Chapter 27

Current and Resistance

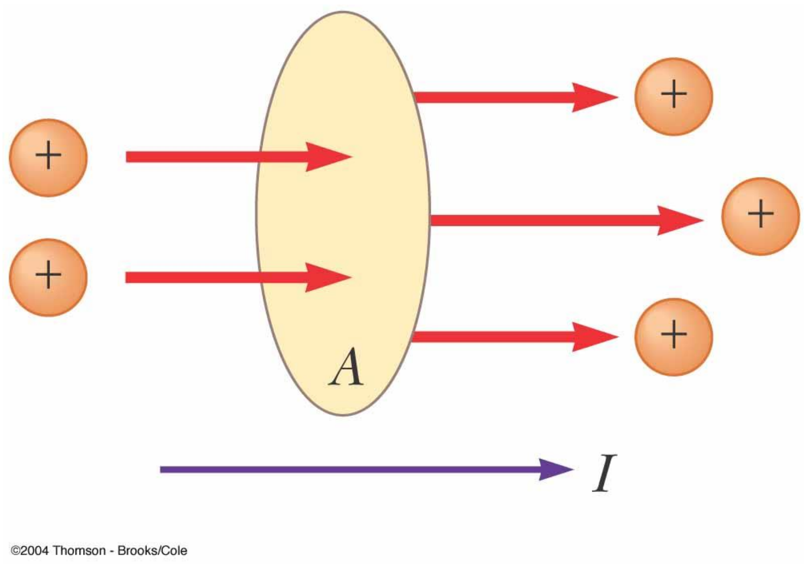
Electric Current

- **Electric current** is the rate of flow of charge through some region of space
- The SI unit of current is the **ampere** (A)
  - 1 A = 1 C / s
- The symbol for electric current is I

Average Electric Current

- Assume charges are moving perpendicular to a surface of area *A*
- If ⊗*Q* is the amount of charge that passes through *A* in time ⊗*t*, then the average current is

$$I_{av} = \frac{\Delta Q}{\Delta t}$$



Instantaneous Electric Current

- If the rate at which the charge flows varies with time, the instantaneous current, I, can be found

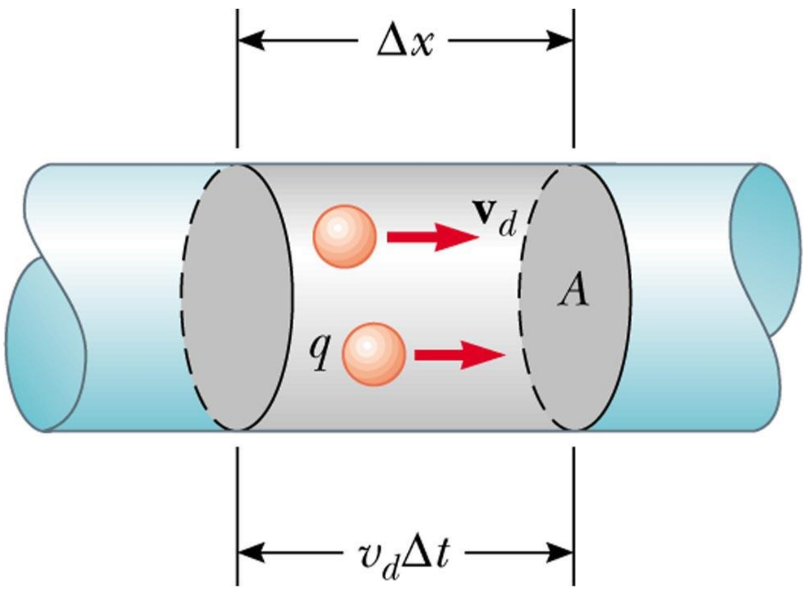
$$I = \frac{dQ}{dt}$$

Direction of Current

- The charges passing through the area could be positive or negative or both
- It is conventional to assign to the current the same direction as the flow of positive charges
- The direction of current flow is opposite the direction of the flow of electrons
- It is common to refer to any moving charge as a *charge carrier*

Current and Drift Speed

- Charged particles move through a conductor of cross-sectional area *A*
- *n* is the number of charge carriers per unit volume
- $nA\,\Delta x$ is the total number of charge carriers

