بسم الله الرحمن الرحيم

King Saud University
College of Science
Physics & Astronomy Dept.

PHYS 103 (GENERAL PHYSICS)

CHAPTER 9: LINEAR MOMENTUM-II

LECTURE NO. 14

THIS PRESENTATION HAS BEEN PREPARED BY: *DR. NASSR S. ALZAYED*

Lecture Outline

- Here is a quick list of the subjects that we will cover in this presentation. It is based on Serway, Ed. 6

9.3 Collisions in One Dimension

- The total kinetic energy of the system of particles may or may not be conserved, depending on the type of collision. In fact, whether or not kinetic energy is conserved is used to classify collisions as either ***elastic or inelastic.***
- An elastic collision between two objects is one in which the total kinetic energy (as well as total momentum) of the system is the same before and after the collision.
- An inelastic collision is one in which the total kinetic energy of the system is not the same before and after the collision (even though the momentum of the system is conserved).
- Inelastic collisions are of two types. When the colliding objects stick together after the collision, the collision is called **perfectly inelastic**, When the colliding objects do not stick together, but some kinetic energy is lost, the collision is called **inelastic**.

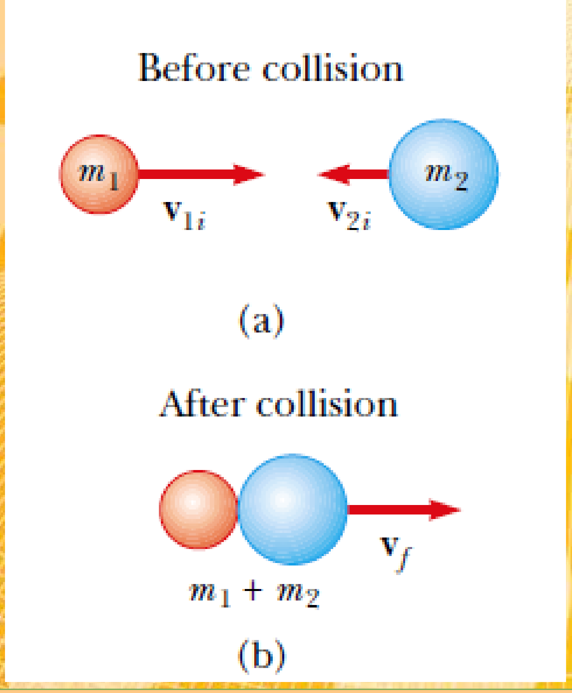
9.3 Perfectly Inelastic Collisions

- Consider two particles of masses m_1 and m_2 moving with initial velocities v_{1i} and v_{2i} along the same straight line, as shown in Figure. The two particles collide head-on, stick together, and then move with some common velocity v_f after the collision.

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f \qquad (9.13)$$

$$\Rightarrow$$

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2} \qquad (9.14)$$



- This is true only if the two objects
  Stick together in one-object.

9.3 Perfectly Elastic Collisions

- For this type of collisions: kinetic energy and liner momentum are conserved:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \qquad (9.15)$$

$$\frac{1}{2} m_1 v_{1i}^{\;2} + \frac{1}{2} m_2 v_{2i}^{\;2} = \frac{1}{2} m_1 v_{1f}^{\;2} + \frac{1}{2} m_2 v_{2f}^{\;2} \qquad (9.16)$$

- We can use (9.15) and (9.16) directly to solve our problems or simplify (9.16) to go directly to some special cases:

$$(9.16) \rightarrow m_1 v_{1i}^{\;2} + m_2 v_{2i}^{\;2} = m_1 v_{1f}^{\;2} + m_2 v_{2f}^{\;2}$$

$$\therefore m_1 (v_{1i}^{\;2} - v_{1f}^{\;2}) = m_2 (v_{2f}^{\;2} - v_{2i}^{\;2})$$

$$\therefore m_1 (v_{1i} - v_{1f})(v_{1i} + v_{1f}) = m_2 (v_{2f} - v_{2i})(v_{2f} + v_{2i}) \qquad (9.17)$$

$$(9.15) \rightarrow m_1 (v_{1i} - v_{1f}) = m_2 (v_{2f} - v_{2i}) \qquad (9.18)$$

