بسم الله الرحمن الرحيم

King Saud University
College of Science
Physics & Astronomy Dept.

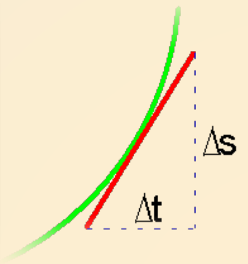
PHYS 103 (GENERAL PHYSICS)

LECTURE NO. 3

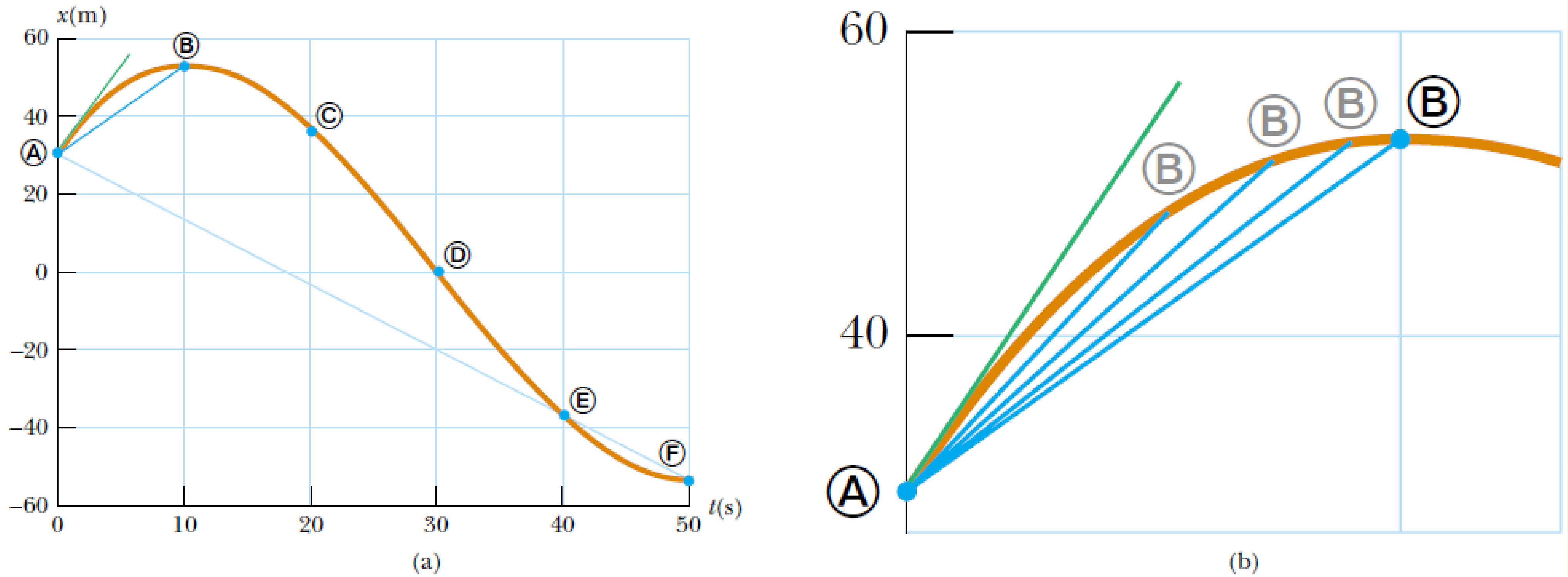
THIS PRESENTATION HAS BEEN PREPARED BY: *DR. NASSR S. ALZAYED*

2.2 Instantaneous Velocity and Speed

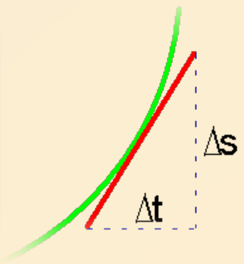
- Often we need to know the velocity of a particle at a particular instant in time, rather than the average velocity over a finite time interval.
- For example, even though you might want to calculate your average velocity during a long automobile trip, you would be especially interested in knowing your velocity at the *instant you noticed the police* car parked alongside the road ahead of you
- With the invention of calculus (at the late 1600s), scientists began to understand how to describe an object's motion at any moment in time.
- Hence, in this lecture we shall learn how to find instantaneous quantities. In particular; we will find: v(t) and a(t) which are the instantaneous velocity and acceleration respectively.



