PHYS 104
1ST semester 1439-1440

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Lecture 18

Chapter 28

Direct Current Circuits

Example 28.4 Find the Equivalent Resistance

Four resistors are connected as shown in Figure 28.9a.

(A) Find the equivalent resistance between points a and c.

Solution The combination of resistors can be reduced in steps, as shown in Figure 28.9. The 8.0-Ω and 4.0-Ω resistors are in series; thus, the equivalent resistance between a and b is 12.0 Ω (see Eq. 28.5). The 6.0-Ω and 3.0-Ω resistors are in parallel, so from Equation 28.7 we find that the equivalent resistance from b to c is 2.0 Ω. Hence, the equivalent resistance from a to c is 14.0 Ω.

(B) What is the current in each resistor if a potential difference of 42 V is maintained between a and c?

Solution The currents in the 8.0-Ω and 4.0-Ω resistors are the same because they are in series. In addition, this is the same as the current that would exist in the 14.0-Ω equivalent resistor subject to the 42-V potential difference. Therefore, using Equation 27.8 ($R = \Delta V/I$) and the result from part (A), we obtain

$$I = \frac{\Delta V_{ac}}{R_{\text{eq}}} = \frac{42\text{ V}}{14.0\ \Omega} = 3.0\text{ A}$$

This is the current in the 8.0-Ω and 4.0-Ω resistors. When this 3.0-A current enters the junction at b, however, it splits, with part passing through the 6.0-Ω resistor (I_1) and part through the 3.0-Ω resistor (I_2). Because the potential difference is ΔV_{bc} across each of these parallel resistors, we see that $(6.0\ \Omega)I_1 = (3.0\ \Omega)I_2$, or $I_2 = 2I_1$. Using this result and the fact that $I_1 + I_2 = 3.0$ A, we find that $I_1 = 1.0$ A and $I_2 = 2.0$ A. We could have guessed this at the start by noting that the current in the 3.0-Ω resistor has to be twice that in the 6.0-Ω resistor, in view of their relative resistances and the fact that the same voltage is applied to each of them.

As a final check of our results, note that $\Delta V_{bc} = (6.0\ \Omega)I_1 = (3.0\ \Omega)I_2 = 6.0$ V and $\Delta V_{ab} = (12.0\ \Omega)I = 36$ V; therefore, $\Delta V_{ac} = \Delta V_{ab} + \Delta V_{bc} = 42$ V, as it must.

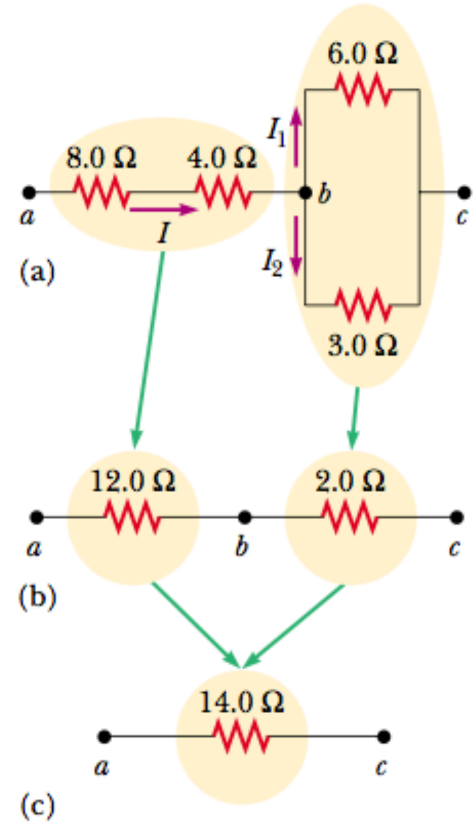


Figure 28.9 (Example 28.4) The original network of resistors is reduced to a single equivalent resistance.

Example 28.6 Three Resistors in Parallel

Interactive

Three resistors are connected in parallel as shown in Figure 28.11a. A potential difference of 18.0 V is maintained between points *a* and *b*.

(A) Find the current in each resistor.

