations of the amplitude A

we have $\int \psi^2(x)\,dx = 1$

$\Psi(x<0) = \Psi(x>a) = 0$ so

$$\int_{-\infty}^{+\infty} \Psi(x)^2 dx = \int_0^a \Psi^2(x)\,dx = 1$$

$$A^2 \int_0^a \sin^2 kx\,dx = 1$$

$$I = \int_0^a \sin^2 kx\,dx$$

$$J = \int_0^a \cos^2 kx\,dx = I$$

$$I + J = 2I = \int_0^a 1\,dx = a$$

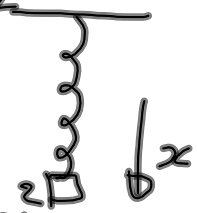
$$I = \frac{a}{2}$$

$A^2 \times \frac{a}{2} = 1$ So $\boxed{A = \sqrt{\frac{2}{a}}}$

chap 5

$\boxed{\text{HARMONIC OSCILLATOR}}$

Classical study:



$$F = -kx$$

$$P.E = -\int F\,dx = \frac{1}{2}kx^2$$

So $P.E = \boxed{V(x) = \frac{1}{2}kx^2}$

and

$$E = \frac{p^2}{2m} + \frac{1}{2}kx^2$$

Quantum study:

The Hamiltonian of the system is:

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + \frac{1}{2}kx^2$$

The $\ddot{S}$ eq become:

$$\psi''(x) + \frac{2m}{\hbar^2}\left(E - \frac{1}{2}kx^2\right)\psi = 0$$

we put:

$$\omega = \sqrt{\frac{k}{m}}\ ;\ \varepsilon = \frac{2E}{\hbar\omega}$$

and $y = \sqrt{\frac{m\omega}{\hbar}}\,x$

$$\psi(x) = \sqrt{\frac{m\omega}{\hbar}}\,u$$

The S̈ eq. become:

$$u'' + (\varepsilon - y^2)u = 0$$

all quantities are dimensionless

For $y \to \infty$; the S̈ eq:

$$u'' - y^2 u = 0$$

The solution is

$$u = H e^{-\frac{y^2}{2}}$$

H constant

Now, for any y, we put:

$$u = H(y)\, e^{-\frac{y^2}{2}}$$

H verify:

$$H'' - 2yH' - H = 0$$

H is the Hermite polynomials $H(y) = H_n$

n is the degres of the polyn.