

THIS PRESENTATION HAS BEEN PREPARED BY: DR. NASSRS, AIVAYED

## Lecture Outline

- Here is a quick list of the subjects that we will cover in this presentation. It is based on Serway, Ed. 6
- Applications on Newton's Laws
- Newton's Second Law Applied to Uniform Circular Motion
- Example 6.2 The Conical Pendulum
- Example 6.4 What Is the Maximum Speed of the Car?
- Example 6.5 The Banked Exit Ramp
- Lecture Summary
- Interactive Quiz
- Interactive Flash
- End of Presentation


## Newton's Sec ond Law

- Applying Newton’s Second Law to Uniform Circular Motion we get:

$$
a_{c}=\frac{v^{2}}{r}
$$

## (centripetal acceleration)

- The acceleration is called centripetal acceleration because ac is directed toward the center of the circle
$\downarrow \mathbf{a}_{\mathrm{c}}$ is always perpendicular to $\mathbf{v}$
- If we apply Newton's second law along the radial direction, we find that the net force causing the centripetal acceleration can be evaluated:

$$
\begin{equation*}
\sum \boldsymbol{F}=m a_{c}=m \frac{v^{2}}{r} \tag{6.1}
\end{equation*}
$$



## Circular Motion

$\Rightarrow$ A force causing a centripetal acceleration acts toward the center of the circular path and causes a change in the direction of the velocity vector.

- If that force should vanish, the object would no longer move in its circular path; instead, it would move along a straight-line path tangent to the circle.
- This idea is illustrated in the figure for the ball whirling at the end of a string in a horizontal plane. If the string breaks at some instant, the ball moves along the straight-line path tangent to the circle at the point where the string breaks.



## Example 6.2 The Conical Pendulum

$>$ A small object of mass $m$ is suspended from a string of length $L$. The object revolves with constant speed $v$ in a horizontal circle of radius $r$, as shown in Figure. (Because the string sweeps out the surface of a cone, the system is known as a conical pendulum.) Find an expression for $v$.

- Solution:
- We shall apply Newton's $2^{\text {nd }}$ law as we did before.
- We need first to analyze forces and apply the law in x , then y directions.

$$
\begin{align*}
& \sum \boldsymbol{F}_{x}=m a_{x}  \tag{1}\\
& \sum \boldsymbol{F}_{y}=m a_{y} \tag{2}
\end{align*}
$$



## Example 6.2 (continued)

- Solving we get:

$$
\begin{align*}
& (1) \Rightarrow T \cos \theta=m g  \tag{3}\\
& (2) \Rightarrow T \sin \theta=\mathrm{ma}_{\mathrm{c}}=m \frac{v^{2}}{r}  \tag{4}\\
& (4) \div(3): \Rightarrow \frac{T \sin \theta}{T \cos \theta}=\frac{m \frac{v^{2}}{r}}{m g} \Rightarrow \tan \theta=\frac{v^{2}}{g r} \\
& \therefore \mathrm{v}=\sqrt{g r \tan \theta}  \tag{5}\\
& \because r=L \sin \theta \\
& \therefore(5) \Rightarrow \mathrm{v}=\sqrt{L g \sin \theta \tan \theta}
\end{align*}
$$

## Example 6.4 A caron a Curve

- A 1 500-kg car moving on a flat, horizontal road negotiates a curve, as shown in Figure. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is 0.500 , find the maximum speed the car can have and still make the turn successfully.

- Solution: In this case, the force that enables the car to remain in its circular path is the force of static friction. (Static because no slipping occurs at the point of contact between road and tires.
- We shall apply Newton's $2^{\text {nd }}$ law.

(b)



## Example 6.4 (continued)

- Solving we get:

$$
\begin{align*}
& m=1500 \mathrm{~kg}, r=35 \mathrm{~m}, \mu_{s}=0.5 \\
& \because \sum \boldsymbol{F}_{x}=m a_{x} \tag{1}
\end{align*}
$$

$$
\begin{equation*}
\therefore f_{s}=m \frac{v^{2}}{r} \tag{2}
\end{equation*}
$$

$$
\begin{equation*}
\because f_{s}=\mu_{s} n=\mu_{s}(m g) \tag{3}
\end{equation*}
$$

$$
\therefore \mu_{s}(m g)=m T \frac{v^{2}}{r}
$$

$$
\begin{equation*}
\Rightarrow v=\sqrt{r \mu_{s} g} \tag{4}
\end{equation*}
$$

$$
\therefore v=\sqrt{(35)(0.5)(9.8)}=13.1 \mathrm{~m} / \mathrm{s}
$$

## Example 6.5 The Banked Exit Ramp

- A civil engineer wishes to design a curved exit ramp for a highway in such a way that a car will not have to rely on friction to round the curve without skidding. In other words, a car moving at the designated speed can negotiate the curve even when the road is covered with ice. Such a ramp is usually banked. Suppose the designated speed for the ramp is to be 13.4 and the radius of the curve is 50.0 m . At what angle should the curve be banked?
- Solution: We shall apply Newton's $2^{\text {nd }}$ law:

$$
\begin{align*}
& \sum F_{x}=m a_{x}  \tag{1}\\
& \sum F_{y}=m a_{y} \tag{2}
\end{align*}
$$



## Example 6.5 (continued)

- Solving we get:

$$
\begin{equation*}
r=50 \mathrm{~m}, v=13.4 \mathrm{~m} / \mathrm{s} \tag{3}
\end{equation*}
$$

(1) $\Rightarrow n \sin \theta=m \frac{v^{2}}{r}$
(2) $\Rightarrow n \cos \theta=m g$
$(3) \div(4) \Rightarrow: \tan \theta=\frac{v^{2}}{g r}$
$\therefore \theta=\tan ^{-1}\left(\frac{v^{2}}{g r}\right)=\tan ^{-1}\left(\frac{13.4^{2}}{(50)(9.8)}\right)=20.1^{\circ}$

- A driver who attempts to negotiate the curve at a speed greater than 13.4 $\mathrm{m} / \mathrm{s}$ has to depend on friction to keep from sliding up the bank.



## Lecture Summary

- Newton's second law applied to a particle moving in uniform circular motion states that the net force causing the particle to undergo a centripetal acceleration is:

$$
\begin{equation*}
\sum \boldsymbol{F}=m a_{c}=m \frac{v^{2}}{r} \tag{6.1}
\end{equation*}
$$

- A particle moving in a uniform circular motion has the centripetal acceleration give by:

$$
a_{c}=\frac{v^{2}}{r}
$$

(centripetal acceleration)

## Interactive Quiz

| My Quiz |  |  |
| :--- | :--- | :--- | :--- |
| Question 4 of 16 | Point Value: $20 /$ Total Points: 10 out of 160 |  |
| Match the following items: |  |  |
| Item 1 |  |  |
| Item 2 3 |  |  |
| Item 4 |  |  |
| Answer |  |  |

Click the Quiz button on
iCnrina Drn tnolhar tn odit unur

Interactive Fash