Solution The resistors are in parallel, and so the potential difference across each must be 18.0 V. Applying the relationship $\Delta V = IR$ to each resistor gives

$$I_1 = \frac{\Delta V}{R_1} = \frac{18.0\ \text{V}}{3.00\ \Omega} = \boxed{6.00\ \text{A}}$$

$$I_2 = \frac{\Delta V}{R_2} = \frac{18.0\ \text{V}}{6.00\ \Omega} = \boxed{3.00\ \text{A}}$$

$$I_3 = \frac{\Delta V}{R_3} = \frac{18.0\ \text{V}}{9.00\ \Omega} = \boxed{2.00\ \text{A}}$$

(B) Calculate the power delivered to each resistor and the total power delivered to the combination of resistors.

Solution We apply the relationship $\mathcal{P} = I^2R$ to each resistor and obtain

$$3.00\text{-}\Omega: \quad \mathcal{P}_1 = I_1{}^2R_1 = (6.00\ \text{A})^2(3.00\ \Omega) = \boxed{108\ \text{W}}$$

$$6.00\text{-}\Omega: \quad \mathcal{P}_2 = I_2{}^2R_2 = (3.00\ \text{A})^2(6.00\ \Omega) = \boxed{54.0\ \text{W}}$$

$$9.00\text{-}\Omega: \quad \mathcal{P}_3 = I_3{}^2R_3 = (2.00\ \text{A})^2(9.00\ \Omega) = \boxed{36.0\ \text{W}}$$

This shows that the smallest resistor receives the most power. Summing the three quantities gives a total power of 198 W.

(C) Calculate the equivalent resistance of the circuit.

Solution We can use Equation 28.8 to find R_{eq}:

$$\frac{1}{R_{eq}} = \frac{1}{3.00\ \Omega} + \frac{1}{6.00\ \Omega} + \frac{1}{9.00\ \Omega}$$

$$R_{eq} = \frac{18.0\ \Omega}{11.0} = \boxed{1.64\ \Omega}$$

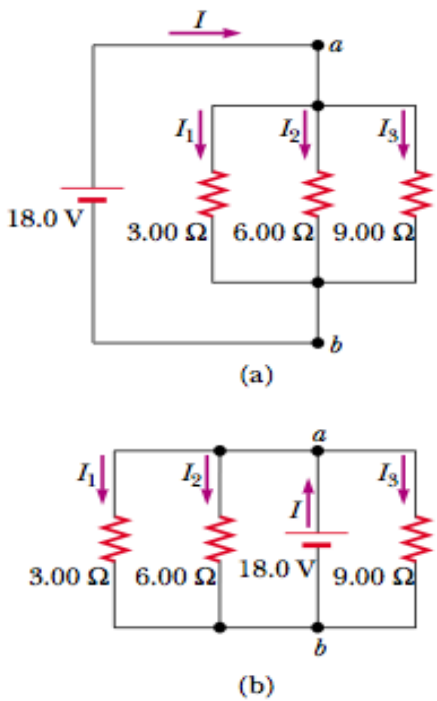


Figure 28.11 (Example 28.6) (a) Three resistors connected in parallel. The voltage across each resistor is 18.0 V. (b) Another circuit with three resistors and a battery. Is this equivalent to the circuit in part (a) of the figure?

28.3 Kirchhoff's Rules

- It is not possible to reduce a circuit to a single loop. The procedure for analyzing more complex circuits is greatly simplified if we use two principles called **Kirchhoff 's rules:**

1. **Junction rule.** The sum of the currents entering any junction in a circuit must equal the sum of the currents leaving that junction:

$$\sum I_{\text{in}} = \sum I_{\text{out}} \qquad (28.9)$$

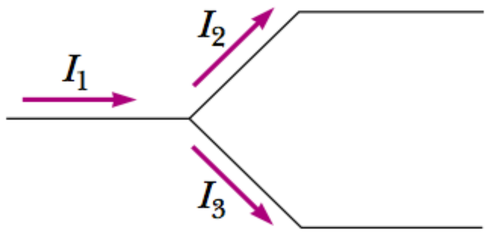
2. **Loop rule.** The sum of the potential differences across all elements around any closed circuit loop must be zero:

$$\sum_{\substack{\text{closed}\\ \text{loop}}} \Delta V = 0 \qquad (28.10)$$

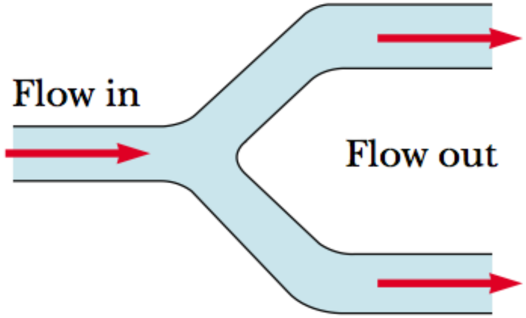
28.3 Kirchhoff's Rules

- **Junction rule**
  - Kirchhoff's first rule is a statement of conservation of electric charge.
  - All charges that enter a given point in a circuit must leave that point because charge cannot build up at a point.

$$I_1 = I_2 + I_3$$



(a)



(b)

- **Loop rule**
  - Kirchhoff's second rule follows from the law of conservation of energy.

28.3 Kirchhoff's Rules

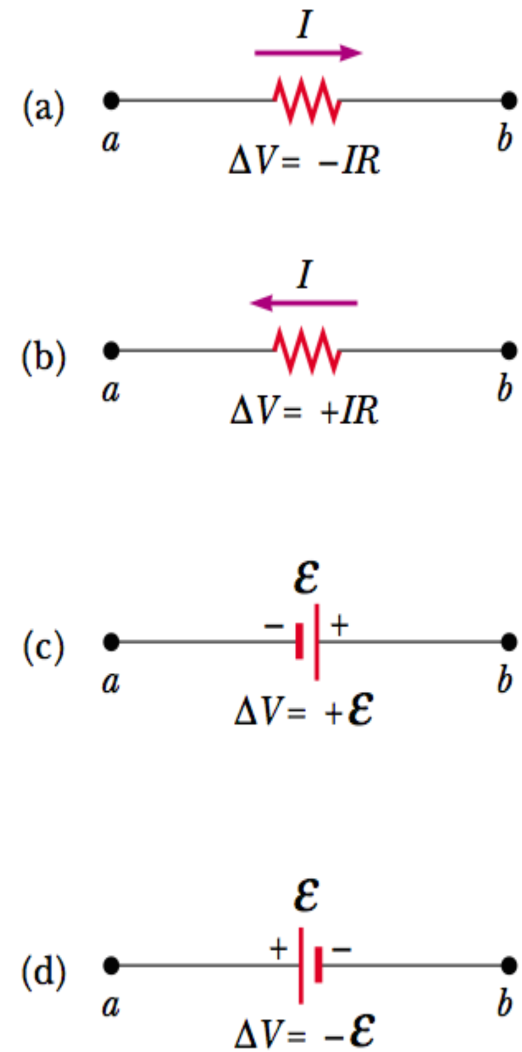
- **Loop rule**
- Rules for determining the potential differences across a resistor and a battery:

(a) Because charges move from the high-potential end of a resistor toward the low-potential end, if a resistor is traversed in the direction of the current, the potential difference ΔV across the resistor is $-IR$

(b) If a resistor is traversed in the direction *opposite* the current, the potential difference ΔV across the resistor is $+IR$.

(c) If a source of emf (assumed r=0) is traversed in the direction of the emf (from – to +), the potential difference ΔV is $+\varepsilon$. The emf of the battery increases the electric potential as we move through it in this direction.

(d) If a source of emf (assumed r=0) is traversed in the direction opposite the emf (from + to -), the potential difference ΔV is $-\varepsilon$. The emf of the battery reduces the electric potential as we move through it.



28.3 Kirchhoff's Rules

- In general, in order to solve a particular circuit problem, the number of independent equations you need to obtain from the two rules equals the number of unknown currents.
- **Any capacitor acts as an open branch in a circuit;** that is, the current in the branch containing the capacitor is zero under steady-state condition-that is, the currents in the various branches are constant.

