PHYS 104
1^{ST} semester 1439-1440

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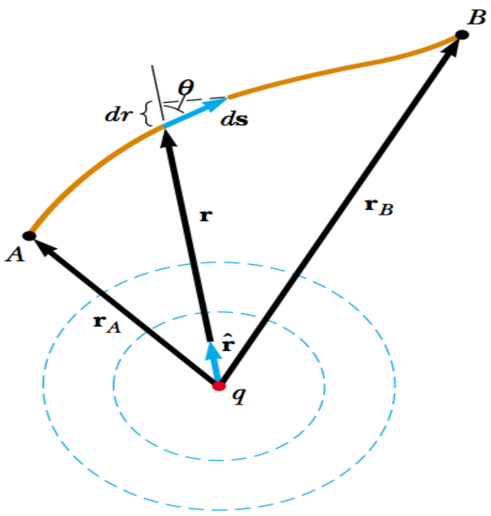
Lecture 12

25.3 Electric Potential and Potential Energy Due to Point Charges



- The electric potential at a point located a distance *r* from the charge, then the potential difference
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$$V_B - V_A = -\int_A^B \mathbf{E} \cdot d\mathbf{s}$$

$$\mathbf{E} \cdot d\mathbf{s} = k_e \frac{q}{r^2} \hat{\mathbf{r}} \cdot d\mathbf{s}$$

Because the magnitude of $\hat{\mathbf{r}}$ is 1, the dot product $\hat{\mathbf{r}} \cdot d\mathbf{s} = ds \cos \theta$, where θ is the angle between $\hat{\mathbf{r}}$ and $d\mathbf{s}$. Furthermore, $ds \cos \theta$ is the projection of $d\mathbf{s}$ onto $\mathbf{r}$; thus, $ds \cos \theta = dr$. That is, any displacement $d\mathbf{s}$ along the path from point A to point B produces a change dr in the magnitude of $\mathbf{r}$, the position vector of the point relative to the charge creating the field. Making these substitutions, we find that $\mathbf{E} \cdot d\mathbf{s} = (k_e q / r^2)\, dr$; hence, the expression for the potential difference becomes

$$V_B - V_A = -k_e q \int_{r_A}^{r_B} \frac{dr}{r^2} = \frac{k_e q}{r}\bigg]_{r_A}^{r_B}$$

$$V_B - V_A = k_e q \left[\frac{1}{r_B} - \frac{1}{r_A} \right]$$

25.3 Electric Potential and Potential Energy Due to Point Charges

- This equation shows us that the integral of **E**.*d***s** is *independent* of the path between points *A* and *B*.
- It is customary to choose the reference of electric potential for a point charge to be $V = 0$ at $r_A = \infty$. With this reference choice, the electric potential created by a point charge at any distance r from the charge is

$$V = k_e \frac{q}{r}$$

25.3 Electric Potential and Potential Energy Due to Point Charges

- The total electric potential at some point *P* due to several point charges is the sum of the potentials due to the individual charges.

$$V = k_e \sum_i \frac{q_i}{r_i}$$

- Note that the sum is an algebraic sum of scalars rather than a vector sum.

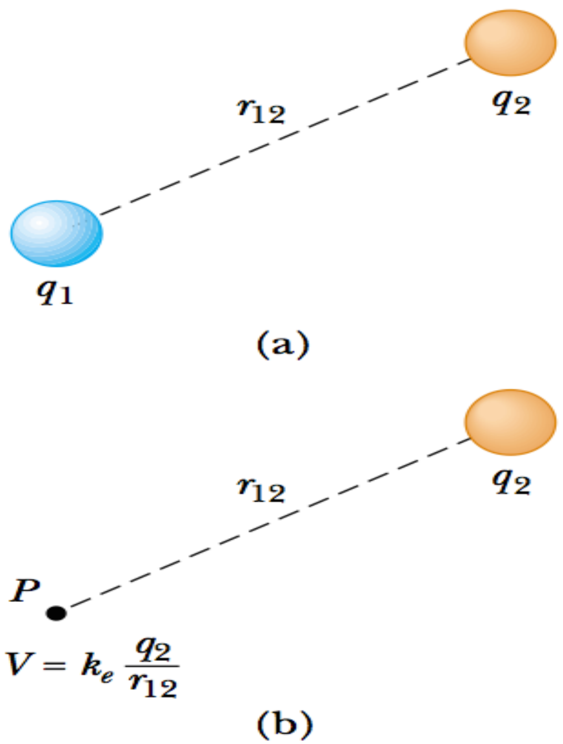
25.3 Electric Potential and Potential Energy Due to Point Charges