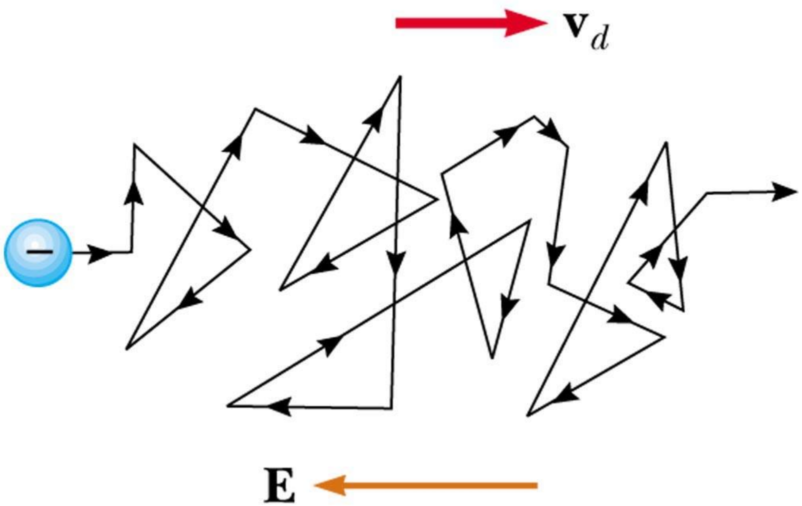


Current and Drift Speed, cont

- The total charge is the number of carriers times the charge per carrier, q
  - $\Delta Q = (nA\,\Delta x)q$
- The drift speed, v_d, is the speed at which the carriers move
  - $v_d = \Delta x / \Delta t$
- Rewritten: $\Delta Q = (nAv_d\,\Delta t)q$
- Finally, current, $I_{av} = \Delta Q/\Delta t = nqv_dA$

Charge Carrier Motion in a Conductor

- The zigzag black line represents the motion of a charge carrier in a conductor
  - The net drift speed is small
- The sharp changes in direction are due to collisions
- The net motion of electrons is opposite the direction of the electric field



Motion of Charge Carriers, cont.

- In spite of all the collisions, the charge carriers slowly move along the conductor with a drift velocity, v_d
- Changes in the electric field that drives the free electrons travel through the conductor with a speed near that of light
  - This is why the effect of flipping a switch is effectively instantaneous

Drift Velocity, Example 27.1

Example 27.1 Drift Speed in a Copper Wire

The 12-gauge copper wire in a typical residential building has a cross-sectional area of $3.31 \times 10^{-6}\ \text{m}^2$. If it carries a current of 10.0 A, what is the drift speed of the electrons? Assume that each copper atom contributes one free electron to the current. The density of copper is $8.95\ \text{g/cm}^3$.

Drift Velocity, Example

- Assume a copper wire, with one free electron per atom contributed to the current
- The drift velocity for a 12-gauge copper wire carrying a current of 10.0 A is

  2.22×10^{-4} m/s
  - This is a typical order of magnitude for drift velocities

Current Density

- ***J*** is the **current density** of a conductor
- It is defined as the current per unit area
  - $\mathbf{J} = I / A = nq\mathbf{v}_d$
  - This expression is valid only if the current density is uniform and *A* is perpendicular to the direction of the current
- *J* has SI units of A/m^2
- The current density is in the direction of the positive charge carriers

Conductivity

- A current density **J** and an electric field **E** are established in a conductor whenever a potential difference is maintained across the conductor
- $\mathbf{J} = \sigma \mathbf{E}$
- The constant of proportionality, σ, is called the **conductivity** of the conductor

Ohm’s Law

- **Ohm’s law** states that for many materials, the ratio of the current density to the electric field is a constant σ that is independent of the electric field producing the current
  - Most metals obey Ohm’s law
  - Mathematically, $\mathbf{J} = \sigma\,\mathbf{E}$
  - Materials that obey Ohm’s law are said to be *ohmic*

Ohm's Law, cont.

- Not all materials follow Ohm's law
  - Materials that do not obey Ohm's law are said to be *nonohmic*
- Ohm's law is not a fundamental law of nature
- Ohm's law is an empirical relationship valid only for certain materials

Resistance

- In a conductor, the voltage applied across the ends of the conductor is proportional to the current through the conductor
- The constant of proportionality is called the **resistance** of the conductor

$$R = \frac{\Delta V}{I}$$

Resistance, cont.

