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Parameters Estimation for a Linear Exponential Distribution Based on Grouped Data

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Abstract

This paper presents estimations of the parameters included in linear exponential distribution, based on grouped and censored data. The methods of maximum likelihood, regression and Bayes are discussed. The maximum likelihood method does not provide closed forms for the estimations, thus numerical procedure is used. The regression estimates of the parameters are used as guess values to get the maximum likelihood estimates of the parameters. Reliability measures for the linear exponential distribution are calculated. Testing the goodness of fit for the exponential distribution against the linear exponential distribution is discussed. Relevant reliability measures of the linear exponential distribution are also evaluated. A set of real data is employed to illustrate the results given in this paper.

Mathematics Subject Classification: 60E05, 62G05

Keywords: Lifetime data, point estimate, interval estimate, maximum likelihood estimate, regression, Bayes method

1 Introduction

ACRONYMS¹

cdf	cumulative distribution function
pdf	probability density function
sf	survival function
jpdf	joint probability density function
MLE	maximum likelihood estimate
LSRE	least square estimate
BE	Bayes estimate
C.I.	confidence interval
LED(α, β)	linear exponential distribution with parameters α, β
ED(α)	exponential distribution with parameter α
RD(β)	Rayleigh distribution with parameter β
$\hat{\alpha}_M, \hat{\beta}_M$	MLE of the parameter α, β
$\hat{\alpha}_B, \hat{\beta}_B$	BE of the parameter α, β
$\hat{\alpha}_L, \hat{\beta}_L$	LSRE of the parameter α, β
w.r.t.	with respect to

Tests, in reliability analysis, can be performed for units or systems either continuously or intermittently inspection for failure. If the life test can be based on continuous inspection throughout the experiment, then the sample life lengths, i.e. data, is said to be complete. In other hand, the data from intermittent inspections are known by grouped data. The number of failures expresses this data in each inspection interval is used more frequently than the first one, since it, generally, costs less and requires fewer efforts. Moreover, intermittent inspections is some times the only possible or available for units or their system, Ehrenfeld [5].

It is known that the exponential distribution (ED) is the most frequently used distribution in reliability theory and applications. The mean and its confidence limit of this distribution have been estimated based on group and censored data by Seo and Yum [13] and Chen and Mi [4], respectively.

The LED(α, β) has the following cdf

$$F(x; \alpha, \beta) = 1 - \exp \left\{ -\alpha x - \frac{\beta}{2} x^2 \right\}, \quad x > 0, \alpha, \beta > 0. \quad (1.1)$$

The corresponding sf is

$$\bar{F}(x; \alpha, \beta) = \exp \left\{ -\alpha x - \frac{\beta}{2} x^2 \right\}, \quad (1.2)$$

¹The singular and plural of an acronym are always spelled the same.

the pdf is

$$f(x; \alpha, \beta) = (\alpha + \beta x) \exp \left\{ -\alpha x - \frac{\beta}{2} x^2 \right\}, \quad (1.3)$$

the hazard rate function is

$$h(x; \alpha, \beta) = \alpha + \beta x, \quad (1.4)$$

and the mean time to failure is

$$\text{MTTF} = \frac{K(\alpha^2/(2\beta))}{\sqrt{\beta/2}} - \frac{\alpha}{\beta}, \quad (1.5)$$

where

$$K(\nu) = \exp\{\nu\} \int_{\nu}^{\infty} x^{1/2} \exp\{-x\} dx.$$

The LED(α, β) generalizes an ED(α) when $\beta = 0$, and RD(β) when $\alpha = 0$. Also, LED(α, β) can be considered as a mixture of an exponential and Rayleigh distributions.

The LED has many applications in applied statistics and reliability analysis. Broadbent [2], uses the LED to describe the service of milk bottles that are filled in a dairy, circulated to customers, and returned empty to the dairy. The Linear exponential model was also used by Carbone et al. [3] to study the survival pattern of patients with plasmacytic myeloma. The type-2 censored data is used by Bain [1] to discuss the least square estimates of the parameters α and β and by Pandey et al. [11] to study the Bayes estimators of (α, β).

In this paper, we use the grouped and censored data to estimate the parameters of LED(α, β). We provide the MLE, least square estimates and Bayes estimates of the unknown parameters. Also, estimates of some reliability measures for the LED are provided. The MLE of α and β do not have closed forms. Thus, an iterative procedure is needed. The LSRE can be used as guess values for the iterative procedure to get the MLE. In fact numerical iterative procedure has been used for the ED by Ehrenfeld [5] and Nelson [10] among many others.