9.3 Perfectly Elastic Collisions (continued)

- To obtain our final result, we divide Equation 9.17 by Equation 9.18 and obtain:

$$v_{1i} + v_{1f} = v_{2i} + v_{2f}$$

$$v_{1i} - v_{2i} = -(v_{1f} - v_{2f}) \qquad (9.19)$$

- Suppose that the masses and initial velocities of both particles are known:

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} + \left(\frac{2m_2}{m_1 + m_2}\right) v_{2i} \qquad (9.20)$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} + \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_{2i} \qquad (9.21)$$

- Let us consider some special cases. If $m_1 = m_2$, then Equations 9.20 and 9.21 show us that $v_{1f} = v_{2i}$ and $v_{2f} = v_{1i}$.

9.3 Perfectly Elastic Collisions (continued)

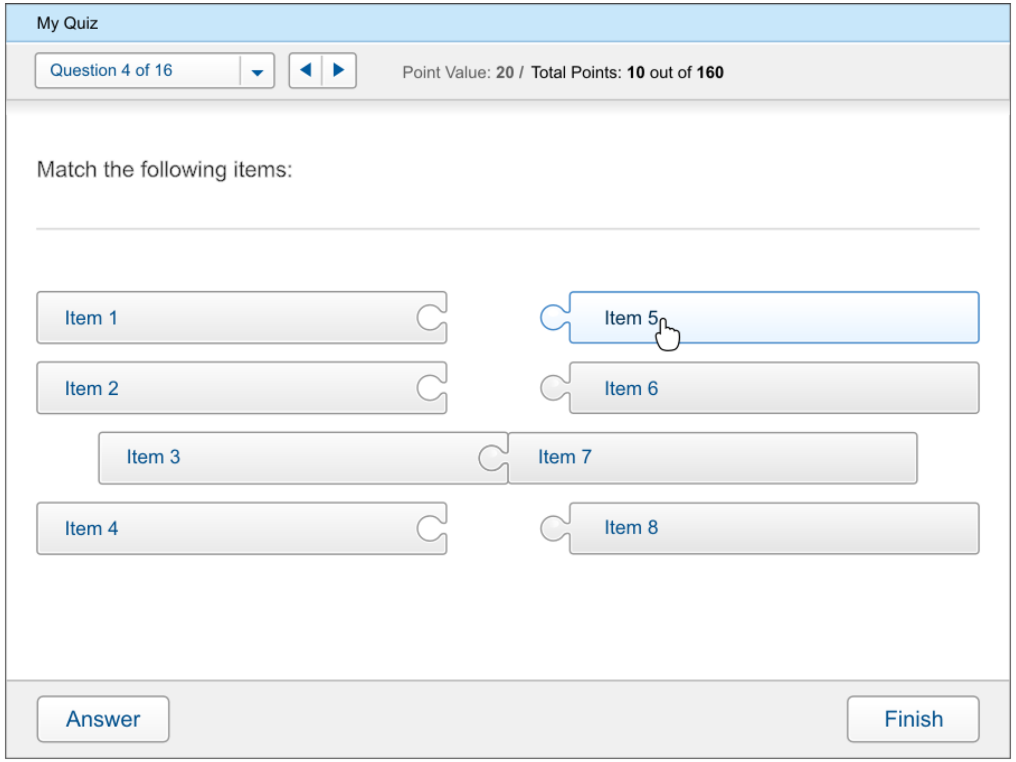
- If m_2 is initially at rest ➔ $v_{2i} = 0$
- and (9.20) (9.21) becomes:

$$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} \qquad (9.22)$$

$$v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} \qquad (9.23)$$

- If m_1 is much greater than m_2 and $v_{2i} = 0$, we see from Equations 9.22 and 9.23 that $v_{1f} \approx v_{1i}$ and $v_{2f} \approx 2v_{1i}$. That is, when a very heavy particle collides head-on with a very light one that is initially at rest, the heavy particle continues its motion unaltered after the collision and the light particle rebounds with a speed equal to about twice the initial speed of the heavy particle.

