Precalculus 2
Section 10.6 Parametric Equations

Parametric Equations

- Write parametric equations.
- Graph parametric equations.
- Determine an equivalent rectangular equation for parametric equations.
- Determine parametric equations for a rectangular equation.
- Determine the location of a moving object at a specific time.

Writing Parametric Equations for a Line

In the equation $y = 2x - 3$, x is the independent variable and y is the dependent variable. In parametric equations, t is the independent variable and x and y are both dependent variables. If we set the independent variables x and t equal, we can write two parametric equations in terms of t.

Writing Parametric Equations for a Line

Given $y = 2x - 3$, then

$x = t$

$y = 2t - 3$

What about $y = 4x^2 - 3$?

$y = t^2 - 3$

$t = \sqrt{4x^2} = 2x$, thus $x = t/2$ or

$y = 4t^2 - 3$

$x = t$

Parametric Equations for a line passing through two points.

- <u>Example</u>: Find parametric equations of the line through ***(-1,3)*** and ***(1,1)***
- Choose one of the endpoints as the starting point. We'll select ***(-1,3).***
- Consider the ***x*** values. How far does ***-1*** have to move to get to ***1***?
- Obviously, ***1 - (-1)*** or ***2*** units. So ***x = -1 + 2t.***
- Consider the ***y*** values. How far does ***3*** have to move to get to ***1***?
- Obviously, ***1 - 3*** or ***-2*** units. So ***y = 3 - 2t***

- A **vector** is a ray with direction and magnitude (length).

- Example: Find a vector equation of the line through ***(-1,3)*** and ***(1,1)***.
- It helps to draw the line first so you can see what it looks like.
- Choose one of the endpoints as the starting point. We'll select ***(-1,3)***
- The direction vector of the line is the vector (represented by an ordered pair) that tells you how to move from one point to another. We know we have to move the ***x 2*** units and the ***y -2*** units, so our direction vector is ***(2,-2)***.
- The vector equation is ***(-1,3) + (2,-2)t***.
(-1,3) is the starting point and ***(2,-2)*** is the direction vector.

A vector equation for a Line passing through a point and parallel to a vector.

For a line passing through (-2, -3) and parallel to vector *a* = (5, 4) we can write

$(x + 2, y + 3) = t\,(5, 4).$

Therefore, $(x – x_1, y - y_1) = t\,(a_1, a_2)$

Graphing Parametric Equations

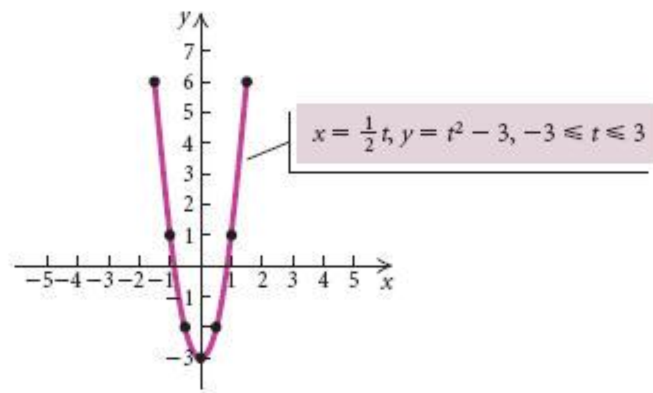
We have graphed *plane curves* that are composed of sets of ordered pairs (x, y) in the rectangular coordinate plane. Now we discuss a way to represent plane curves in which x and y are functions of a third variable t.

One method will be to construct a table in which we choose values of t and then determine the values of x and y.

