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King Saud University
College of Science
Physics & Astronomy Dept.

PHYS 103 (GENERAL PHYSICS)

CHAPTER 4: VECTORS

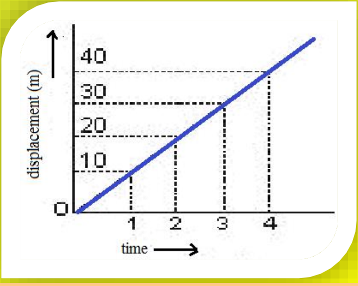
LECTURE NO. 5

THIS PRESENTATION HAS BEEN PREPARED BY: DR. NASSR S. ALZAYED

Lecture Outline

- Here is a quick list of the subjects that we will cover in this presentation. It is based on Serway, Ed. 6

- *4.1 The Position, Velocity, and Acceleration Vectors*
- *4.2 Two-Dimensional Motion with Constant Acceleration*
- *Examples*
- *Lecture Summary*
- *Activities (Interactive Flashes)*
- *Quizzes*

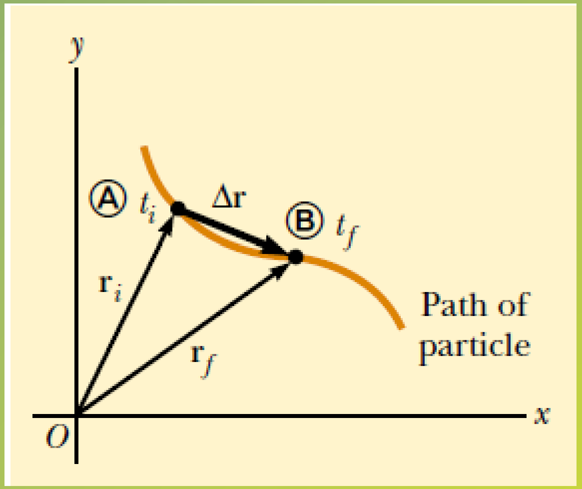


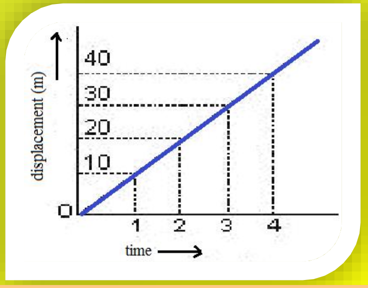
4.1 The Position, Velocity, and Acceleration Vectors

- In the figure: when the particle moves from position *A* to position *B*, we can say that particle moved from position : $\mathbf{r}_i$ to position $\mathbf{r}_f$.
- $\mathbf{r}_i$ and $\mathbf{r}_f$ are: initial and final position vectors respectively.
- We can express the *DISPLACEMENT* that was made by the particle as:

$$\Delta r = r_f - r_i \qquad (4.1)$$

- We have already discussed the displacement in lecture No. 3. Please see the difference between the path (distance) of the particle (orange line) and displacement. Distance is not a vector while displacement is a vector.





4.1 The Average Velocity

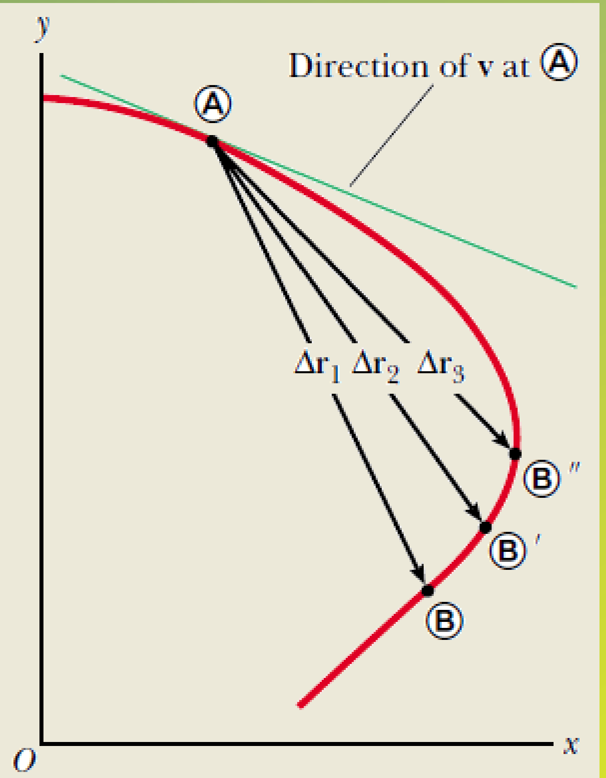
- The average velocity is defined as:

$$\overline{v} = \frac{\Delta r}{\Delta t} = \frac{r_f - r_i}{t_f - t_i} \qquad (4.2)$$