Interactive Quiz



Click the **Quiz** button on iSpring Pro toolbar to edit your quiz

Example 9.6 Carry Collision Insurance!

- An 1800-kg car stopped at a traffic light is struck from the rear by a 900-kg car, and the two become entangled, moving along the same path as that of the originally moving car. If the smaller car were moving at 20.0 m/s before the collision, *what is the velocity of the entangled cars after the collision?*
- ***Solution:***

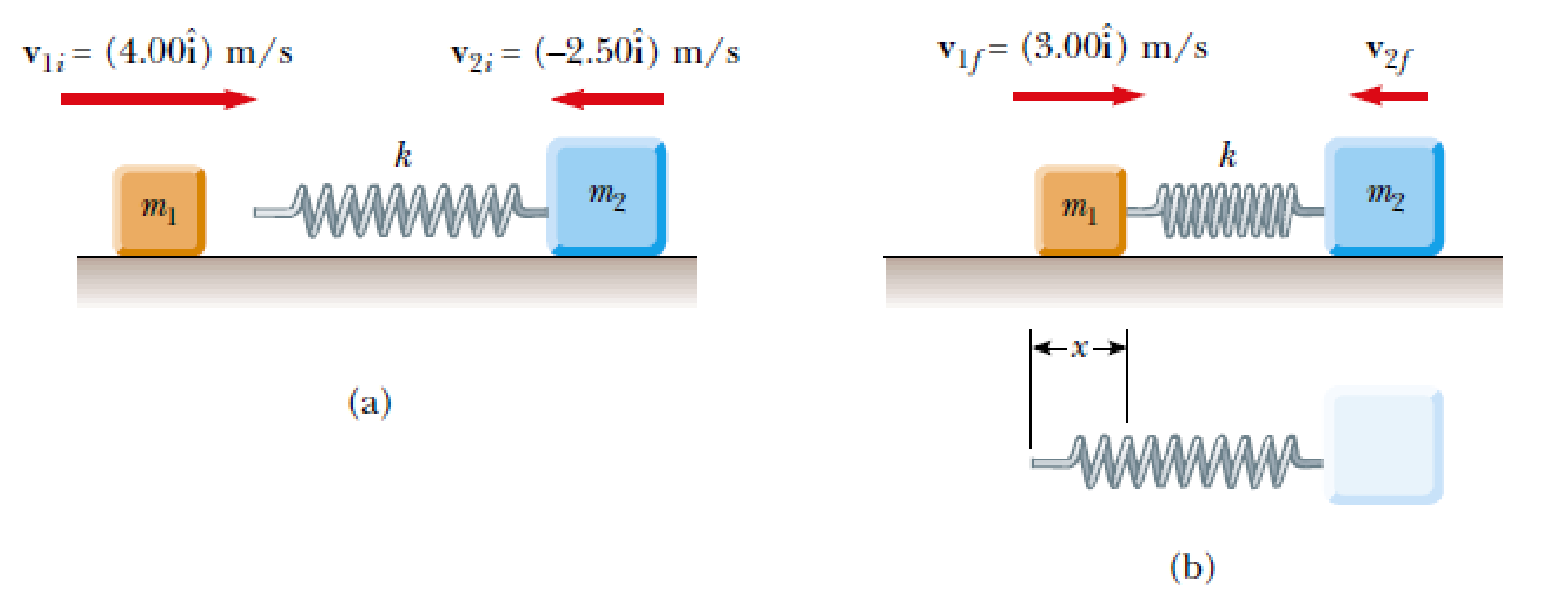
$$\because m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\therefore (1800)(0) + (900)(20) = (1800 + 900) v_f$$

$$\Rightarrow v_f = \frac{900 \times 20}{2700} = 6.67\, m/s$$

Example 9.8 A Two-Body Collision with a Spring

- A block of mass m_1 = 1.60 kg initially moving to the right with a speed of 4.00 m/s on a frictionless horizontal track collides with a spring attached to a second block of mass m_2 =2.10 kg initially moving to the left with a speed of 2.50 m/s. The spring constant is 600 N/m.
- *(A) Find the velocities of the two blocks after the collision*