- We now consider the potential energy of a system of **two charged particles**.
- If V_2 is the electric potential at a point P due to charge q_2, then the work an external agent must do to bring a second charge q_1 from infinity to P is $q_1 V_2$.
- The potential energy of the system

$$U = k_e \frac{q_1 q_2}{r_{12}}$$

- If we have removed the charge q_1, a potential due to charge q_2 as

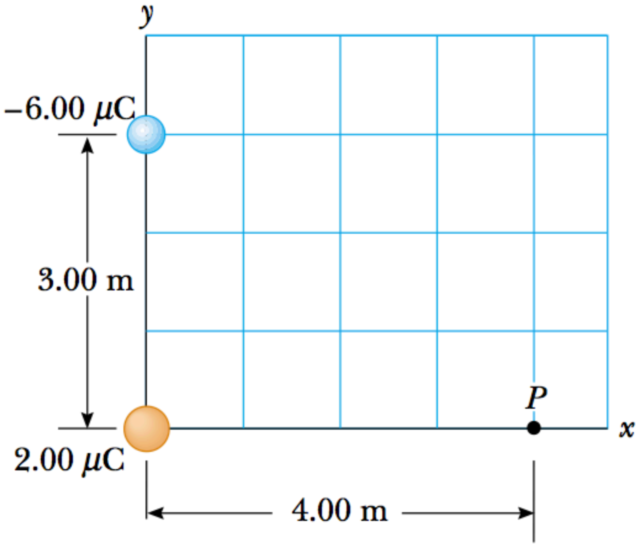
$$V = U/q_1 = k_e\, q_2/r_{12}.$$



Example 25.3 **The Electric Potential Due to Two Point Charges**

A charge $q_1 = 2.00\ \mu C$ is located at the origin, and a charge $q_2 = -6.00\ \mu C$ is located at (0, 3.00) m, as shown in Figure 25.12a.

(A) Find the total electric potential due to these charges at the point *P*, whose coordinates are (4.00, 0) m.



(a)

Solution For two charges, the sum in Equation 25.12 gives

$$V_P = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$V_P = (8.99 \times 10^9\ \mathrm{N \cdot m^2/C^2})$$

$$\times\left(\frac{2.00 \times 10^{-6}\ \mathrm{C}}{4.00\ \mathrm{m}} - \frac{6.00 \times 10^{-6}\ \mathrm{C}}{5.00\ \mathrm{m}}\right)$$

$$= \boxed{-6.29 \times 10^3\ \mathrm{V}}$$

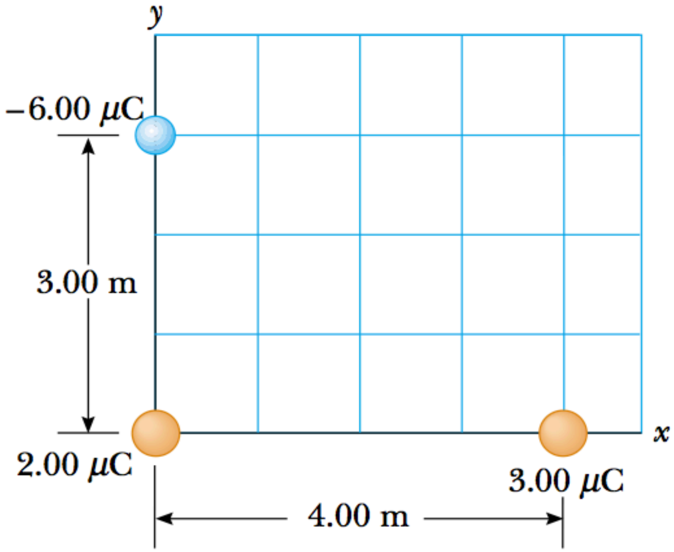
Example 25.3 **The Electric Potential Due to Two Point Charges**

(B) Find the change in potential energy of the system of two charges plus a charge $q_3 = 3.00\ \mu C$ as the latter charge moves from infinity to point P (Fig. 25.12b).