2.2 Instantaneous Velocity and Speed

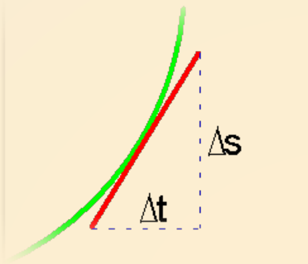


- **Figure 2.3**: (a) Representing the motion a car. (b) An enlargement of the upper-left-hand corner of the graph shows how the blue line between positions A and B approaches the green tangent line as point B is moved closer to point A.



2.2 Instantaneous Velocity and Speed

- Please watch this YouTube to get clear vision.



2.2 Instantaneous Velocity and Speed

- We define the instantaneous velocity for a particle moves on x-axis as:

$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad (2.5)$$

- There are 3 cases of v_x as:

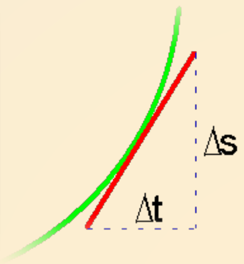
  $v_x > 0$ (Solpe is +): motion is to the right $\rightarrow$

  $v_x < 0$ (Slope is -) : motion is to the left $\leftarrow$

  $v_x = 0$ (at maxima or minima) particle is momentarily at rest

- From here on, we use the word *velocity* to designate instantaneous velocity. When it is average velocity we are interested in, we shall always use the adjective *average*.

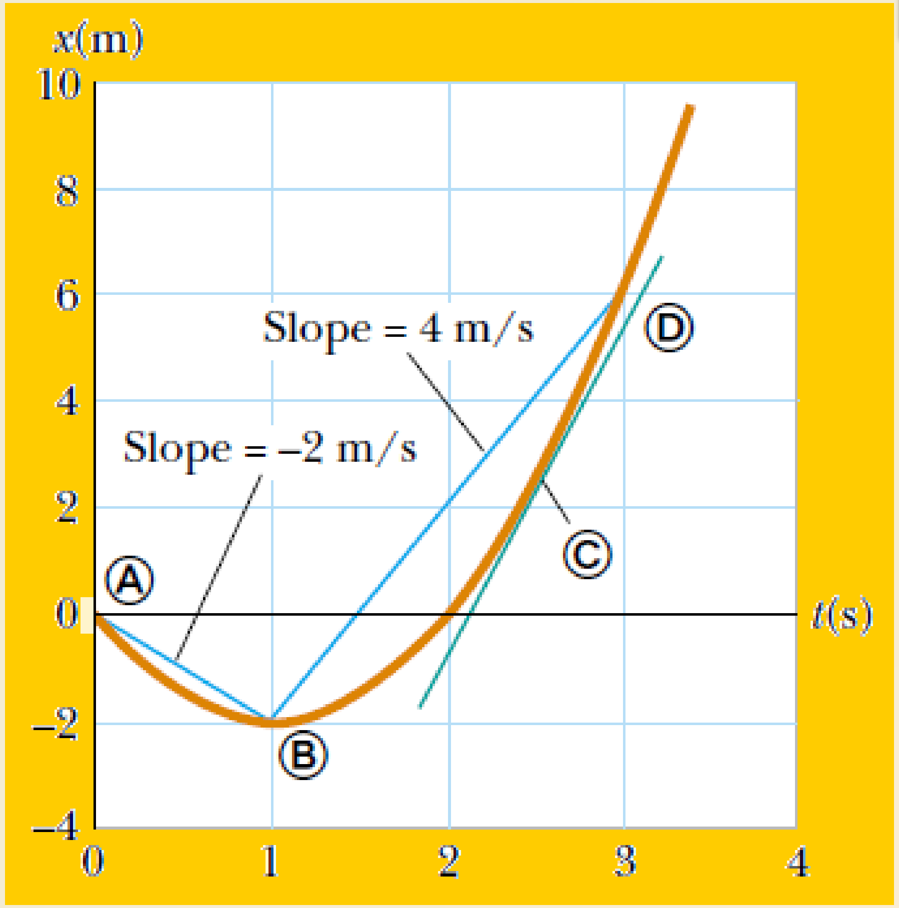
- The instantaneous speed is defined as the magnitude of its inst. velocity. It has no direction associated with it and hence carries *no algebraic sign*