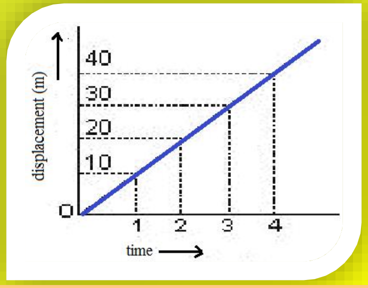
- We can get the instantaneous velocity (velocity as a function of time) as follows:

$$v = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{dr}{dt} \qquad (4.3)$$

- the instantaneous velocity equals the derivative of the position vector with respect to time



- The direction of the instantaneous velocity vector is along a line tangent to the path at that point and in the direction of motion.
- The magnitude of the instantaneous velocity vector $\upsilon = |\overline{v}|$ is called the speed, which is a scalar quantity.



4.1 The Average acceleration

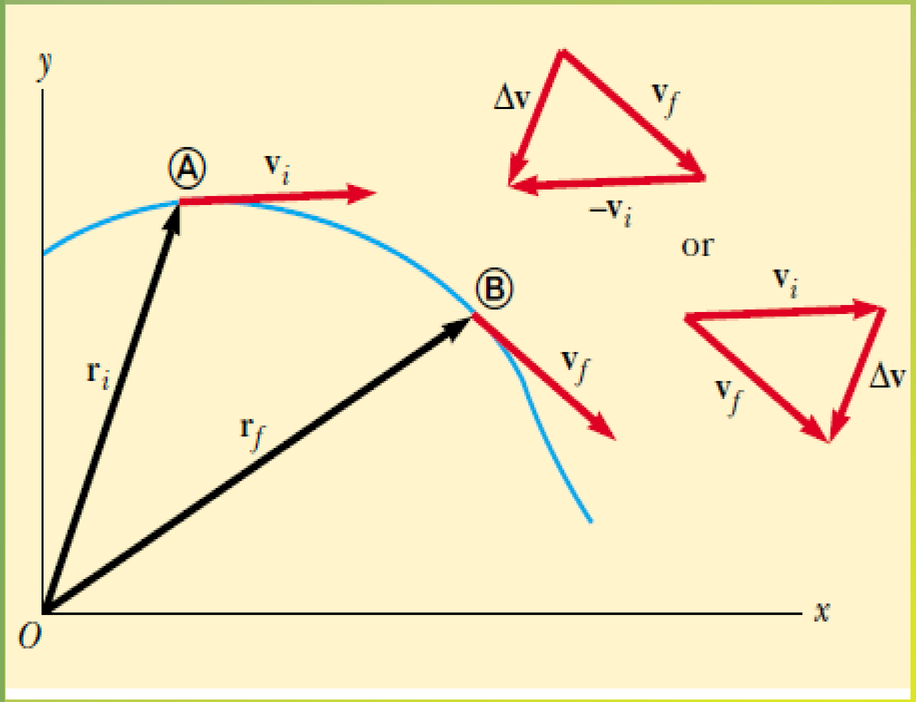
- The average acceleration is defined as:

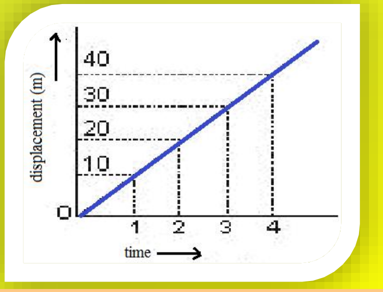
$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \qquad (4.4)$$

- We can get the instantaneous acceleration (acceleration as a function of time) as follows:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \qquad (4.5)$$

- Note: the magnitude of the velocity vector (the speed) may change with time as in straight-line (one-dimensional) motion.
- the direction of the velocity vector may change with time even if its magnitude (speed) remains constant, as in curved-path (2-d) motion.





