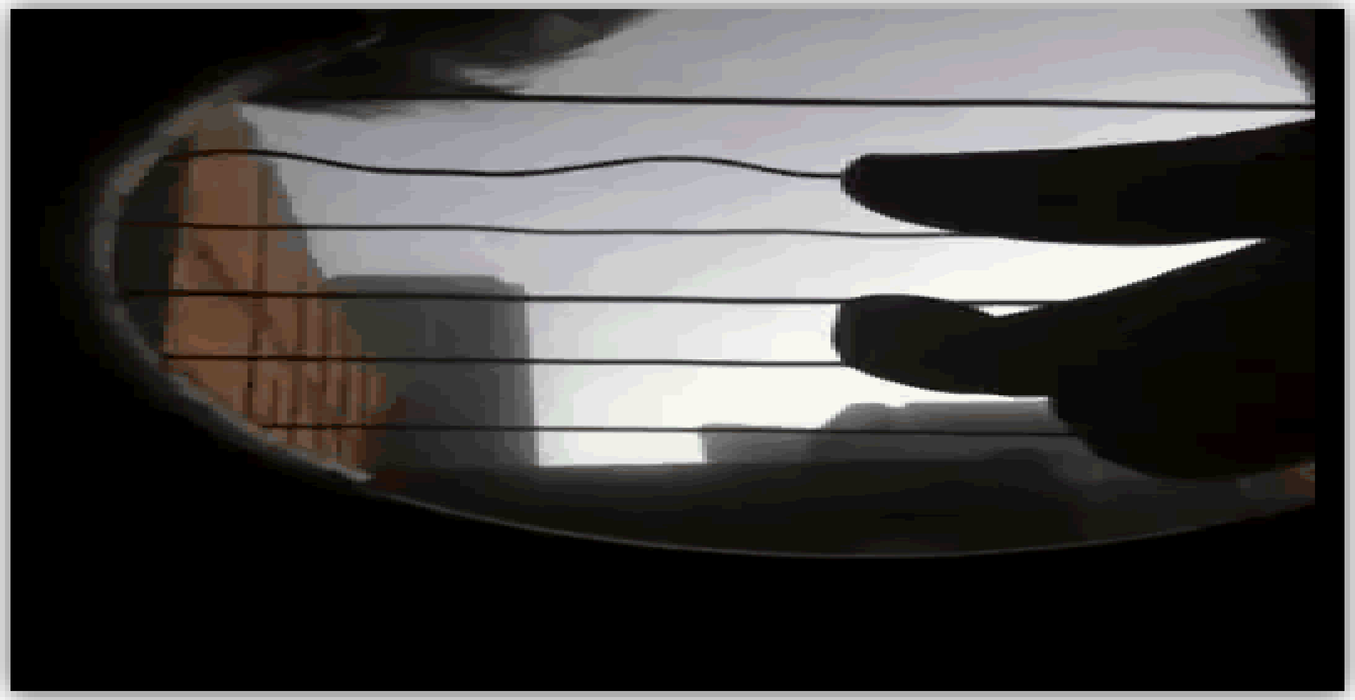
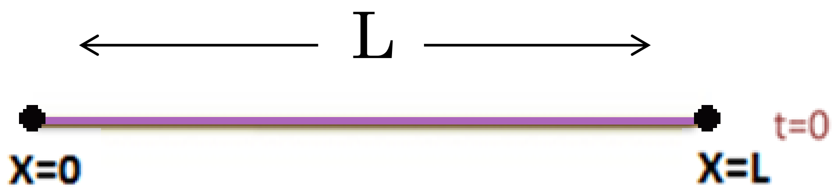
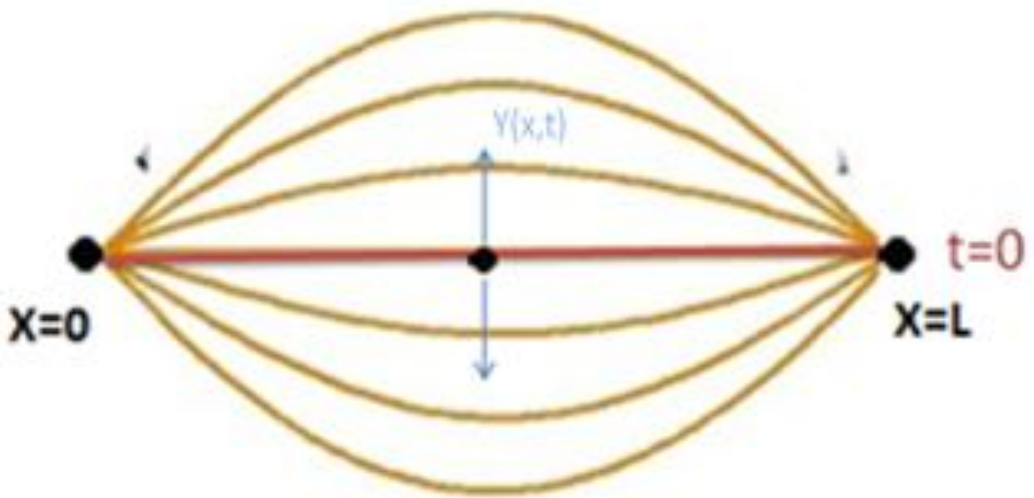
إذا شد خيط وثبت من نقطتين المسافة بينهما ℓ وحدثت إزاحة للخيط على الصورة

$y = k(\ell x - x^2)$ عند $t = 0$ ، أوجد الإزاحة عند أى لحظة.