The rest of the paper is organized as follows. In section 2, we present the notations and assumptions used throughout this paper. MLE of the parameters are discussed in section 3. Also, the asymptotic confidence intervals of the unknown parameters are given in section 3. Section 4 presents the LSRE of the parameters α and β . Bayes estimators of α and β are discussed in section 5. Testing for the goodness of fit of ED against the LED based on the estimated likelihood ratio test statistics is discussed in section 6. A set of real data is applied and a conclusion is drawn in section 7.

2 Model and assumptions

Assumptions:

1. N independent and identical experimental units are put on a life test at time zero.
2. The lifetime of each unit follows a $LED(\alpha, \beta)$ with cdf given by (1.1).
3. The inspection times $0 < t_1 < t_2 < \dots < t_k < \infty$ are predetermined.
4. The test is terminated at the predetermined time t_k . That is, the data is of Type-I censoring.
5. $t_0 = 0$ and $t_{k+1} = \infty$.
6. The number of failures in $(t_i, t_{i+1}]$ are recorded.

The data collected from the above test scheme consist of number of failures n_i in the interval $(t_{i-1}, t_i]$, $i = 1, 2, \dots, k$ and the number of units tested without failing up to t_k , n_{k+1} (censored units).

3 Maximum likelihood estimators

Based on the data collected in the previous section, the likelihood function takes the following form

$$L = C \prod_{i=1}^k [P\{t_{i-1} < T \leq t_i\}]^{n_i} [P\{T > t_k\}]^{n_{k+1}} \quad (3.1)$$

where $C = \frac{n!}{\prod_{\ell=1}^{k+1} n_{\ell}!}$ is a constant with respect to the parameters α and β .

But

$$P\{t_{i-1} < T \leq t_i\} = F(t_i) - F(t_{i-1})$$

and

$$P\{T > t_k\} = \bar{F}(t_k)$$

Then based on assumption 2, we have

$$L = C \prod_{i=1}^{k+1} P_i^{n_i} \quad (3.2)$$

where

$$P_i = \bar{F}(t_{i-1}) - \bar{F}(t_i), i = 1, 2, \dots, k + 1, \tag{3.3}$$

It notable to recall that $\bar{F}(t_0) = 1$ and $\bar{F}(t_{k+1}) = 0$.

The log-likelihood function becomes

$$\mathcal{L} = \ln C - n_{k+1} \left[\alpha t_k + \frac{\beta}{2} t_k^2 \right] + \sum_{i=1}^k n_i \ln \left[e^{-\alpha t_{i-1} - \frac{\beta}{2} t_{i-1}^2} - e^{-\alpha t_i - \frac{\beta}{2} t_i^2} \right], \tag{3.4}$$

The partial derivatives of \mathcal{L} are

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \alpha} &= -t_k n_{k+1} + \sum_{i=1}^k n_i \frac{t_i e^{-\alpha t_i - \frac{\beta}{2} t_i^2} - t_{i-1} e^{-\alpha t_{i-1} - \frac{\beta}{2} t_{i-1}^2}}{\left[e^{-\alpha t_{i-1} - \frac{\beta}{2} t_{i-1}^2} - e^{-\alpha t_i - \frac{\beta}{2} t_i^2} \right]^2}, \\ \frac{\partial \mathcal{L}}{\partial \beta} &= -\frac{1}{2} t_k^2 n_{k+1} + \frac{1}{2} \sum_{i=1}^k n_i \frac{t_i^2 e^{-\alpha t_i - \frac{\beta}{2} t_i^2} - t_{i-1}^2 e^{-\alpha t_{i-1} - \frac{\beta}{2} t_{i-1}^2}}{\left[e^{-\alpha t_{i-1} - \frac{\beta}{2} t_{i-1}^2} - e^{-\alpha t_i - \frac{\beta}{2} t_i^2} \right]^2}. \end{aligned} \tag{3.5}$$

Setting $\frac{\partial \mathcal{L}}{\partial \alpha} = 0$ and $\frac{\partial \mathcal{L}}{\partial \beta} = 0$, we get the likelihood equations, which should be solved to get the MLE of the parameters α and β . As it seems the likelihood equations have no closed form solution in α and β . Therefore a numerical technique method should be used to get the solution.