4.2 Two-D Motion with Cons. Acceleration

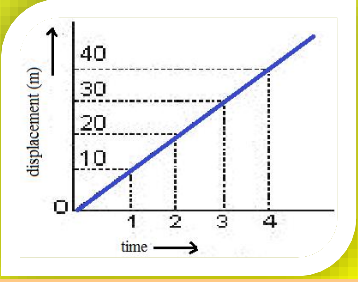
- The Let us consider 2-dimensional motion during which the acceleration remains constant in both magnitude and direction.
- The position vector for a particle moving in the xy plane can be written:

$$r = x\hat{i} + y\hat{j} \qquad (4.6)$$

- Please note that: *r*, *x* and *y* are time-dependant. They change with time as the particle moves.
- the velocity of the particle can be derived as:

$$v = \frac{dr}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j} \qquad (4.7)$$

- Because *a* is assumed constant, its components a_x and a_y also are also constants.
- Hence, for every component; we can use Table 2.2 (Chapter 2)



4.2 2-D Motion x and y equations

- We will have 2 sets of Equations; one for each direction.
- For x-direction; we have:

$$v_{xf} = v_{xi} + a_x t$$

$$x_f = v_{xi} t + \frac{1}{2} a_x t^2$$

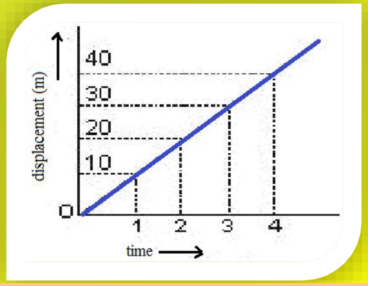
$$v_{xf}^2 = v_{xi}^2 + 2 a_x x_f$$

- For y-direction; we have:

$$v_{yf} = v_{yi} + a_y t$$

$$y_f = v_{yi} t + \frac{1}{2} a_y t^2$$

$$v_{yf}^2 = v_{yi}^2 + 2 a_y y_f$$



4.2 2-D Motion velocity and position vectors

- Hence, vectors of velocity ***v*** and position ***r*** are.

$$\because v = v_x \hat{i} + v_y \hat{j}$$

$$\therefore v_f = (v_{xi} + a_x t)\hat{i} + (v_{yi} + a_y t)\hat{j}$$

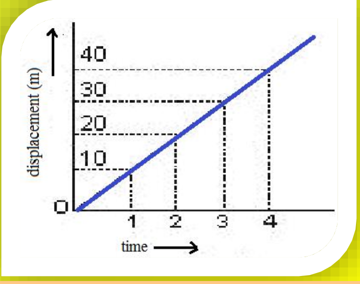
$$= (v_{xi}\hat{i} + v_{yi}\hat{j}) + (a_x \hat{i} + a_y \hat{j})\mathrm{t}$$

$$= v_i + at \qquad (4.8)$$

$$r_f = (v_{xi} t + \frac{1}{2} a_x t^2)\hat{i} + (v_{yi} t + \frac{1}{2} a_y t^2)\hat{j}$$

$$= (v_{xi}\hat{i} + v_{yi}\hat{j})t + \frac{1}{2}(a_x \hat{i} + a_y \hat{j})t^2$$

$$= v_i t + \frac{1}{2} a t^2 \qquad (4.9)$$



Example 4.1 Motion in a Plane

- A particle starts from the origin at t = 0 with an initial velocity having an *x* component of 20 m/s and a *y* component of -15 m/s. The particle moves in the *xy* plane with an *x* component of acceleration only, given by $a_x = 4.0\ m/s^2$.

(A) Determine the components of the velocity vector at any time and the total velocity vector at any time.

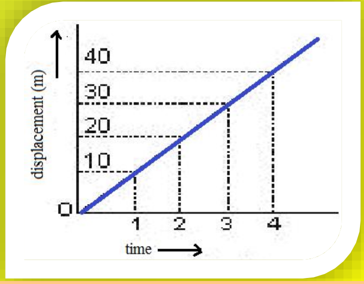
Solution:

We want to find v_{xf} and v_{yf} as functions of t then find **v**(t).

$$v_{xf} = v_{xi} + a_x t = (20 + 4t)\ \text{m/s}$$

$$v_{yf} = v_{yi} + a_y t = (-15 + 0t) = -15\ \text{m/s}$$

$$\Rightarrow v_f = v_x \hat{i} + v_y \hat{j} = [(20 + 4t)\hat{i} - 15\hat{j}]\ \text{m/s}$$



Example 4.1 Motion in a Plane (continued)

(B) Calculate the velocity and speed of the particle at t = 5.0 s.