Solution When the charge q_3 is at infinity, let us define $U_i = 0$ for the system, and when the charge is at P, $U_f = q_3 V_P$; therefore,

$$\Delta U = q_3 V_P - 0 = (3.00 \times 10^{-6}\ \mathrm{C})(-6.29 \times 10^{3}\ \mathrm{V})$$

$$= \boxed{-1.89 \times 10^{-2}\ \mathrm{J}}$$



(b)

SUMMARY

When a positive test charge q_0 is moved between points A and B in an electric field $\mathbf{E}$, the **change in the potential energy of the charge–field system** is

$$\Delta U = -q_0 \int_A^B \mathbf{E} \cdot d\mathbf{s} \qquad (25.1)$$

The **electric potential** $V = U/q_0$ is a scalar quantity and has the units of J/C, where 1 J/C ≡ 1 V.

The **potential difference** ΔV between points A and B in an electric field $\mathbf{E}$ is defined as

$$\Delta V \equiv \frac{\Delta U}{q_0} = -\int_A^B \mathbf{E} \cdot d\mathbf{s} \qquad (25.3)$$

The potential difference between two points A and B in a uniform electric field $\mathbf{E}$, where $\mathbf{s}$ is a vector that points from A to B and is parallel to $\mathbf{E}$ is

$$\Delta V = -Ed \qquad (25.6)$$

where $d = |\mathbf{s}|$.

An **equipotential surface** is one on which all points are at the same electric potential. Equipotential surfaces are perpendicular to electric field lines.

If we define $V = 0$ at $r_A = \infty$, the electric potential due to a point charge at any distance r from the charge is

$$V = k_e \frac{q}{r} \qquad (25.11)$$

We can obtain the electric potential associated with a group of point charges by summing the potentials due to the individual charges.

The **potential energy associated with a pair of point charges** separated by a distance r_{12} is

$$U = k_e \frac{q_1 q_2}{r_{12}} \qquad (25.13)$$

This energy represents the work done by an external agent when the charges are brought from an infinite separation to the separation r_{12}. We obtain the potential energy of a distribution of point charges by summing terms like Equation 25.13 over all pairs of particles.

Chapter 26
Capacitance and Dielectrics

26.1 Definition of Capacitance

- Consider two conductors carrying charges of equal magnitude and opposite sign.
- Such a combination of two conductors is called a **capacitor.**
- The conductors are called *plates.*
- A potential difference ΔV exists between the conductors due to the presence of the charges.

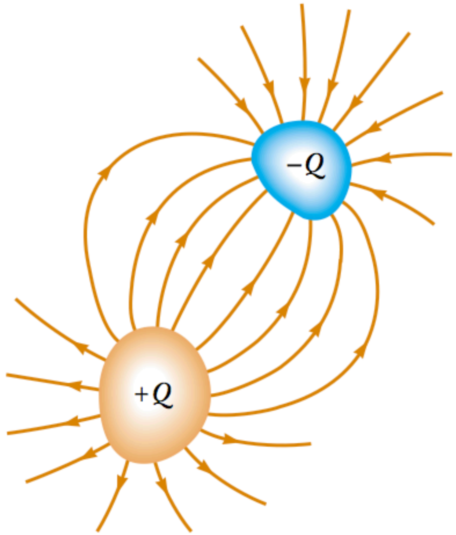


Figure 26.1 A capacitor consists of two conductors. When the capacitor is charged, the conductors carry charges of equal magnitude and opposite sign.

The **capacitance** C of a capacitor is defined as the ratio of the magnitude of the charge on either conductor to the magnitude of the potential difference between the conductors:

$$C \equiv \frac{Q}{\Delta V} \tag{26.1}$$

26.1 Definition of Capacitance

- The SI unit of capacitance is the **farad** (F)

