PHYS 104
1ST semester 1439-1440

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Lecture 23

Chapter 30

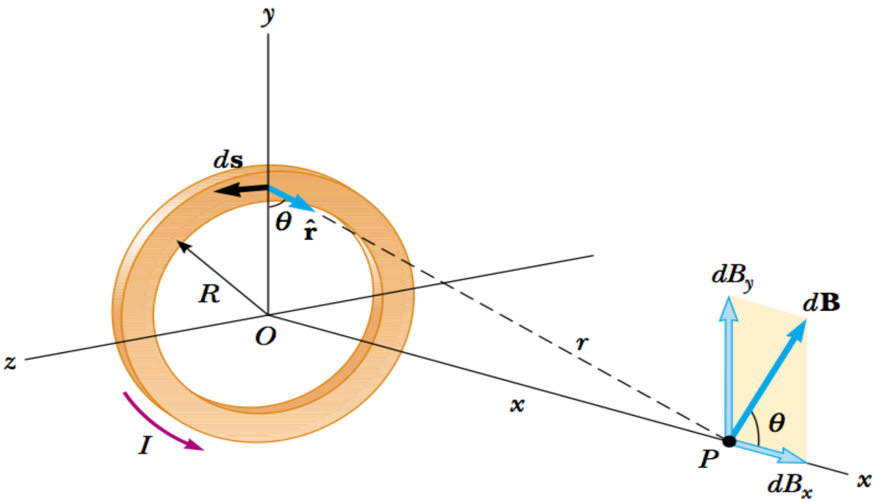
Sources of the Magnetic Field

- **30.1** The Biot–Savart Law
- **30.2** The Magnetic Force Between Two Parallel Conductors
- **30.3** Ampère's Law
- **30.4** The Magnetic Field of a Solenoid
- **30.5** Magnetic Flux
- **30.6** Gauss's Law in Magnetism

Example 30.3 **Magnetic Field on the Axis of a Circular Current Loop**

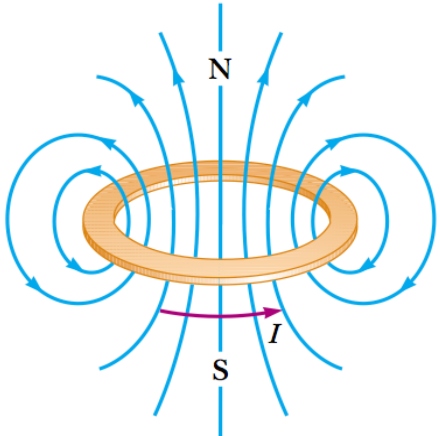
- Consider a circular wire loop of radius *R* located in the *yz* plane and carrying a steady current *I*. Calculate the magnetic field at an axial point *P* a distance *x* from the center of the loop.

$$B_x = \frac{\mu_0 I R}{4\pi(x^2 + R^2)^{3/2}} \oint ds = \boxed{\frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}}$$



- To find the magnetic field at the center of the loop, we set x = 0

$$B = \frac{\mu_0 I}{2R} \qquad (\text{at } x = 0)$$



30.3 Ampère's Law

The line integral of $\mathbf{B} \cdot d\mathbf{s}$ around any closed path equals $\mu_0 I$, where I is the total steady current passing through any surface bounded by the closed path.

$$\oint \mathbf{B} \cdot d\mathbf{s} = \mu_0 I \qquad (30.13)$$

Quick Quiz 30.4 Rank the magnitudes of $\oint \mathbf{B} \cdot d\mathbf{s}$ for the closed paths in Figure 30.10, from least to greatest.