PROBLEM-SOLVING HINTS

Kirchhoff's Rules

- Draw a circuit diagram, and label all the known and unknown quantities. You must assign a *direction* to the current in each branch of the circuit. Although the assignment of current directions is arbitrary, you must adhere rigorously to the assigned directions when applying Kirchhoff's rules.
- Apply the junction rule to any junctions in the circuit that provide new relationships among the various currents.
- Apply the loop rule to as many loops in the circuit as are needed to solve for the unknowns. To apply this rule, you must correctly identify the potential difference as you imagine crossing each element while traversing the closed loop (either clockwise or counterclockwise). Watch out for errors in sign!
- Solve the equations simultaneously for the unknown quantities. Do not be alarmed if a current turns out to be negative; *its magnitude will be correct and the direction is opposite to that which you assigned.*

Example 28.8 A Single-Loop Circuit

A single-loop circuit contains two resistors and two batteries, as shown in Figure 28.16. (Neglect the internal resistances of the batteries.)

(A) Find the current in the circuit.

Solution We do not need Kirchhoff's rules to analyze this simple circuit, but let us use them anyway just to see how they are applied. There are no junctions in this single-loop circuit; thus, the current is the same in all elements. Let us assume that the current is clockwise, as shown in Figure 28.16. Traversing the circuit in the clockwise direction, starting at a, we see that $a \to b$ represents a potential difference of $+\mathcal{E}_1$, $b \to c$ represents a potential difference of $-IR_1$, $c \to d$ represents a potential difference of $-\mathcal{E}_2$, and $d \to a$ represents a potential difference of $-IR_2$. Applying Kirchhoff's loop rule gives

$$\sum \Delta V = 0$$

$$\mathcal{E}_1 - IR_1 - \mathcal{E}_2 - IR_2 = 0$$

Solving for I and using the values given in Figure 28.16, we obtain

$$(1) \qquad I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = \frac{6.0\ \text{V} - 12\ \text{V}}{8.0\ \Omega + 10\ \Omega} = \boxed{-0.33\ \text{A}}$$

The negative sign for I indicates that the direction of the current is opposite the assumed direction. Notice that the emfs in the numerator subtract because the batteries have opposite polarities in Figure 28.16. In the denominator, the resistances add because the two resistors are in series.

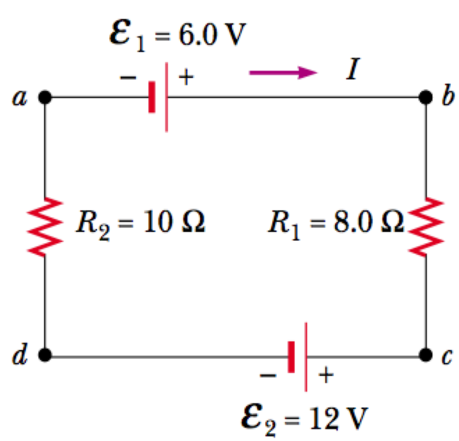


Figure 28.16 (Example 28.8) A series circuit containing two batteries and two resistors, where the polarities of the batteries are in opposition.

(B) What power is delivered to each resistor? What power is delivered by the 12-V battery?

Solution Using Equation 27.23,

$$\mathcal{P}_1 = I^2R_1 = (0.33\ \text{A})^2(8.0\ \Omega) = \boxed{0.87\ \text{W}}$$

$$\mathcal{P}_2 = I^2R_2 = (0.33\ \text{A})^2(10\ \Omega) = \boxed{1.1\ \text{W}}$$

Hence, the total power delivered to the resistors is $\mathcal{P}_1 + \mathcal{P}_2 = 2.0$ W.

The 12-V battery delivers power $I\mathcal{E}_2 = 4.0$ W. Half of this power is delivered to the two resistors, as we just calculated. The other half is delivered to the 6-V battery, which is being charged by the 12-V battery. If we had included the internal resistances of the batteries in our analysis, some of the power would appear as internal energy in the batteries; as a result, we would have found that less power was being delivered to the 6-V battery.

What If? What if the polarity of the 12.0-V battery were reversed? How would this affect the circuit?