- SI units of resistance are *ohms* (Ω)
  - $1\ \Omega = 1\ V / A$
- Resistance in a circuit arises due to collisions between the electrons carrying the current with the fixed atoms inside the conductor

Resistivity

- The inverse of the conductivity is the **resistivity**:
  - $\rho = 1 / \sigma$
- Resistivity has SI units of ohm-meters ($\Omega \cdot$ m)
- Resistance is also related to resistivity:

$$R = \rho \frac{\ell}{A}$$

Resistivity Values

Table 27.1

Resistivities and Temperature Coefficients of Resistivity for Various Materials

Material	Resistivity[a] ($\Omega \cdot m$)	Temperature Coefficient[b] $\alpha[(°C)^{-1}]$
Silver	1.59×10^{-8}	3.8×10^{-3}
Copper	1.7×10^{-8}	3.9×10^{-3}
Gold	2.44×10^{-8}	3.4×10^{-3}
Aluminum	2.82×10^{-8}	3.9×10^{-3}
Tungsten	5.6×10^{-8}	4.5×10^{-3}
Iron	10×10^{-8}	5.0×10^{-3}
Platinum	11×10^{-8}	3.92×10^{-3}
Lead	22×10^{-8}	3.9×10^{-3}
Nichrome[c]	1.50×10^{-6}	0.4×10^{-3}
Carbon	3.5×10^{-5}	-0.5×10^{-3}
Germanium	0.46	-48×10^{-3}
Silicon	640	-75×10^{-3}
Glass	10^{10} to 10^{14}	
Hard rubber	$\sim 10^{13}$	
Sulfur	10^{15}	
Quartz (fused)	75×10^{16}	

[a] All values at 20°C.

[b] See Section 27.4.

[c] A nickel–chromium alloy commonly used in heating elements.

Resistance and Resistivity, Summary

- Every ohmic material has a characteristic resistivity that depends on the properties of the material and on temperature
- The resistance of a material depends on its geometry and its resistivity
- An ideal conductor would have zero resistivity
- An ideal insulator would have infinite resistivity

Example 27.2

EXAMPLE 27.2 The Resistance of a Conductor

Calculate the resistance of an aluminum cylinder that is 10.0 cm long and has a cross-sectional area of 2.00×10^{-4} m^2. Repeat the calculation for a cylinder of the same dimensions and made of glass having a resistivity of $3.0 \times 10^{10}\ \Omega \cdot \text{m}$.

Example 27.3

EXAMPLE 27.3 **The Resistance of Nichrome Wire**

(a) Calculate the resistance per unit length of a 22-gauge Nichrome wire, which has a radius of 0.321 mm.

(b) If a potential difference of 10 V is maintained across a 1.0-m length of the Nichrome wire, what is the current in the wire?

Example 27.3

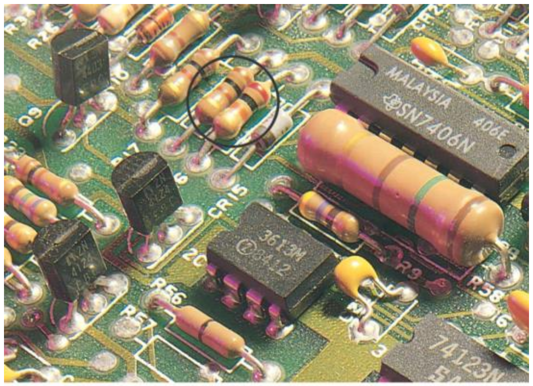
Exercise What is the resistance of a 6.0-m length of 22-gauge Nichrome wire? How much current does the wire carry when connected to a 120-V source of potential difference?

Exercise Calculate the current density and electric field in the wire when it carries a current of 2.2 A.

Resistors

- Most circuits use elements called resistors
- Resistors are used to control the current level in parts of the circuit
- Resistors can be *composite* or *wire-wound*