It is worthwhile to mention here that, the following models can be derived as special cases from the model discussed here:

1. The exponential distribution model, studied by Seo and Yum [13]. Setting $\beta = 0$, the LED model reduces to exponential distribution model with parameter α , and (3.5) reduces to

$$0 = -t_k n_{k+1} + \sum_{i=1}^k n_i \frac{t_i e^{-\alpha t_i} - t_{i-1} e^{-\alpha t_{i-1}}}{\left[e^{-\alpha t_{i-1}} - e^{-\alpha t_i} \right]^2}. \tag{3.6}$$

the MLE of α can be obtained by solving (3.6) w.r.t. α .

2. The Rayleigh distribution model. Setting $\alpha = 0$, the LED model reduces to Rayleigh distribution model with parameter β , and (3.5) reduces to

$$0 = -t_k^2 n_{k+1} + \sum_{i=1}^k n_i \frac{t_i^2 e^{-\frac{\beta}{2} t_i^2} - t_{i-1}^2 e^{-\frac{\beta}{2} t_{i-1}^2}}{\left[e^{-\frac{\beta}{2} t_{i-1}^2} - e^{-\frac{\beta}{2} t_i^2} \right]^2}. \tag{3.7}$$

the MLE of β can be obtained by solving (3.7) w.r.t. β .

Asymptotic confidence bounds: Since the MLE of the element of the vector of unknown parameters $\theta = (\alpha, \beta)$, are not obtained in closed forms, then it is not possible to derive the exact distributions of the MLE of these parameters. Thus we derive approximate confidence intervals of the parameters based on the asymptotic distributions of the MLE of the parameters. It is known that the asymptotic distribution of the MLE $\hat{\theta}$ is given by, see Miller (1981),

$$\left((\hat{\alpha}_1 - \alpha), (\hat{\beta}_1 - \beta) \right) \rightarrow N_2(0, \mathbf{I}^{-1}(\boldsymbol{\alpha}, \boldsymbol{\beta})) \quad (3.8)$$

where $\mathbf{I}^{-1}(\boldsymbol{\alpha}, \boldsymbol{\beta})$ is the variance covariance matrix of the unknown parameters $\theta = (\alpha, \beta)$. The elements of the 2×2 matrix \mathbf{I}^{-1} , $I_{ij}(\alpha, \beta)$, $i, j = 1, 2$, can be approximated by $I_{ij}(\alpha, \hat{\beta})$, where

$$I_{ij}(\theta) = - \left. \frac{\partial^2 \mathcal{L}(\theta)}{\partial \theta_i \partial \theta_j} \right|_{\theta=\hat{\theta}} \quad (3.9)$$

From (3.4), we get the following

$$\frac{\partial^2 \mathcal{L}}{\partial \alpha^2} = \sum_{i=1}^{k+1} n_i \frac{P_i P_{i,\alpha^2} - [P_{i,\alpha}]^2}{P_i^2}, \quad (3.10)$$

$$\frac{\partial^2 \mathcal{L}}{\partial \beta^2} = \sum_{i=1}^{k+1} n_i \frac{P_i P_{i,\beta^2} - P_{i,\alpha} P_{i,\beta}}{P_i^2}, \quad (3.11)$$

$$\frac{\partial^2 \mathcal{L}}{\partial \alpha \partial \beta} = \sum_{i=1}^{k+1} n_i \frac{P_i P_{i,\alpha\beta} - P_{i,\alpha} P_{i,\beta}}{P_i^2}, \quad (3.12)$$

where

$$P_i = S_{i-1} - S_i, \quad i = 1, 2, \dots, k+1,$$

$S_0 = 1$, $S_{k+1} = 0$, and for $i = 2, 3, \dots, k$, $S_i = \bar{F}(t_i) = \exp \left\{ -\alpha t_i - \frac{\beta}{2} t_i^2 \right\}$,

$$P_{i,\alpha} = -t_{i-1} S_{i-1} + t_i S_i,$$

$$P_{i,\alpha^2} = t_{i-1}^2 S_{i-1} - t_i^2 S_i,$$

$$P_{i,\beta} = -\frac{1}{2} t_{i-1}^2 S_{i-1} + \frac{1}{2} t_i^2 S_i,$$

$$P_{i,\beta^2} = \frac{1}{4} t_{i-1}^4 S_{i-1} - \frac{1}{4} t_i^4 S_i,$$

$$P_{i,\alpha\beta} = \frac{1}{2} t_{i-1}^3 S_{i-1} - \frac{1}{2} t_i^3 S_i.$$

Therefore, the approximate $100(1 - \gamma)\%$ two sided confidence intervals for α and β are, respectively, given by

$$\hat{\boldsymbol{\alpha}} \pm Z_{\gamma/2} \sqrt{\mathbf{I}_{11}^{-1}(\hat{\boldsymbol{\alpha}})}, \quad \hat{\boldsymbol{\beta}} \pm Z_{\gamma/2} \sqrt{\mathbf{I}_{22}^{-1}(\hat{\boldsymbol{\beta}})}$$