Solution: We want to get **v** as (vector), its direction, and find the Speed:

$$v_f = v_x \hat{i} + v_y \hat{j} = [(20 + 4t)\hat{i} - 15\hat{j}]\ \text{m/s}$$

$$\therefore v_f = [(20 + 4 \times 5)\hat{i} - 15\hat{j}]\ \text{m/s}$$

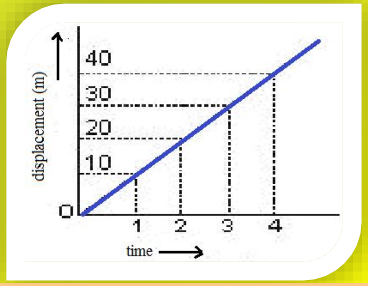
$$\therefore v_f = (40\hat{i} - 15\hat{j})\ \text{m/s}$$

for direction:

$$\because \theta = \tan^{-1}\left(\frac{v_{yf}}{v_{xf}}\right) = \tan^{-1}\left(\frac{-15}{40}\right) = -21^o$$

for speed: we find magnetude of v:

$$|v_f| = \sqrt{v_{fx}^2 + v_{fy}^2} = \sqrt{(40)^2 + (-15)^2} = 43\, m/s$$



Example 4.1 Motion in a Plane (continued)

(C) Determine the x and y coordinates of the particle at any time t and the position vector at this time.

Solution: We want to get *x* and *y* then **r** as (vector):

$$\because x_f = v_{xi}t + \frac{1}{2}a_x t^2 \Rightarrow x_f = 20t + \frac{1}{2}(4)t^2 = (20t + 2t^2)\,m$$

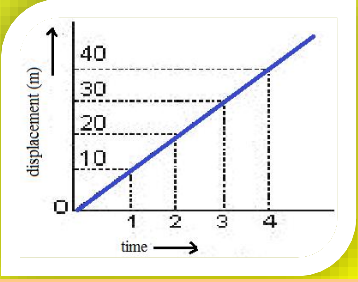
$$\because y_f = v_{yi}t + \frac{1}{2}a_y t^2 \Rightarrow y_f = -15t + \frac{1}{2}(0)t^2 = (-15t)\,m$$

$$\because r_f = x_f\,\hat{i} + y_f\,\hat{j} \Rightarrow r_f = [(20t + 2t^2)\hat{i} + -15t\hat{j}]\,m$$

to find magnetude of r_f at t=5 s:

$$|r_f| = \sqrt{x_{fx}^2 + y_{fy}^2} = \sqrt{150^2 + (-75)^2} = 170\,m$$

- *Please note: the last value is NOT the DISTANCE.*



Lecture Summary

- Displacement of a particle in 2-D is:

$$\Delta r = r_f - r_i \qquad (4.1)$$

- The average velocity is defined as:

$$\overline{v} = \frac{\Delta r}{\Delta t} = \frac{r_f - r_i}{t_f - t_i} \qquad (4.2)$$

- instantaneous velocity:

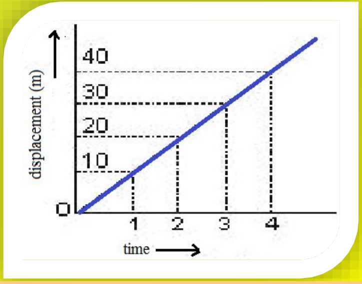
$$v = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{dr}{dt} \qquad (4.3)$$

- The average acceleration is defined as:

$$\overline{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i} \qquad (4.4)$$

- the instantaneous acceleration:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \qquad (4.5)$$



Lecture Summary (continued)

- Constant Acceleration motion of a particle in 2-D:

$$v_{xf} = v_{xi} + a_x t \qquad v_{yf} = v_{yi} + a_y t$$

$$x_f = v_{xi} t + \frac{1}{2} a_x t^2 \qquad y_f = v_{yi} t + \frac{1}{2} a_y t^2$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x x_f \qquad v_{yf}^2 = v_{yi}^2 + 2a_y y_f$$