Resistor Values



- Values of resistors are commonly marked by colored bands

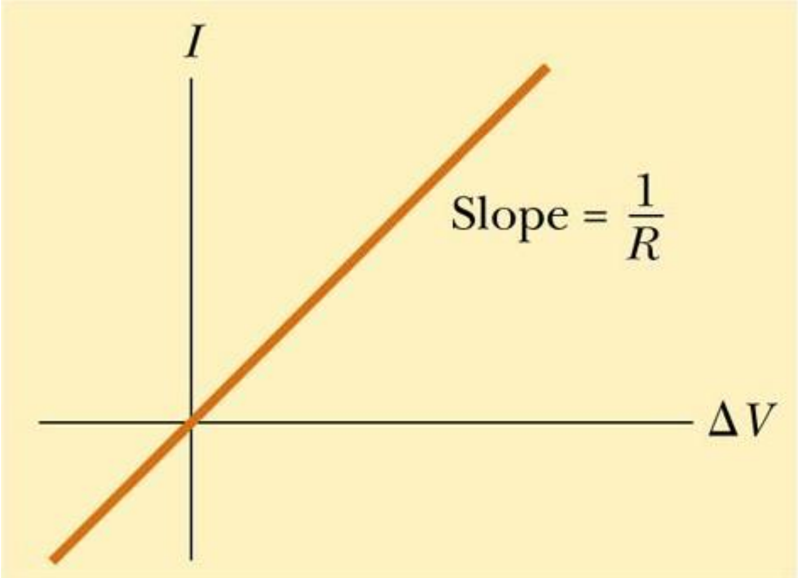
Table 27.2

Color Coding for Resistors

Color	Number	Multiplier	Tolerance
Black	0	1	
Brown	1	10^1	
Red	2	10^2	
Orange	3	10^3	
Yellow	4	10^4	
Green	5	10^5	
Blue	6	10^6	
Violet	7	10^7	
Gray	8	10^8	
White	9	10^9	
Gold		10^{-1}	5%
Silver		10^{-2}	10%
Colorless			20%

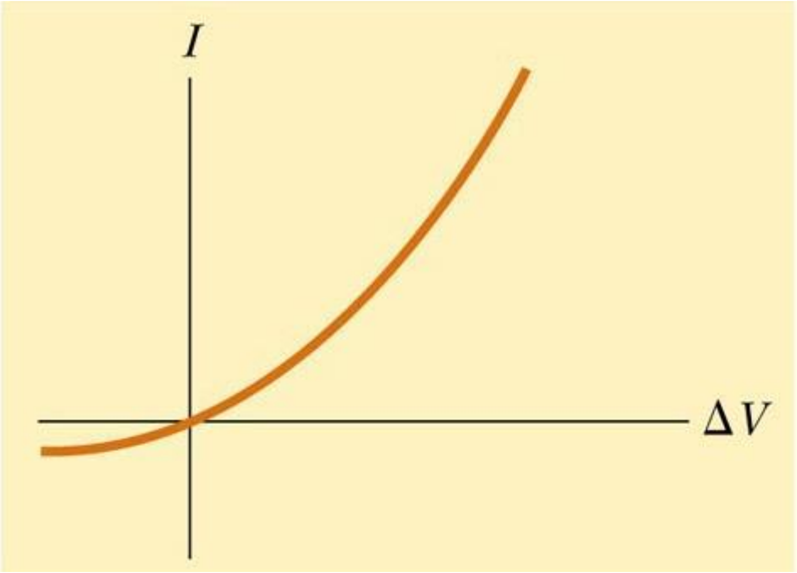
Ohmic Material, Graph

- An ohmic device
- The resistance is constant over a wide range of voltages
- The relationship between current and voltage is linear
- The slope is related to the resistance



Nonohmic Material, Graph

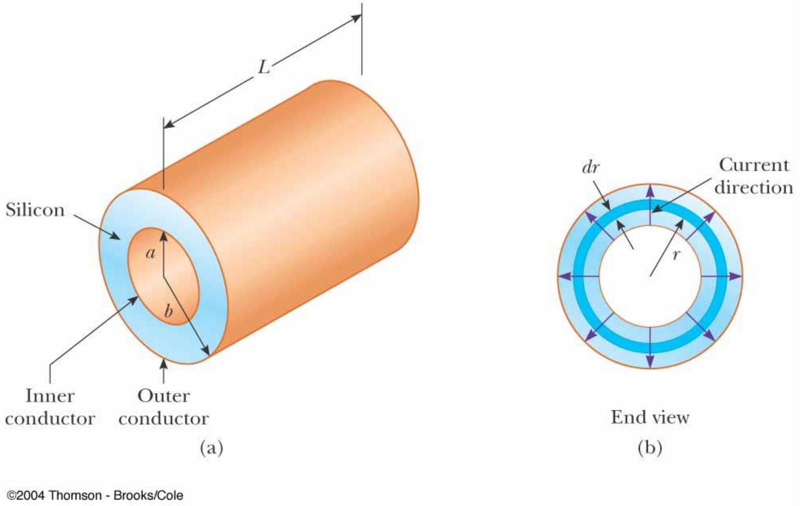
- Nonohmic materials are those whose resistance changes with voltage or current
- The current-voltage relationship is nonlinear
- A diode is a common example of a nonohmic device



Resistance of a Cable, Example

- Assume the silicon between the conductors to be concentric elements of thickness *dr*
- The resistance of the hollow cylinder of silicon is

$$dR = \frac{\rho}{2\pi r L} dr$$



Resistance of a Cable, Example, cont.