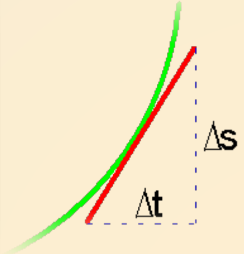
2.2 Instantaneous Velocity and Speed

▶ ***Example: 2.3:***

A particle moves along the x axis. Its position varies with time according to the expression $x = -4t + 2t^2$ where x is in meters and t is in seconds. The position–time graph for this motion is shown in the figure. Note that the particle moves in the negative x direction for the first second of motion, is momentarily at rest at the moment t = 1 s, and moves in the positive x direction at times t > 1 s.



Position–time graph for a particle



2.2 Instantaneous Velocity and Speed

- ***Example: 2.3: (continues)***

(a) Determine the displacement of the particle in the time intervals t = 0 to t = 1 s and t = 1 s to t = 3 s

1^{st} interval (0 to 1 s):

$$\because x(0) = -4(0) + 2(0) = 0\,m = x_i$$

$$\because x(1) = -4(1) + 2(1) = -2\,m = x_f$$

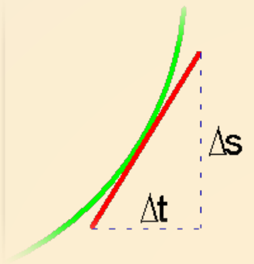
$$\therefore \Delta x = x_f - x_i = -2 - 0 = -2m$$

2^{nd} interval (1 to 3 s):

$$\because x(1) = -4(1) + 2(1) = -2\,m = x_i$$

$$\because x(3) = -4(3) + 2(9) = 6\,m = x_f$$

$$\therefore \Delta x = x_f - x_i = 6 - (-2) = 8m$$



2.2 Instantaneous Velocity and Speed

- ***Example: 2.3: (continues)***

(b) Calculate the average velocity during these two time intervals.

1st interval (0 to 1 s):

$$\because \bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t} = \frac{-2}{1} = -2m/s$$

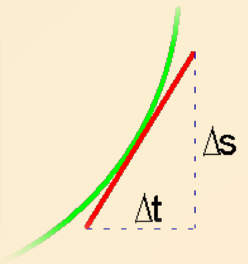
2nd interval (1 to 3 s):

$$\because \bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t} = \frac{8}{2} = 4m/s$$

(c) Find the instantaneous velocity of the particle at t = 2.5 s.