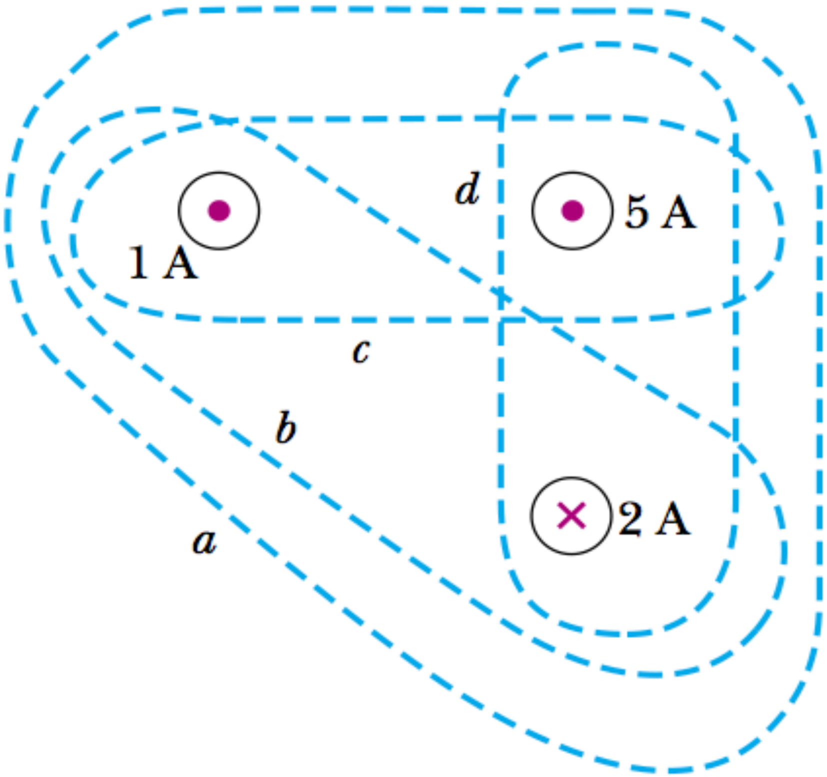
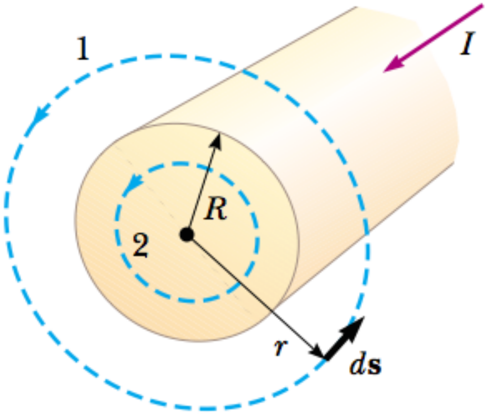


Figure 30.10 (Quick Quiz 30.4) Four closed paths around three current-carrying wires.

Example 30.4 The **Magnetic Field Created by a Long Current-Carrying Wire**

- A long, straight wire of radius R carries a steady current I that is uniformly distributed through the cross section of the wire (Fig. 30.12). Calculate the magnetic field a distance r from the center of the wire in the regions $r \geq R$ and $r < R$.



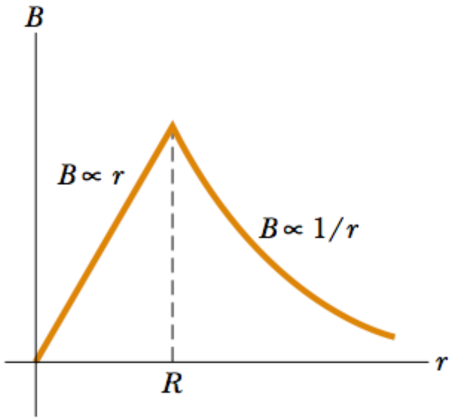
- For the r ≥ R ,

$$\oint \mathbf{B} \cdot d\mathbf{s} = B \oint ds = B(2\pi r) = \mu_0 I$$

$$B = \boxed{\frac{\mu_0 I}{2\pi r}} \qquad (\text{for } r \geq R)$$

- For the r < R ,

$$B = \boxed{\left(\frac{\mu_0 I}{2\pi R^2}\right) r} \qquad (\text{for } r < R)$$

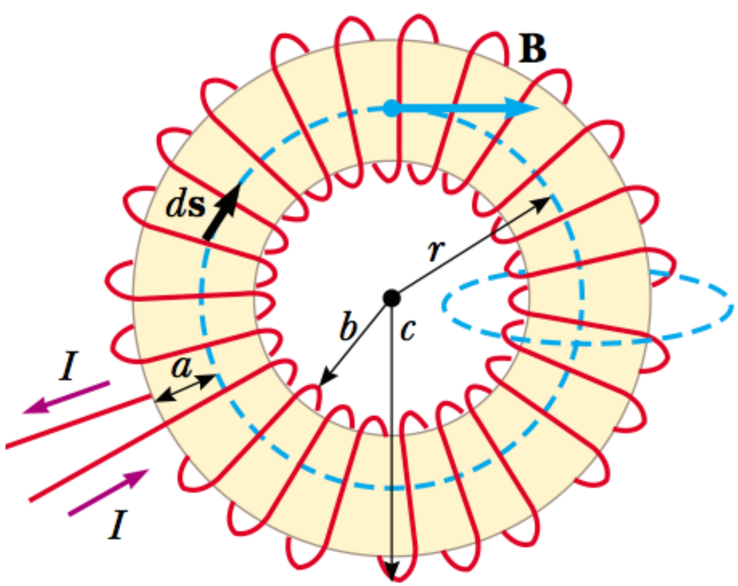


Example 30.5 The **Magnetic Field Created by a Toroid**

A device called a *toroid* (Fig. 30.14) is often used to create an almost uniform magnetic field in some enclosed area. The device consists of a conducting wire wrapped around a ring (a *torus*) made of a nonconducting material. For a toroid having N closely spaced turns of wire, calculate the magnetic field in the region occupied by the torus, a distance r from the center.