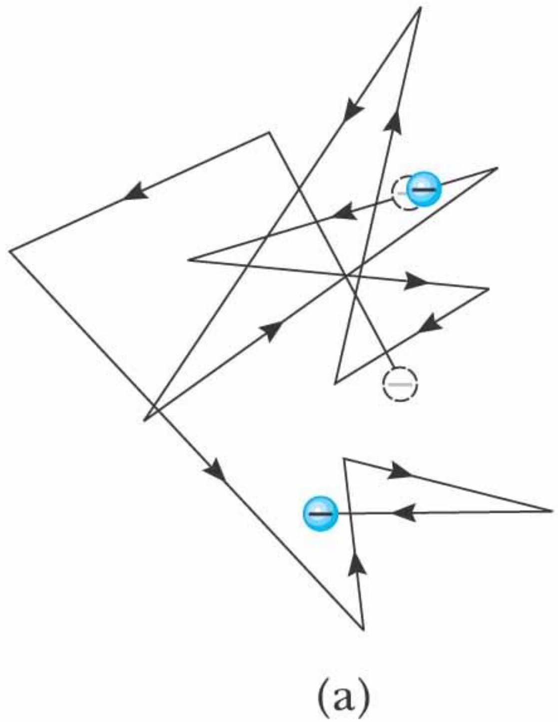
- The total resistance across the entire thickness is

$$R = \int_a^b dR = \frac{\rho}{2\pi L} ln\left(\frac{b}{a}\right)$$

- This is the radial resistance of the cable
- This is fairly high, which is desirable since you want the current to flow along the cable and not radially out of it

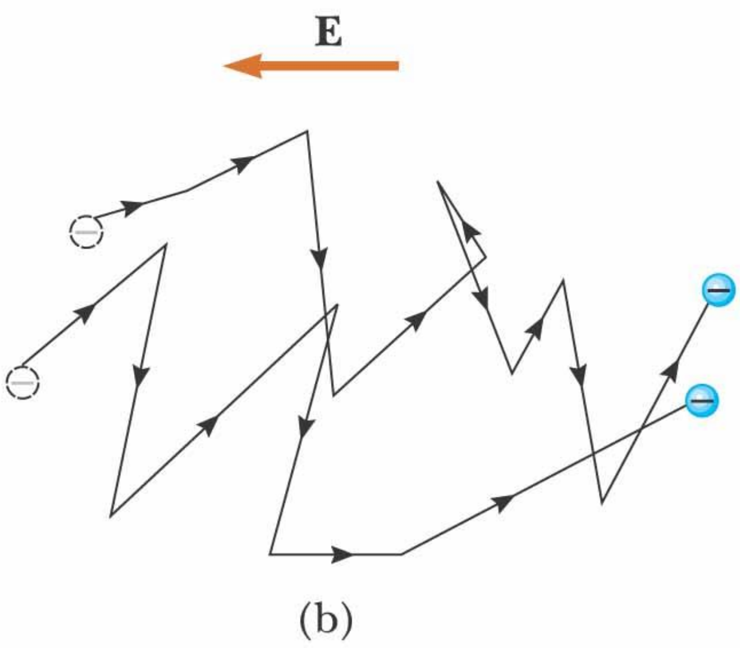
Electrical Conduction – A Model

- The diagram shows a description of the motion of free electrons in a conductor
- The motion is random
- There is no net displacement after many collisions
- The drift velocity is zero



Conduction Model, 2

- An electric field is applied
- The field modifies the motion of the charge carriers
- The electrons drift in the direction opposite of **E**



Conduction Model, 3

- Assumptions:
  - The motion of an electron after a collision is independent of its motion before the collision
  - The excess energy acquired by the electrons in the field is lost to the atoms of the conductor during the collision
  - The energy given up to the atoms increases their vibration and therefore the temperature of the conductor increases