Example 9.8 (Continued)

- ***Solution:***

$$\because m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\therefore (1.6)(4)+(2.10)(-2.5)=(1.6)v_{1f}+(2.10)v_{2f} \qquad (1)$$

$$(9.19) \rightarrow v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$$

$$\therefore (4) - (-2.5) = -v_{1f} + v_{2f}$$

$$\therefore 6.5 = -v_{1f} + v_{2f} \qquad (2)$$

$$(2) \times 1.6 \rightarrow 10.4 = (1.6)(-v_{1f}) + (1.6)(v_{2f}) \qquad (3)$$

$$(1) + (3) : 11.55 = 3.7 v_{2f}$$

$$\Rightarrow v_{2f} = \frac{11.55}{3.7} = 3.12\, m/s \qquad (4)$$

$$(4) \text{ in } (2) : v_{1f} = -3.38\, m/s \qquad (5)$$

Example 9.8 (Continued)

- *(B) During the collision, at the instant block 1 is moving to the right with a velocity of +3.00 m/s, determine the velocity of block 2.*

$$\because m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\therefore (1.6)(4)+(2.10)(-2.5)=(1.6)(3)+(2.10)v_{2f} \Rightarrow v_{2f} = -1.74\, m/s$$

- *(C) Determine the distance the spring is compressed at that instant.*

$$\because K_i + U_i = K_f + U_f$$

$$\therefore \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 + 0 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2 + \frac{1}{2} k x^2$$

$$\Rightarrow \frac{1}{2}(1.6)(4)^2 + \frac{1}{2}(2.1)(-2.5)^2 = \frac{1}{2}(1.6)(3)^2 + \frac{1}{2}(2.1)(-1.74)^2 + \frac{1}{2}(600)x^2$$

$$\therefore x = \sqrt{\frac{8.98 \times 2}{600}} = 0.173\, m$$

PROBLEM-SOLVING HINTS

- Set up a coordinate system and define your velocities with respect to that system.
- In your sketch of the coordinate system, draw and label all velocity vectors and include all the given information.
- Write expressions for the *x and y components of the momentum of each* object before and after the collision.
- Write expressions for the total momentum of the system in the *x direction* before and after the collision and equate the two.
- If the collision is inelastic, kinetic energy of the system is *not conserved*, *and* additional information is probably required.
- If the collision is *perfectly* inelastic, the final velocities of the two objects are equal. Solve the momentum equations for the unknown quantities.
- If the collision is *elastic*, *kinetic energy of the system is conserved*, *and you can* equate the total kinetic energy before the collision to the total kinetic energy after the collision to obtain an additional relationship between the velocities.

Lecture Summary

- Perfectly Inelastic Collisions:

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f \qquad (9.13)$$

- Perfectly Elastic Collisions:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \qquad (9.15)$$

$$\frac{1}{2} m_1 {v_{1i}}^2 + \frac{1}{2} m_2 {v_{2i}}^2 = \frac{1}{2} m_1 {v_{1f}}^2 + \frac{1}{2} m_2 {v_{2f}}^2 \qquad (9.16)$$

- When two particles collide, the total momentum of the isolated system before the collision always equals the total momentum after the collision, regardless of the nature of the collision. An inelastic collision is one for which the total kinetic energy of the system is not conserved. A perfectly inelastic collision is one in which the colliding bodies stick together after the collision. An elastic collision is one in which the kinetic energy of the system is conserved.

Interactive Flash

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THE END!