Answer While we could repeat the Kirchhoff's rules calculation, let us examine Equation (1) and modify it accordingly. Because the polarities of the two batteries are now in the same direction, the signs of $\mathcal{E}_1$ and $\mathcal{E}_2$ are the same and Equation (1) becomes

$$I = \frac{\mathcal{E}_1 + \mathcal{E}_2}{R_1 + R_2} = \frac{6.0\text{ V} + 12\text{ V}}{8.0\ \Omega + 10\ \Omega} = 1.0\text{ A}$$

The new powers delivered to the resistors are

$$\mathcal{P}_1 = I^2 R_1 = (1.0\text{ A})^2(8.0\ \Omega) = 8.0\text{ W}$$

$$\mathcal{P}_2 = I^2 R_2 = (1.0\text{ A})^2(10\ \Omega) = 10\text{ W}$$

This totals 18 W, nine times as much as in the original circuit, in which the batteries were opposing each other.

Example 28.9 Applying Kirchhoff's Rules

Interactive

Find the currents I_1, I_2, and I_3 in the circuit shown in Figure 28.17.

Solution Conceptualize by noting that we cannot simplify the circuit by the rules of adding resistances in series and in parallel. (If the 10.0-V battery were taken away, we could reduce the remaining circuit with series and parallel combinations.) Thus, we categorize this problem as one in which we must use Kirchhoff's rules. To analyze the circuit, we arbitrarily choose the directions of the currents as labeled in Figure 28.17. Applying Kirchhoff's junction rule to junction *c* gives

$$(1) \qquad I_1 + I_2 = I_3$$

We now have one equation with three unknowns—I_1, I_2, and I_3. There are three loops in the circuit—*abcda, befcb,* and *aefda*. We therefore need only two loop equations to determine the unknown currents. (The third loop equation would give no new information.) Applying Kirchhoff's loop rule to loops *abcda* and *befcb* and traversing these loops clockwise, we obtain the expressions

$$(2) \qquad abcda \quad 10.0\text{ V} - (6.0\ \Omega)I_1 - (2.0\ \Omega)I_3 = 0$$

$$(3) \quad befcb \quad -14.0\text{ V} + (6.0\ \Omega)I_1 - 10.0\text{ V} - (4.0\ \Omega)\ I_2 = 0$$

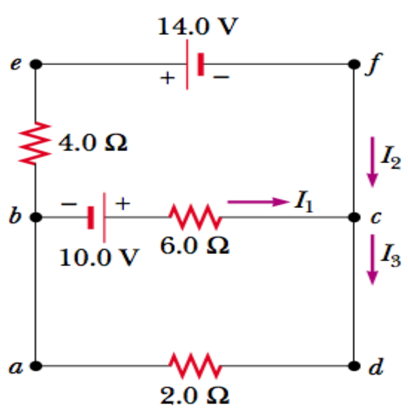


Figure 28.17 (Example 28.9) A circuit containing different branches.

Note that in loop *befcb* we obtain a positive value when traversing the 6.0-Ω resistor because our direction of travel is opposite the assumed direction of I_1. Expressions (1), (2), and (3) represent three independent equations with three unknowns. Substituting Equation (1) into Equation (2) gives

$$10.0\text{ V} - (6.0\ \Omega)I_1 - (2.0\ \Omega)\ (I_1 + I_2) = 0$$

$$(4) \qquad 10.0\text{ V} = (8.0\ \Omega)I_1 + (2.0\ \Omega)I_2$$

Dividing each term in Equation (3) by 2 and rearranging gives

$$(5) \qquad -12.0\text{ V} = -(3.0\ \Omega)I_1 + (2.0\ \Omega)I_2$$

Subtracting Equation (5) from Equation (4) eliminates I_2, giving

$$22.0\text{ V} = (11.0\ \Omega)I_1$$

$$I_1 = \boxed{2.0\text{ A}}$$

Using this value of I_1 in Equation (5) gives a value for I_2:

$$(2.0\ \Omega)I_2 = (3.0\ \Omega)I_1 - 12.0\text{ V}$$
$$= (3.0\ \Omega)(2.0\text{ A}) - 12.0\text{ V} = -6.0\text{ V}$$

$$I_2 = \boxed{-3.0\text{ A}}$$

Finally,

$$I_3 = I_1 + I_2 = \boxed{-1.0\text{ A}}$$

To finalize the problem, note that I_2 and I_3 are both negative. This indicates only that the currents are opposite the direction we chose for them. However, the numerical values are correct. What would have happened had we left the current directions as labeled in Figure 28.17 but traversed the loops in the opposite direction?