Conduction Model, 4

- The force experienced by an electron is $\mathbf{F} = q\mathbf{E}$
- From Newton's Second Law, the acceleration is $\mathbf{a} = \mathbf{F} / m_e = q\mathbf{E} / m_e$
- Applying a motion equation $\mathbf{v}_f = \mathbf{v}_i + \mathbf{a}t$ or $\mathbf{v}_f = \mathbf{v}_i + (q\mathbf{E}/m_e)t$
- Since the initial velocities are random, their average value is zero

Conduction Model, 5

- Let τ be the average time interval between successive collisions
- The average value of $\mathbf{v}_f$ is the drift velocity
- $\mathbf{v}_{f\,\text{avg}} = \mathbf{v}_d = (q\mathbf{E}/m_e)\tau$
- This is also related to the current density: $J = nqv_d = (nq^2E\,/\,m_e)\tau$
  - n is the number of charge carriers per unit volume

Conduction Model, final

- Using Ohm's Law, expressions for the conductivity and resistivity of a conductor can be found:

$$\sigma = \frac{nq^2\tau}{m_e} \qquad \rho = \frac{1}{\sigma} = \frac{m_e}{nq^2\tau}$$

- Note, the conductivity and the resistivity do not depend on the strength of the field
- The average time is also related to the free mean path: $\tau = \ell / v_{av}$

Resistance and Temperature

- Over a limited temperature range, the resistivity of a conductor varies approximately linearly with the temperature

$$\rho = \rho_o[1 + \alpha(T - T_o)]$$

  - ρ_o is the resistivity at some reference temperature T_o
    - T_o is usually taken to be 20° C
    - α is the **temperature coefficient of resistivity**
      - SI units of α are $^oC^{-1}$

Temperature Variation of Resistance

- Since the resistance of a conductor with uniform cross sectional area is proportional to the resistivity, you can find the effect of temperature on resistance

$$R = R_o[1 + \alpha(T - T_o)]$$

Example 27.6

EXAMPLE 27.6 **A Platinum Resistance Thermometer**

A resistance thermometer, which measures temperature by measuring the change in resistance of a conductor, is made from platinum and has a resistance of 50.0 Ω at 20.0°C. When immersed in a vessel containing melting indium, its resistance increases to 76.8 Ω. Calculate the melting point of the indium.

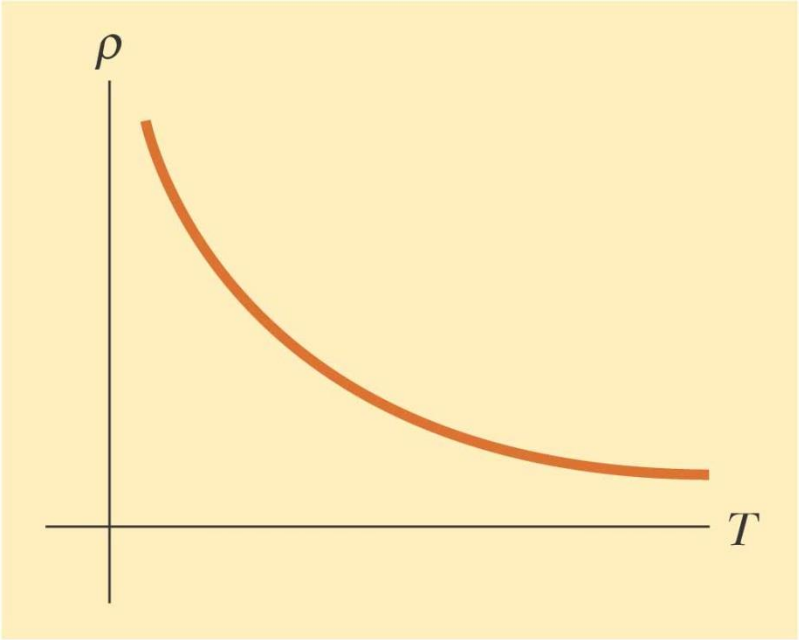
Example 27.6

$$\Delta T = \frac{R - R_0}{\alpha R_0} = \frac{76.8\ \Omega - 50.0\ \Omega}{[3.92 \times 10^{-3}\ (^\circ\text{C})^{-1}](50.0\ \Omega)} = 137^\circ\text{C}$$

Because $T_0 = 20.0^\circ\text{C}$, we find that T, the temperature of the melting indium sample, is 157°C.