$$\because v_x(t) = -4t + 2t^2$$

$$\therefore v_x(2.5) = -4(2.5) + 2(2.5)^2 = 6\,m/s$$



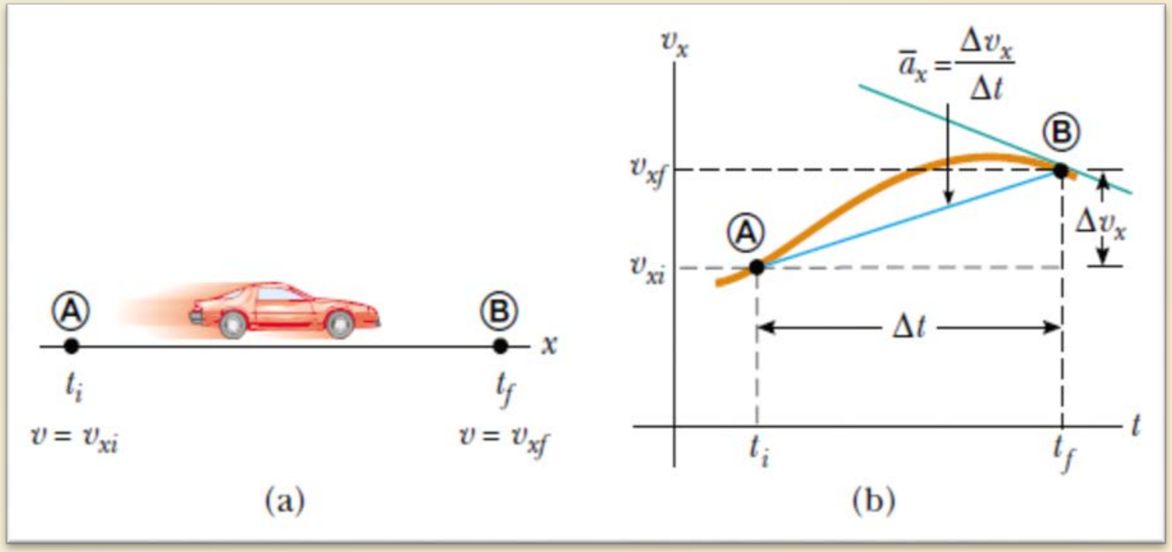
2.3 Acceleration

- Just like with the case of velocity; Acceleration (a) can be and average or instantaneous.
- Average Acceleration is defined as:

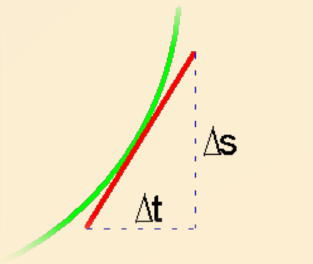
$$\bar{a}_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i} \qquad (2.6)$$

- instantaneous acceleration is

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \qquad (2.7)$$



- When the object's velocity and acceleration are in the *same direction*, the object is speeding up. On the other hand, when the object's velocity and acceleration are in *opposite directions*, the object is slowing down.



2.3 Acceleration

- ***Example: 2.5:***

The velocity of a particle moving along the x axis varies in time according to the expression $v_x = (40 - 5t^2)$ m/s, where t is in seconds.