L
t=0
X=0
X=L


Y(x,t)
t=0
X=0
X=L

معادلة الموجة هي:

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

With condition:

$$y(0,t)=0 \quad , \quad y(t,0)=0$$

$$y = f_1(x).f_2(t)$$

$$\therefore y_{tt} = f_1 . f_2'' \quad , \quad y_{xx} = f_1''.f_2$$

$$f_1 f_2'' = c^2 f_1'' f_2$$

$$\therefore \frac{f_1''}{f_1} = \frac{f_2''}{c^2 f_2} = -\alpha^2$$

$$\therefore f_1'' + \alpha^2 f_1 = 0$$

$$f_2'' + \alpha^2 c^2 f_2 = 0$$

$$\therefore f_1 = c_1 \cos \alpha x + c_2 \sin \alpha x$$

$$f_2 = c_3 \cos \alpha ct + c_4 \sin \alpha ct$$

$$\therefore y = (c_1 \cos \alpha x + c_2 \sin \alpha x)(c_3 \cos \alpha ct + c_4 \sin \alpha ct)$$

$$\because y(0,t)=0$$

$$\therefore c_1(c_3\cos\alpha ct+c_4\sin\alpha ct)=0$$

$$\therefore c_1=0$$

$$\therefore y=c_2\sin\alpha x\,(c_3\cos\alpha ct+c_4\sin\alpha ct)$$

$$\because\left(\frac{\partial y}{\partial t}\right)_{t=0}=0 \qquad , \quad \frac{\partial y}{\partial t}=c_2\sin\alpha x\,(c_3c\alpha\sin\alpha ct+c_4c\alpha\cos\alpha ct)$$

$$\therefore c_2\sin(\alpha x)(c_4c\alpha)=0 \qquad ,\textit{where } t=0$$

$$\textit{but } c_2\neq 0 \longrightarrow c_4=0$$

$$\therefore y=c_2c_3\sin(\alpha x)\cos(\alpha ct)$$

$$i.e:\ y=A\sin(\alpha x)\cos(\alpha c\,t) \qquad ,\textit{where} \qquad A=c_2c_3$$

$$\because y(\ell,t)=0$$

$$\therefore A\sin(\alpha\ell)\cos((\alpha c t)=0$$

$$\therefore \sin(\alpha\ell)=0 \quad \Rightarrow \quad \alpha\ell=n\pi \;,\; n=1,2,\ldots$$

$$\therefore \alpha=\frac{n\pi}{\ell}$$

$$\therefore y(x,t)=A_n\sin(\frac{n\pi}{\ell}x)\cos(\frac{n\pi c}{\ell}t)$$

$$\therefore y=\sum_{n=1}^{\infty}A_n\sin(\frac{n\pi}{\ell}x)\cos(\frac{n\pi c}{\ell}t)$$

$$\because y(x,0)=k(\ell x-x^2)$$

$$\therefore (\ell x-x^2)=\sum_{n=1}^{\infty}B_n\sin(\frac{n\pi}{\ell}x)$$,where $A_n=B_n/k$

$$B_n = \frac{2}{\ell}\int_0^{\ell}(\ell x - x^2)\sin(\frac{n\pi}{\ell}x)dx$$

$$\int x\sin(mx)dx = -\frac{1}{m}x\cos(mx) + \frac{1}{m^2}\sin(mx)$$

$$\int x^2\sin(mx)dx = -\frac{1}{m}x^2\cos(mx) + \frac{2x}{m^2}\sin(mx) + \frac{2}{m^3}\cos(mx)$$

$$\therefore B_n = \frac{2}{\ell}\{(\ell x - x^2)(-\cos(\frac{n\pi}{\ell}x)(\frac{\ell}{n\pi}) + (\ell - 2x)(\sin(\frac{n\pi}{\ell}x))(\frac{\ell^2}{n^2\pi^2}) -$$

$$2\cos(\frac{n\pi}{\ell}x)(\frac{\ell^3}{n^3\pi^3})$$

$$= -2\frac{\ell^3}{n^3\pi^3}[(-1)^n - 1](\frac{2}{\ell})$$

$$= \begin{cases} \frac{8\ell^3}{n^3\pi^3} & , n \text{ odd} \\ 0 & , n \text{ even} \end{cases}$$

$$\therefore y(x,t) = \sum_{n=1}^{\infty} \frac{8\ell^3}{n^3\pi^3} \sin(\frac{n\pi}{\ell}x)\cos(\frac{n\pi c}{\ell}t)$$

The general solution

Where n is odd

عمل الطالبات:

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