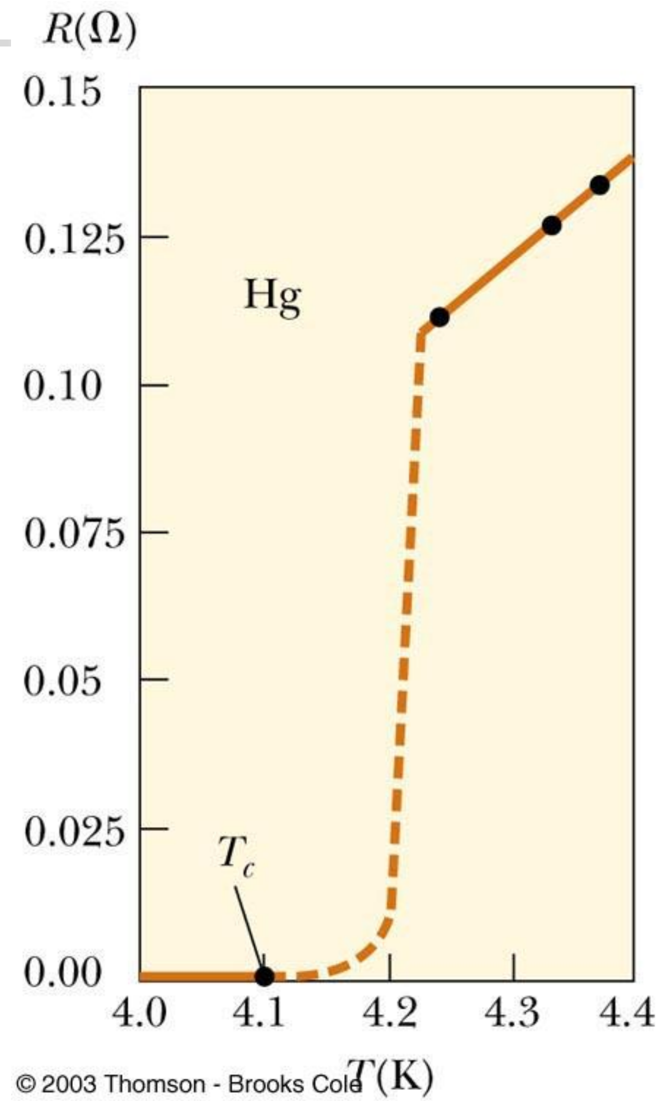
Semiconductors

- Semiconductors are materials that exhibit a decrease in resistivity with an increase in temperature
- α is negative
- There is an increase in the density of charge carriers at higher temperatures



Superconductors

- A class of materials and compounds whose resistances fall to virtually zero below a certain temperature, T_C
  - T_C is called the **critical temperature**
- The graph is the same as a normal metal above T_C, but suddenly drops to zero at T_C

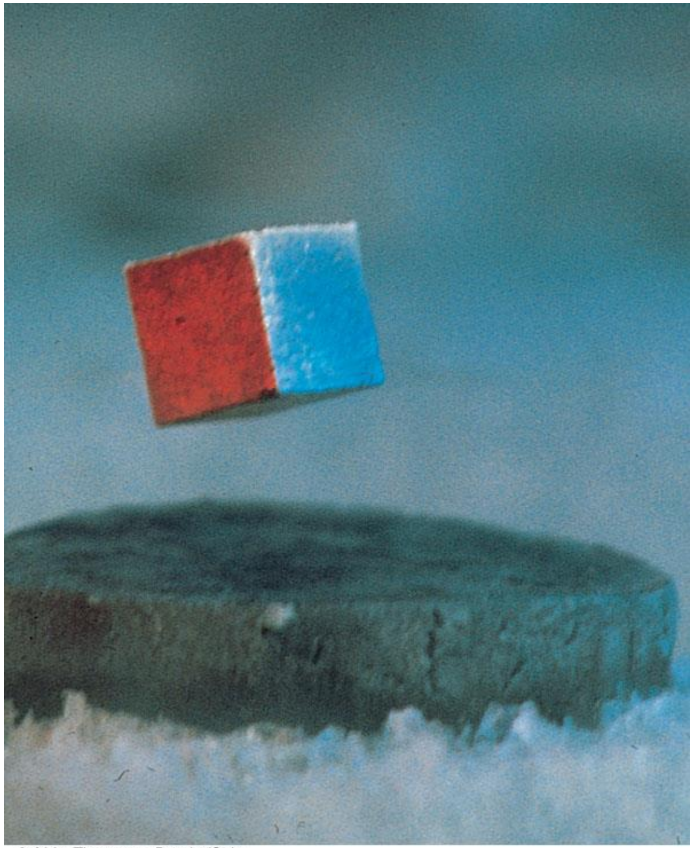


Superconductors, cont

- The value of T_C is sensitive to:
  - chemical composition
  - pressure
  - molecular structure
- Once a current is set up in a superconductor, it persists without any applied voltage
  - Since $R = 0$