- Velocity and position in Vector form in 2-D motion:

$$\because v = v_x \hat{i} + v_y \hat{j}$$

$$\therefore v_f = (v_{xi} + a_x t)\hat{i} + (v_{yi} + a_y t)\hat{j}$$

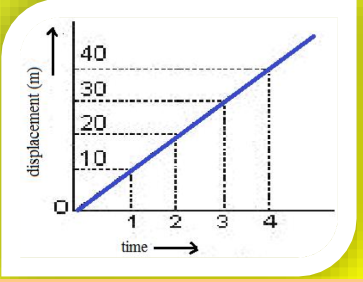
$$= (v_{xi}\hat{i} + v_{yi}\hat{j}) + (a_x \hat{i} + a_y \hat{j})\mathrm{t}$$

$$= v_i + at$$

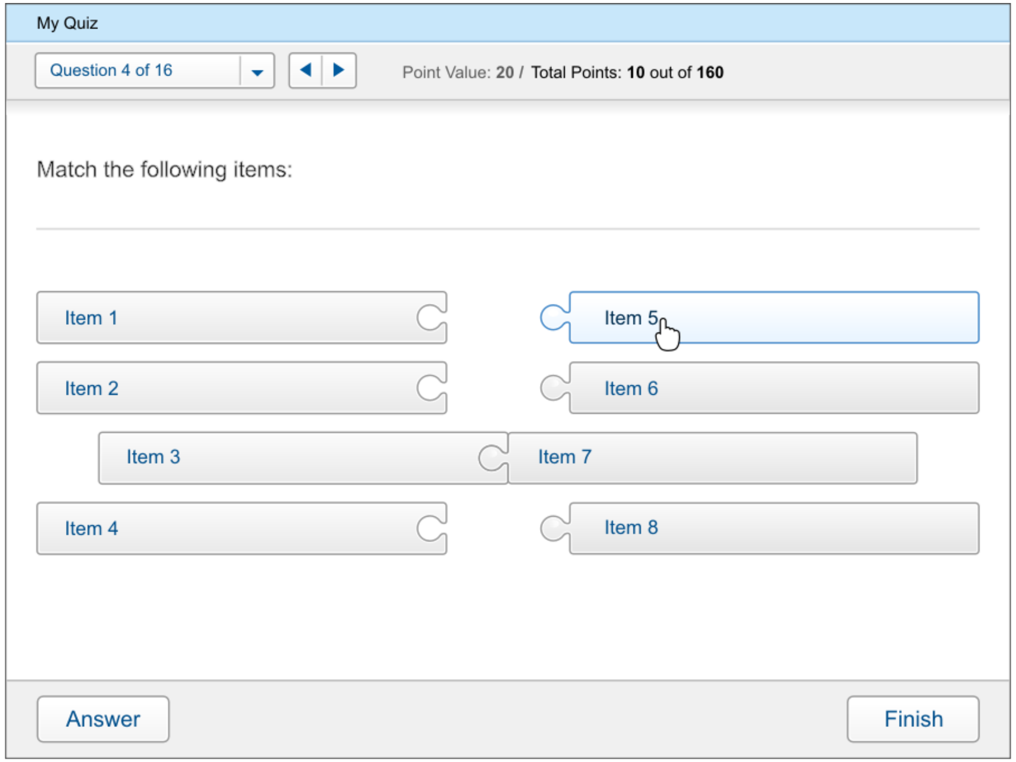
$$r_f = (v_{xi} t + \frac{1}{2} a_x t^2)\hat{i} + (v_{yi} t + \frac{1}{2} a_y t^2)\hat{j}$$

$$= (v_{xi}\hat{i} + v_{yi}\hat{j})t + \frac{1}{2}(a_x \hat{i} + a_y \hat{j})t^2$$

$$= v_i t + \frac{1}{2} a t^2$$



Practice Quiz 4.1 & 4.2

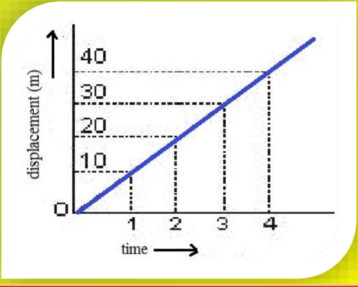


Click the **Quiz** button on iSpring Pro toolbar to edit your quiz

Please Run the Java Applet

- Below; there is a link to a local Java applet. You can run this applet by clicking on the link then following directions.
- from within the applet; please try all possibilities, velocity, position, etc.
- Please use mouse to drag the insect and watch vectors how they change.
- when done, click on Play back and watch.
- *Please Click below, then Save, then run from you computer*

2-D java Applet (click here)

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