Here, $Z_{\gamma/2}$ is the upper $(\gamma/2)$ th percentile of a standard normal distribution.

4 Regression estimators

Multiple regression procedure is used by Pandey et al. (1993) to derive the regression estimations (or least square estimations) of the parameters α and β , based on traditional Type-II censoring data. This procedure can be modified for grouped data discussed here by computing the empirical sf, see [7],

$$\hat{F}(x_{(i)}) = \frac{m_i}{N}, \quad i = 1, 2, \dots, k, \quad (4.1)$$

where $m_i = N - \sum_{\ell=1}^i n_\ell$ is the number of surviving units at the inspection time t_i .

The LSRE of the parameters α and β , can be derived by minimizing the following function with respect to α and β

$$Q = \sum_{i=1}^k \left[y_i - \alpha t_i - \frac{1}{2} \beta t_i^2 \right]^2, \quad (4.2)$$

where $y_i = \ln N - \ln m_i$.

By minimizing Q , we get the LSRE of α and β as

$$\hat{\alpha}_L = \xi_4 K_1 - \xi_3 K_2 \quad \text{and} \quad \hat{\beta}_L = 2 [\xi_2 K_2 - \xi_3 K_1], \quad (4.3)$$

where

$$\xi_j = \frac{\zeta_j}{\zeta_2 \zeta_4 - \zeta_3^2}, \quad \zeta_j = \sum_{i=1}^k t_i^j, \quad K_j = \sum_{i=1}^k t_i^j y_i.$$

5 Bayes estimators

To derive the Bayes estimators for the parameters α and β , we need the following additional assumptions:

1. The parameters α and β are treated as random variables having the following prior jpdf

$$\pi(\alpha, \beta) = a b \exp\{-a\alpha - b\beta\}, \quad \alpha, \beta > 0 \quad (5.1)$$

where a, b are known positive constants.

2. The loss function is quadratic. That is,

$$l\left((\alpha, \beta), (\hat{\alpha}_B, \hat{\beta}_B)\right) = \kappa_1 (\alpha - \hat{\alpha}_B)^2 + \kappa_2 (\beta - \hat{\beta}_B)^2, \quad \kappa_1, \kappa_2 > 0. \quad (5.2)$$

Using the binomial expansion, one can write (3.2) as

$$L = C \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} \cdots \sum_{i_k=0}^{n_k} \mathcal{C}_{i_1, i_2, \dots, i_k}^{n_1, n_2, \dots, n_k} e^{-\alpha T_1^{(i_1, i_2, \dots, i_k)} - \beta T_2^{(i_1, i_2, \dots, i_k)}}, \quad (5.3)$$

where

$$\begin{aligned} \mathcal{C}_{i_1, i_2, \dots, i_k}^{n_1, n_2, \dots, n_k} &= (-1)^{\sum_{\ell=1}^k i_\ell} \binom{n_1}{i_1} \binom{n_2}{i_2} \cdots \binom{n_k}{i_k}, \\ T_1^{(i_1, i_2, \dots, i_k)} &= n_{k+1} t_k + i_1 t_1 + \sum_{j=2}^k [i_j t_j + (n_j - i_j) t_{j-1}], \\ 2T_2^{(i_1, i_2, \dots, i_k)} &= n_{k+1} t_k^2 + i_1 t_1^2 + \sum_{j=2}^k [i_j t_j^2 + (n_j - i_j) t_{j-1}^2]. \end{aligned}$$

For simplicity, we will use T_1 , T_2 instead of $T_1^{(i_1, i_2, \dots, i_k)}$, $T_2^{(i_1, i_2, \dots, i_k)}$, respectively.