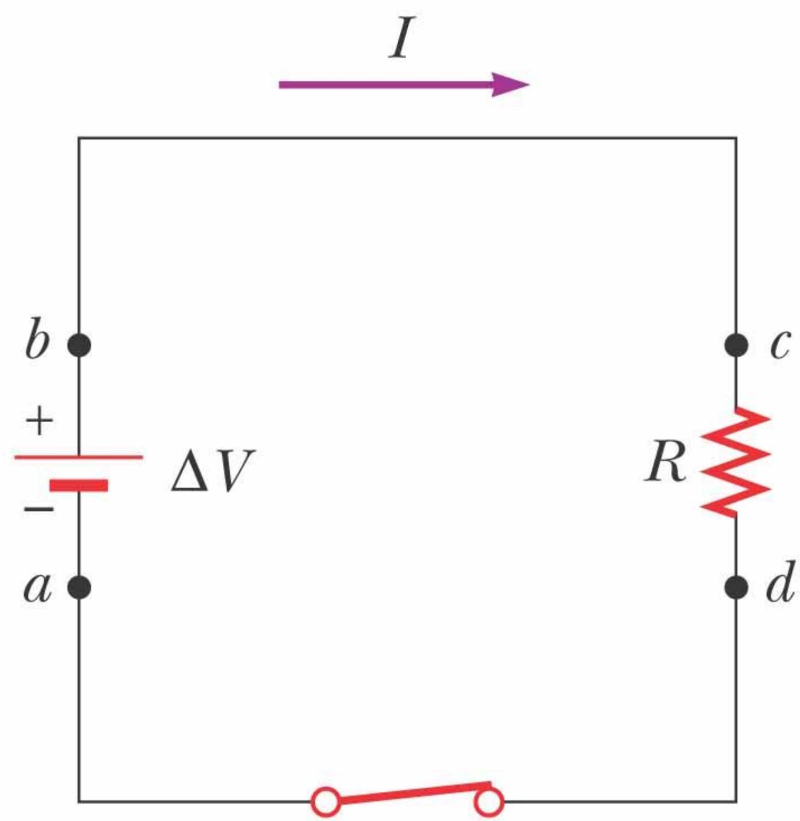
Superconductor Application

- An important application of superconductors is a superconducting magnet
- The magnitude of the magnetic field is about 10 times greater than a normal electromagnet
- Used in MRI units



Electrical Power

- Assume a circuit as shown
- As a charge moves from *a* to *b*, the electric potential energy of the system increases by $Q\Delta V$
  - The chemical energy in the battery must decrease by this same amount



Electrical Power, 2

- As the charge moves through the resistor (*c* to *d*), the system loses this electric potential energy during collisions of the electrons with the atoms of the resistor
- This energy is transformed into internal energy in the resistor
  - Corresponds to increased vibrational motion of the atoms in the resistor

Electric Power, 3

- The resistor is normally in contact with the air, so its increased temperature will result in a transfer of energy by heat into the air
- The resistor also emits thermal radiation
- After some time interval, the resistor reaches a constant temperature
  - The input of energy from the battery is balanced by the output of energy by heat and radiation

Electric Power, 4

- The rate at which the system loses potential energy as the charge passes through the resistor is equal to the rate at which the system gains internal energy in the resistor
- The **power** is the rate at which the energy is delivered to the resistor

Electric Power, final

- The power is given by the equation:

$$\wp = I \Delta V$$

- Applying Ohm's Law, alternative expressions can be found:

$$\wp = I \Delta V = I^2 R = \frac{V^2}{R}$$

- Units: I is in A, R is in Ω, V is in V, and $\wp$ is in W

Electric Power Transmission

- Real power lines have resistance
- Power companies transmit electricity at high voltages and low currents to minimize power losses