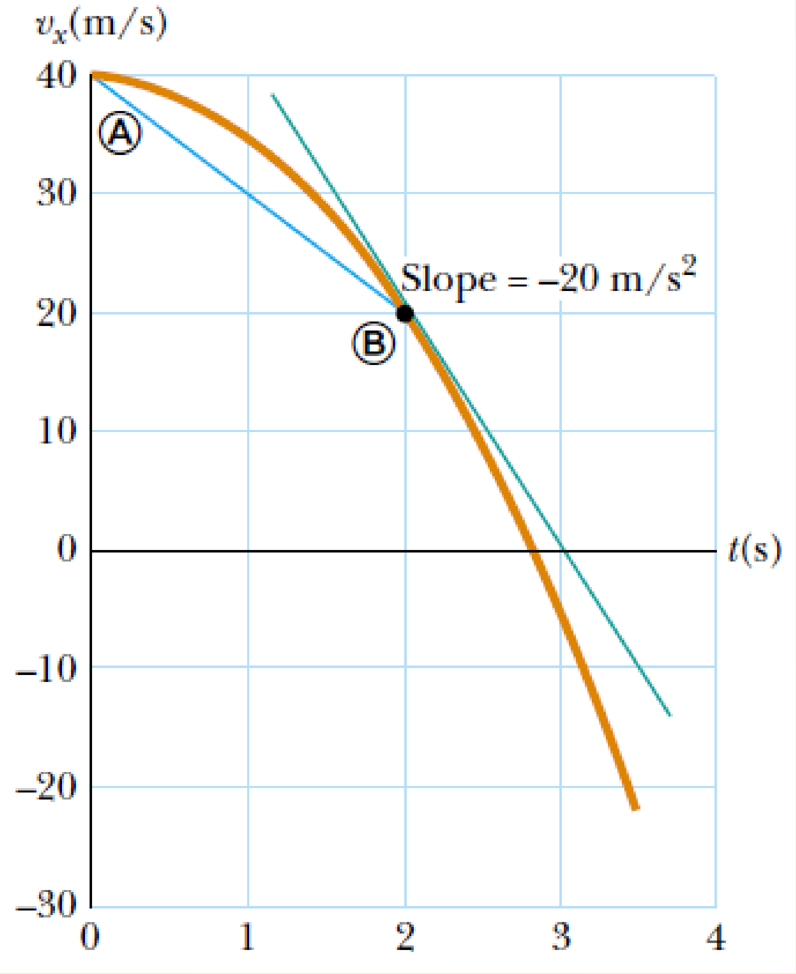
(a) Find the average acceleration in the time interval t = 0 to t = 2.0 s.

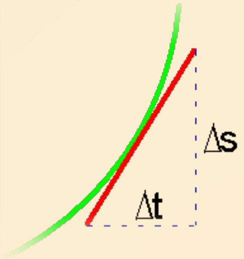
$$\because v_x = 40 - 5t^2$$

$$\therefore v_{xi} = v_x(0) = 40 - 5(0) = 40\,\mathrm{m/s}$$

$$\therefore v_{xf} = v_x(2) = 40 - 5(2)^2 = 20\,\mathrm{m/s}$$

$$\rightarrow a_x = \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{\Delta t} = \frac{20 - 40}{2} = -10 m/s^2$$





- ***Example: 2.5: (continues)***

(b) Determine the acceleration at t = 2.0 s

Solution:

We want to find the instantaneous acceleration at t = 2 s

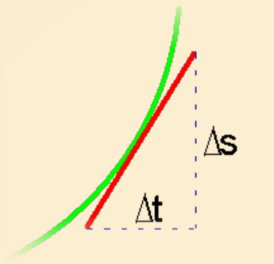
$$\because v_x = 40 - 5t^2$$

$$\because a_x = \frac{dv_x}{dt}$$

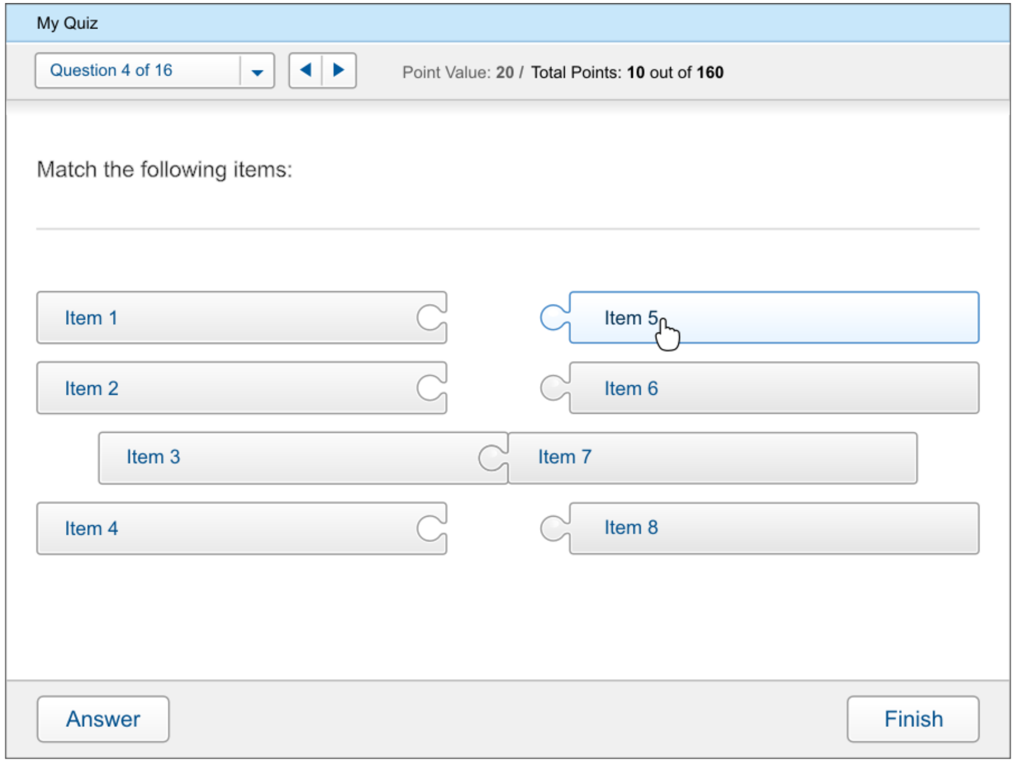
$$\therefore a_x = 0 - 5 \times 2t = -10t$$

$$\therefore a_x(2) = -10(2) = -20 \ \text{m/s}^2$$

Because the velocity of the particle is positive and the acceleration is negative, the particle is slowing down.



Acceleration Quiz



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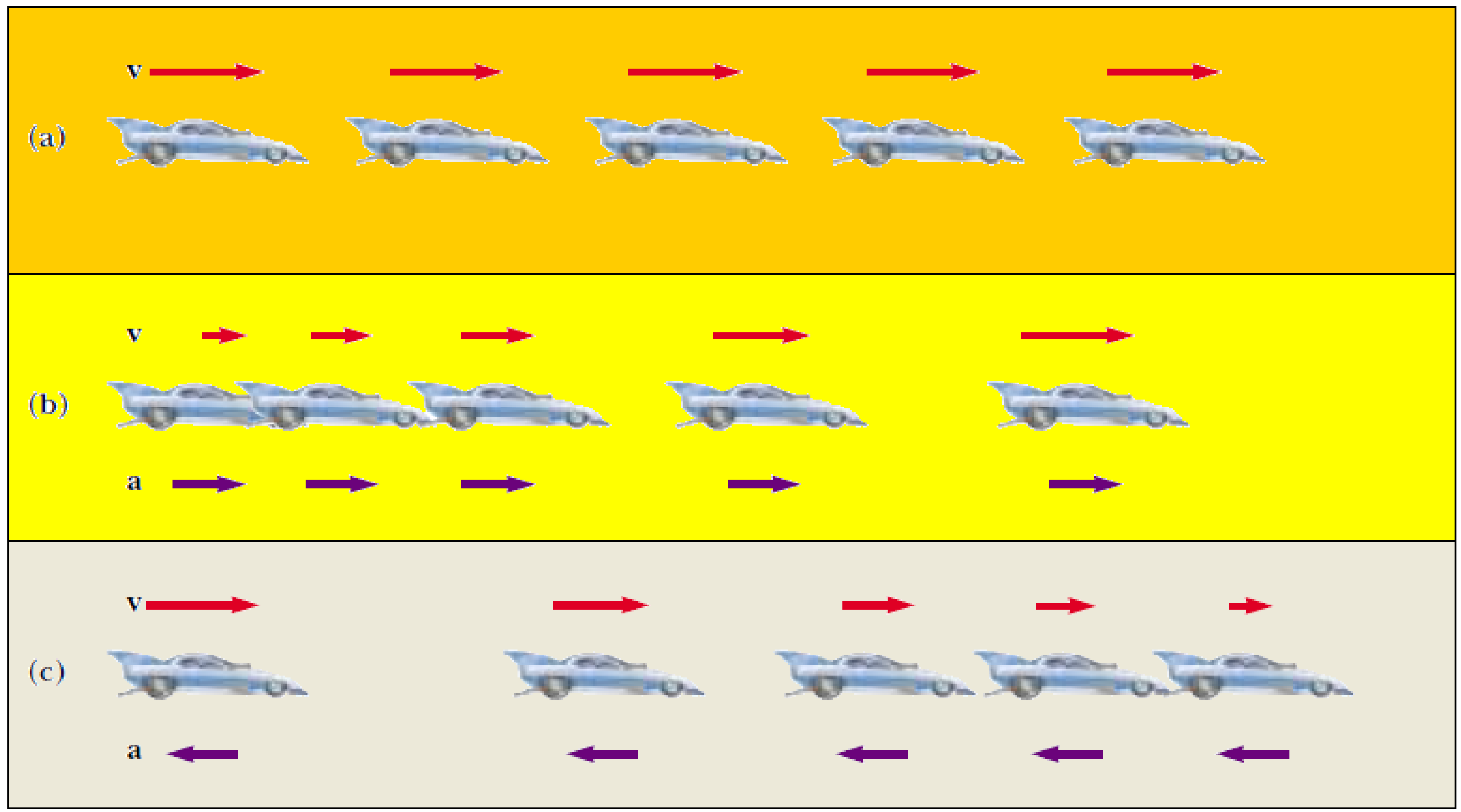
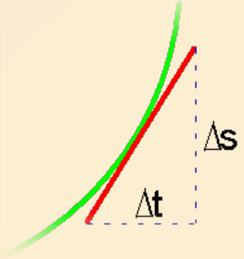
2.4 Motion Diagrams