The posterior jpdf of (α, β) , when the prior jpdf is (5.1), is

$$\pi^*(\alpha, \beta | \mathbf{t}) = \frac{1}{D} \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} \cdots \sum_{i_k=0}^{n_k} \mathcal{C}_{i_1, i_2, \dots, i_k}^{n_1, n_2, \dots, n_k} e^{-\alpha(a+T_1) - \beta(b+T_2)}, \quad \alpha, \beta > 0, \quad (5.4)$$

here D is the normalizing constant, given by

$$D = \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} \cdots \sum_{i_k=0}^{n_k} \frac{\mathcal{C}_{i_1, i_2, \dots, i_k}^{n_1, n_2, \dots, n_k}}{(a + T_1)(b + T_2)}. \quad (5.5)$$

The Bayes estimators for the parameters α , β , under the squared error loss, can be derived as in the following form, Lehman [6],

$$\hat{\alpha}_B = \frac{D\alpha}{D} \text{ and } \hat{\beta}_B = \frac{D\beta}{D}, \quad (5.6)$$

where

$$\begin{aligned} D\alpha &= \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} \cdots \sum_{i_k=0}^{n_k} \frac{\mathcal{C}_{i_1, i_2, \dots, i_k}^{n_1, n_2, \dots, n_k}}{(a + T_1)^2 (b + T_2)}, \\ D\beta &= \sum_{i_1=0}^{n_1} \sum_{i_2=0}^{n_2} \cdots \sum_{i_k=0}^{n_k} \frac{\mathcal{C}_{i_1, i_2, \dots, i_k}^{n_1, n_2, \dots, n_k}}{(a + T_1)(b + T_2)^2}. \end{aligned}$$

6 Goodness of fit

The problem of testing goodness-of-fit of an exponential distribution model against the unrestricted class of alternative is complex. However, by restricting the alternative to a exponential exponential distribution, we can use the usual likelihood ratio test statistic [12] to test the adequacy of an exponential distribution. The following are the null and the alternative hypotheses, respectively,

$$H_0 : \beta = 0, \text{ exponential distribution,}$$

$$H_1 : \beta \neq 0, \text{ linear exponential distribution.}$$

In terms of the MLE, the likelihood ration test statistic for testing H_0 against H_1 is

$$\Lambda = \frac{L(\alpha, \beta = 0)}{L(\alpha, \beta)}. \quad (6.1)$$

Under the null hypothesis, $X_L = -2 \ln(\Lambda) = 2(\mathcal{L}_{LE} - \mathcal{L}_E)$ follows a χ^2 distribution with 1 degree of freedom. Here \mathcal{L}_E and \mathcal{L}_{LE} are the log-likelihood functions under H_0 and H_1 , respectively, after replacing the unknown parameters with their MLE.

7 Data analysis

Using the set of real data presented in Nelson (1982), which is a set of cracking data on 167 independent and identically parts in a machine. The test duration was 63.48 months and 8 unequally spaced inspections were conducted to obtain the number of cracking parts in each interval. The data were

$$(t_1, \dots, t_8) = (6.12, 19.92, 29.64, 35.40, 39.72, 45.24, 52.32, 63.48)$$

and

$$(n_1, \dots, n_9) = (5, 16, 12, 18, 18, 2, 6, 17, 73)$$

Assuming the ED(α) (or under H_0 : $\beta = 0$), the MLE of α and MTTF are obtained as

$$\hat{\alpha} = 1.2097 \times 10^{-2}, \quad \widehat{\text{MTTF}} = 82.6655.$$

The corresponding log-likelihood function is $\mathcal{L}_{ED} = -316.6705$.

Assuming the LED(α, β), the MLE of the parameters α and β are obtained as

$$\hat{\alpha}_M = 4.5273 \times 10^{-3}, \text{ and } \hat{\beta}_M = 2.7688 \times 10^{-4}$$

The LSRE of the parameters α and β are

$$\hat{\alpha}_L = 7.739 \times 10^{-4}, \text{ and } \hat{\beta} = 1.298 \times 10^{-3}$$

The MLE of the MTTF is $\widehat{\text{MTTF}} = 61.4001$.

The corresponding log-likelihood function is $\mathcal{L}_{GED} = -310.01$.

Therefore, the likelihood ratio test statistic is $XL = 2(\mathcal{L}_{GED} - \mathcal{L}_{ED}) = 13.313$ and the p -value is 2.6352×10^{-4} . Thus the LED($4.5273 \times 10^{-3}, 2.7688 \times 10^{-4}$) fits this data much better than ED(1.2097×10^{-2}).