Example 1 Graph Parametric

Graph the curve represented by the equations $x = \frac{1}{2}t, \qquad y = t^2 - 3; \qquad -3 \le t \le 3.$

t	x	y	(x, y)
-3	$-\frac{3}{2}$	6	$\left(-\frac{3}{2}, 6\right)$
-2	-1	1	$(-1, 1)$
-1	$-\frac{1}{2}$	-2	$\left(-\frac{1}{2}, -2\right)$
0	0	-3	$(0, -3)$
1	$\frac{1}{2}$	-2	$\left(\frac{1}{2}, -2\right)$
2	1	1	$(1, 1)$
3	$\frac{3}{2}$	6	$\left(\frac{3}{2}, 6\right)$



The rectangular equation is $y = 4x^2 - 3, \qquad -\frac{3}{2} \le x \le \frac{3}{2}.$

Parametric Equations

If f and g are continuous functions of t on an interval I, then the set of ordered pairs (x, y) such that $x = f(t)$ and $y = g(t)$ is a **plane curve**.

The equations $x = f(t)$ and $y = g(t)$ are **parametric equations** for the curve.

The variable t is the **parameter**.

Determining a Rectangular Equation for Given Parametric Equations

Solve either equation for t.

Then substitute that value of t into the other equation.

Calculate the restrictions on the variables x and y based on the restrictions on t.

Example 2 Find Rectangular

Find a rectangular equation equivalent to

$$x = t^2, \quad y = t - 1; \quad -1 \le t \le 4$$

Solution

$y = t - 1$

$t = y + 1$

Substitute $t = y + 1$ into $x = t^2$.

$x = (y + 1)^2$

Calculate the restrictions:

$-1 \le t \le 4$

$x = t^2; \quad 0 \le x \le 16$

$y = t - 1; \quad -2 \le y \le 3$

The rectangular equation is: $x = (y + 1)^2; \quad 0 \le x \le 16.$

Determining Parametric Equations for a Given Rectangular Equation

Many sets of parametric equations can represent the same plane curve. In fact, there are infinitely many such equations.

The most simple case is to let either x (or y) equal t and then determine y (or x).

Example 3 Find Parametric Equations

Find three sets of parametric equations for the parabola $y = 4 - (x+3)^2$.

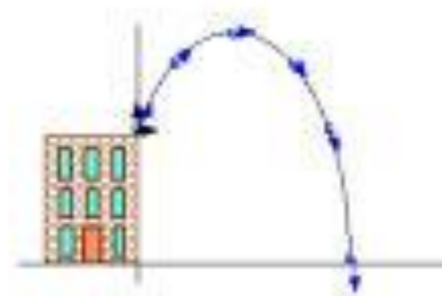
Solution

$$\text{If } x = t, \text{ then } y = 4 - (t+3)^2 = -t^2 - 6t - 5.$$

$$\text{If } x = t - 3, \text{ then } y = 4 - (t - 3 + 3)^2 = -t^2 + 4.$$

$$\text{If } x = \frac{t}{3}, \text{ then } y = 4 - \left(\frac{t}{3} + 3\right)^2 = -\frac{t^2}{9} - 2t - 5.$$

PROJECTILE MOTION



If an object is dropped, thrown, launched etc. at a certain angle and has gravity acting upon it, the equations for its position at time *t* can be written as:

$$x = (v_o \cos\theta)t \qquad y = -\frac{1}{2}gt^2 + (v_o \sin\theta)t + h$$

horizontal position

initial velocity

angle measured from horizontal

time

vertical position

gravitational constant which is 9.8 m/s^2

initial height

Denise Parker was a member of the U.S. Olympic Archery Team in 1988 and in 1992. Denise shoots an arrow with an initial velocity of 65 m/s at an angle of 4.5^o with the horizontal at a target 70 meters away. If Denise holds the bow 1.5 meters above the ground when she shoots the arrow, how far above the ground will the arrow be when it hits the target?

1. Write the parametric equations to model the path of the arrow.

$$x = (v_o \cos\theta)t \qquad\qquad y = -\frac{1}{2}gt^2 + (v_o \sin\theta)t + h$$

2. Then find the amount of time it will take the arrow to travel 70 meters horizontally. This is when it will hit the target.

3. Finally find the vertical position based on step 2.

Answer: The arrow will be about 1.3 meters above the ground when it hits the target.