Motion diagram for a car moving at: (a) constant velocity (b) constant acceleration in the direction of its velocity. (c) constant acceleration in the direction opposite the velocity

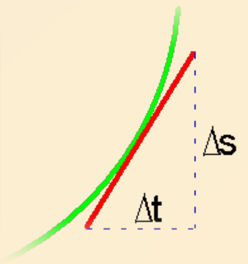


2.3 1-D Motion with Constant Acceleration

- If the acceleration of a particle varies in time, its motion can be complex and difficult to analyze. However, a very common and simple type of one-dimensional motion is that in which the *acceleration is constant*. When this is the case, the average acceleration over any time interval is numerically equal to the instantaneous acceleration a_x at any instant within the interval, and the velocity changes at the same rate throughout the motion.
- In this case we can use a set of equations shown below:

Kinematic Equations for Motion of a Particle Under Constant Acceleration

Equation	Information Given by Equation
$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$	Velocity as a function of position



2.7 Example

▶ ***Example: 2.7:***

A jet lands on an aircraft carrier at 63 m/s.

(a) What is its acceleration if it stops in 2.0 s due to an arresting cable that snags the airplane and brings it to a stop?

$$v_{xf} = 0, v_{xi} = 63 m/s, t = 2 s$$

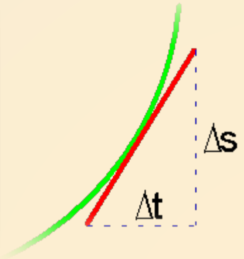
$$\because v_{xf} = v_{xi} + a_x t$$

$$\therefore 0 = 63 + a_x (2) \rightarrow a_x = -\frac{63}{2} = -31 \text{m/s}^2$$