The variance covariance matrix is computed as

$$I^{-1} = \begin{bmatrix} 3.7622 \times 10^{-6} & -1.1632 \times 10^{-7} \\ -1.1632 \times 10^{-7} & 5.5613 \times 10^{-9} \end{bmatrix}$$

Thus, the variances of the MLE of α and β become $\text{Var}(\hat{\alpha}) = 3.7622 \times 10^{-6}$ and $\text{Var}(\hat{\beta}) = 5.5613 \times 10^{-9}$. Therefore, the 95% C.I of α and β , respectively, are

$$[7.2569 \times 10^{-4}, 8.3290 \times 10^{-3}], \quad [1.3072 \times 10^{-4}, 4.2305 \times 10^{-4}].$$

Figure 1 shows the hazard rate functions of the exponential distribution and LED computed when the MLE of the parameters replacing the unknown parameters.

Figure 2 shows the empirical estimate of survival function and estimation of survival function under H_0 and H_1 . Also, we computed the Kolmogorov-Smirnov (K-S) distances of the empirical distribution function and the fitted distribution for the data set. The K-S distance between the empirical survival function and the fitted exponential survival function is 0.0547. The K-S distance between the empirical survival function and the fitted exponential survival function is 0.0298.

Based on the 95% C.I of β and the values of the K-S distances, we get the same conclusion as that we got based on the values of the likelihood ration test statistics and p -value which leads to that the LED($4.5273 \times 10^{-3}, 2.7688 \times 10^{-4}$) fits this data rather than the ED(1.2097×10^{-2}).

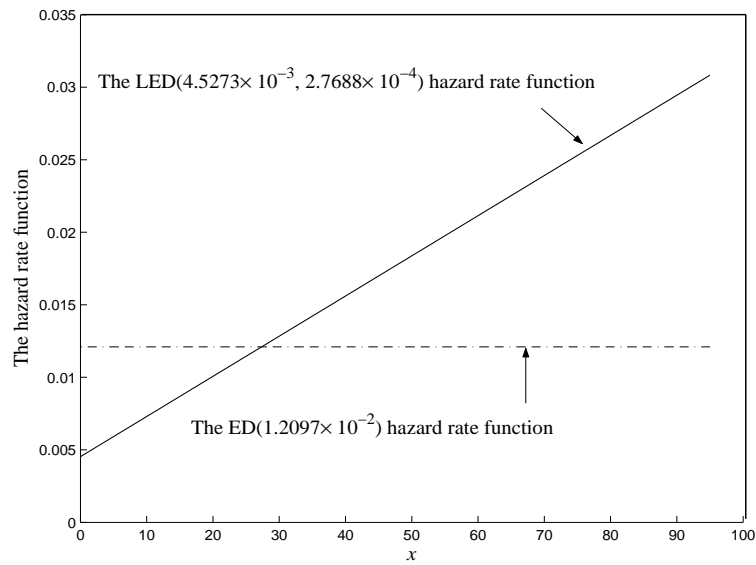


Figure 1. The estimated hazard rate functions.

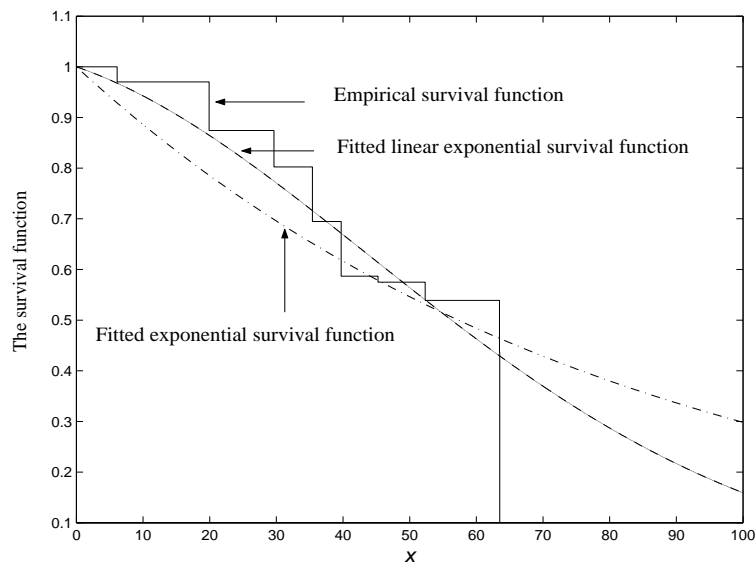


Figure 2. The empirical and fitted survival functions.

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