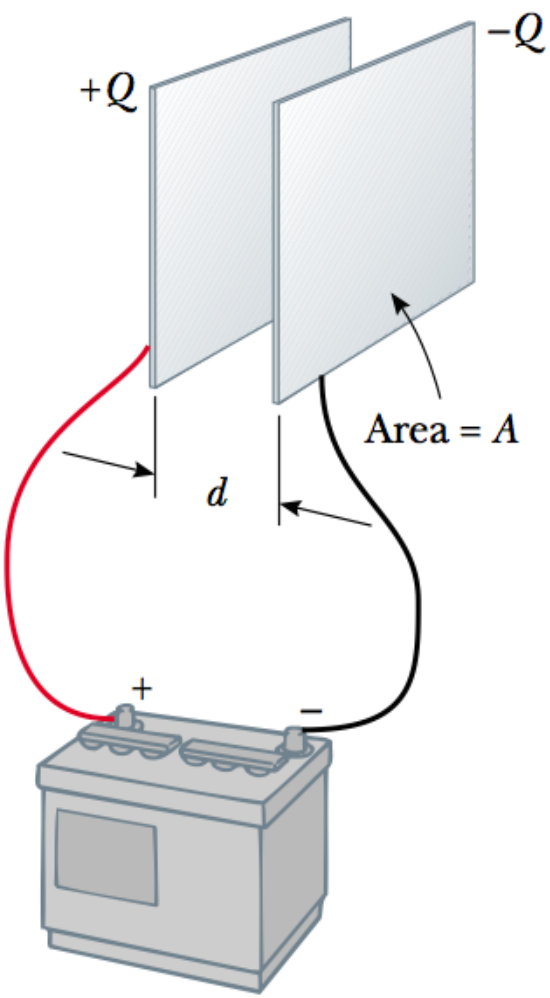
$$1\ \text{F} = 1\ \text{C/V}$$

- The farad is a very large unit of capacitance. In practice, typical devices have capacitances ranging from microfarads (10^{-6} F) to picofarads (10^{-12} F).

Quick Quiz 26.1 A capacitor stores charge Q at a potential difference ΔV. If the voltage applied by a battery to the capacitor is doubled to $2\Delta V$, (a) the capacitance falls to half its initial value and the charge remains the same (b) the capacitance and the charge both fall to half their initial values (c) the capacitance and the charge both double (d) the capacitance remains the same and the charge doubles.

26.1 Definition of Capacitance

- Let us consider a capacitor formed from a pair of parallel plates.
- Each plate is connected to one terminal of a battery, which acts as a source of potential difference.
- The plate connected to the **negative** terminal of the battery. The electric field applies a force on electrons in the wire.
- This force causes the electrons to move onto the plate. This movement continues until the plate, the wire, and the terminal are all at the same electric potential.
- Once this equilibrium point is attained, a potential difference no longer exists between the terminal and the plate, and as a result no electric field is present in the wire, and the movement of electrons stops.
- The plate now carries a **negative charge**.
- A similar process occurs at the other capacitor plate, with electrons moving from the plate to the wire
- Finally, the potential difference across the capacitor plates is the same as that between the terminals of the battery.



26.2 Calculating Capacitance

- For example, imagine a **spherical** charged conductor.
- The electric potential of the sphere of radius *R* is simply k_eQ/R, and setting V=0 for the infinitely large shell, we have

$$C = \frac{Q}{\Delta V} = \frac{Q}{k_e Q / R} = \frac{R}{k_e} = 4\pi\epsilon_0 R$$

- This expression shows that the capacitance of an isolated charged sphere is proportional to its **radius** and is independent of both the charge on the sphere and the potential difference.
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Parallel-Plate Capacitors

- Two parallel metallic plates of equal area *A* are separated by a distance *d*. One plate carries a charge *Q*, and the other carries a charge -*Q* .
- The **surface charge density** on either plate is σ = *Q* /*A.*
- If the plates are very close together, we can assume that the **electric field** is **uniform** between the plates.

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

- The magnitude of the potential difference between the plates

$$\Delta V = Ed = \frac{Qd}{\epsilon_0 A}$$

- The capacitance is

$$C = \frac{Q}{\Delta V} = \frac{Q}{Qd/\epsilon_0 A}$$

$$C = \frac{\epsilon_0 A}{d}$$

- The capacitance of a parallel-plate capacitor is proportional to the area of its plates and inversely proportional to the plate separation.

Parallel-Plate Capacitors

Example 26.1 Parallel-Plate Capacitor

A parallel-plate capacitor with air between the plates has an area $A = 2.00 \times 10^{-4}\ \text{m}^2$ and a plate separation $d = 1.00$ mm. Find its capacitance.

Solution From Equation 26.3, we find that

$$C = \frac{\epsilon_0 A}{d} = \frac{(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)(2.00 \times 10^{-4}\ \text{m}^2)}{1.00 \times 10^{-3}\ \text{m}}$$

$$= 1.77 \times 10^{-12}\ \text{F} = \boxed{1.77\ \text{pF}}$$