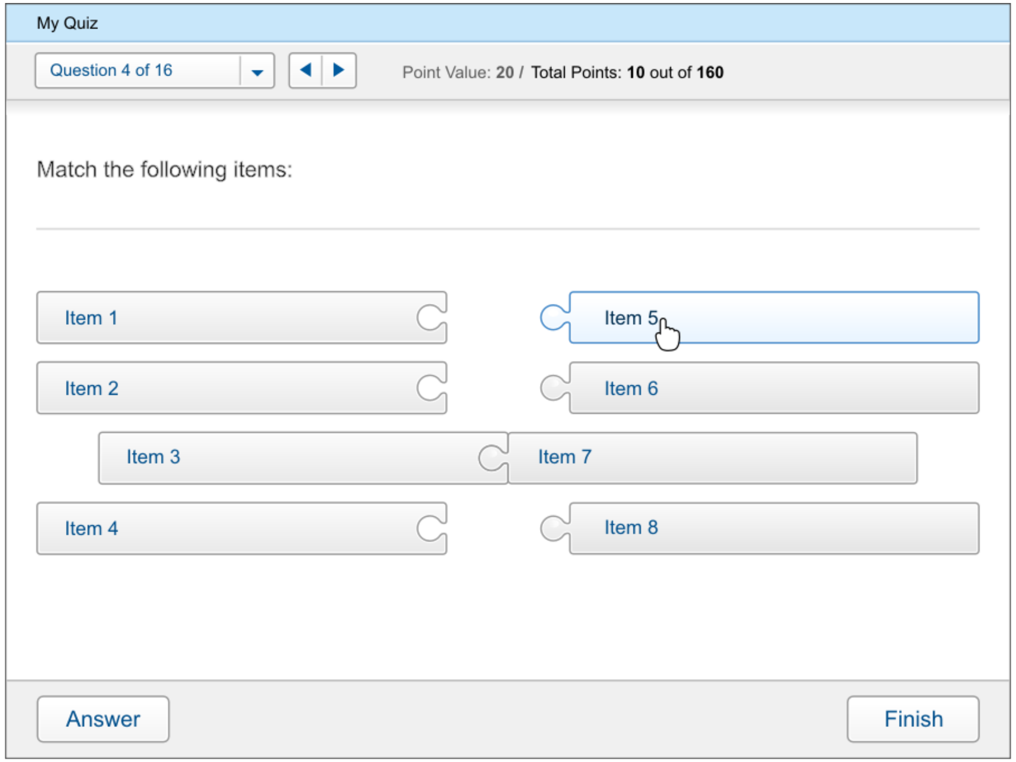
(b) If the plane touches down at position x_i=0, what is the final position of the plane?

$$\because x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2$$

$$\therefore x_f = 0 + 63(2) + \frac{1}{2}(-31)(4) = 64 m$$



v_x -t graph Quiz



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Lecture Summary

- The instantaneous velocity of a particle is defined as:

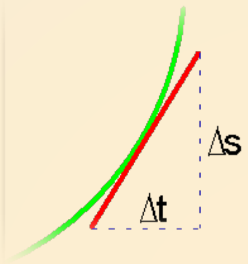
$$v_x \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \qquad (2.5)$$

- The instantaneous speed of a particle is equal to the magnitude of its instantaneous velocity.
- The average acceleration of a particle is defined as:

$$\bar{a}_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i} \qquad (2.6)$$

- The instantaneous acceleration is defined as:

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \qquad (2.7)$$

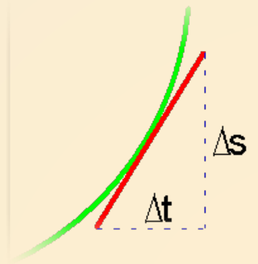


Lecture Summary (continued)

- When the object's velocity and acceleration are in the same direction, the object is speeding up. On the other hand, when the object's velocity and acceleration are in opposite directions, the object is slowing down.
- The equations of kinematics for a particle moving along the x axis with uniform acceleration a_x are:

Kinematic Equations for Motion of a Particle Under Constant Acceleration

Equation	Information Given by Equation
$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
${v_{xf}}^2 = {v_{xi}}^2 + 2a_x(x_f - x_i)$	Velocity as a function of position



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