$$\oint \mathbf{B} \cdot d\mathbf{s} = B \oint ds = B(2\pi r) = \mu_0 N I$$

$$B = \boxed{\frac{\mu_0 N I}{2\pi r}}$$



30.4 The Magnetic Field of a Solenoid

- A **solenoid** is a long wire wound in the form of a helix.
- With this configuration, an uniform magnetic field can be produced in the space surrounded by the turns of wire—which we shall call the *interior* of the solenoid—when the solenoid carries a current.
- The magnetic field lines surrounding a **loosely** wound solenoid in the *interior* are nearly parallel to one another, are uniformly distributed, and are close together, indicating that the field in this space is strong and almost uniform.

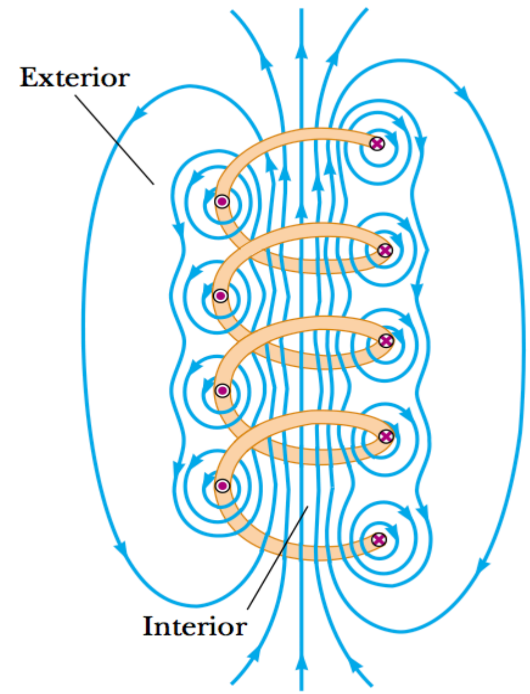
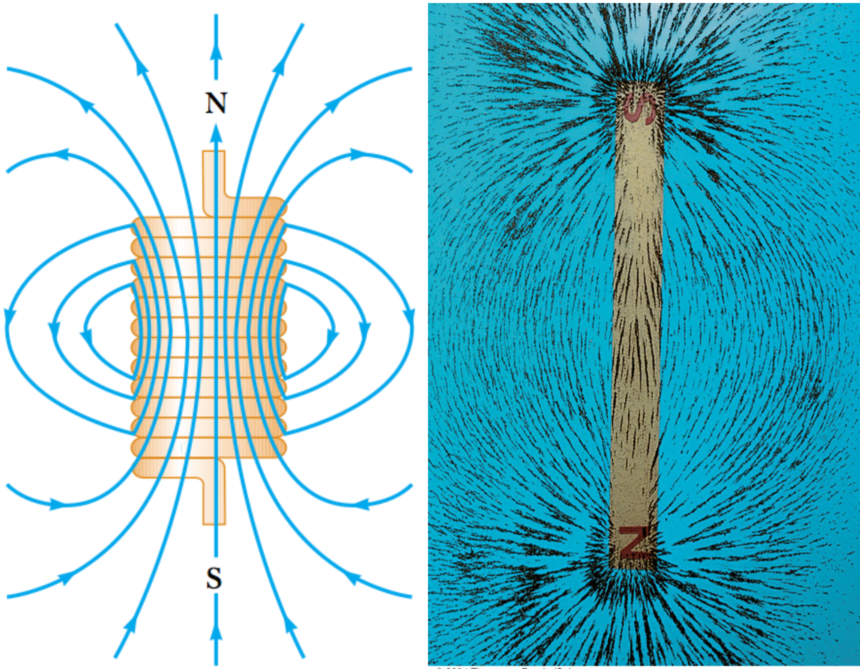


Figure 30.17 The magnetic field lines for a loosely wound solenoid.

30.4 The Magnetic Field of a Solenoid

- When the turns are closely spaced, each can be approximated as a circular loop, and the net magnetic field is the vector sum of the fields resulting from all the turns.

- The field line distribution is similar to that surrounding a bar magnet.
- Hence, one end of the solenoid behaves like the north pole of a magnet, and the opposite end behaves like the south pole.



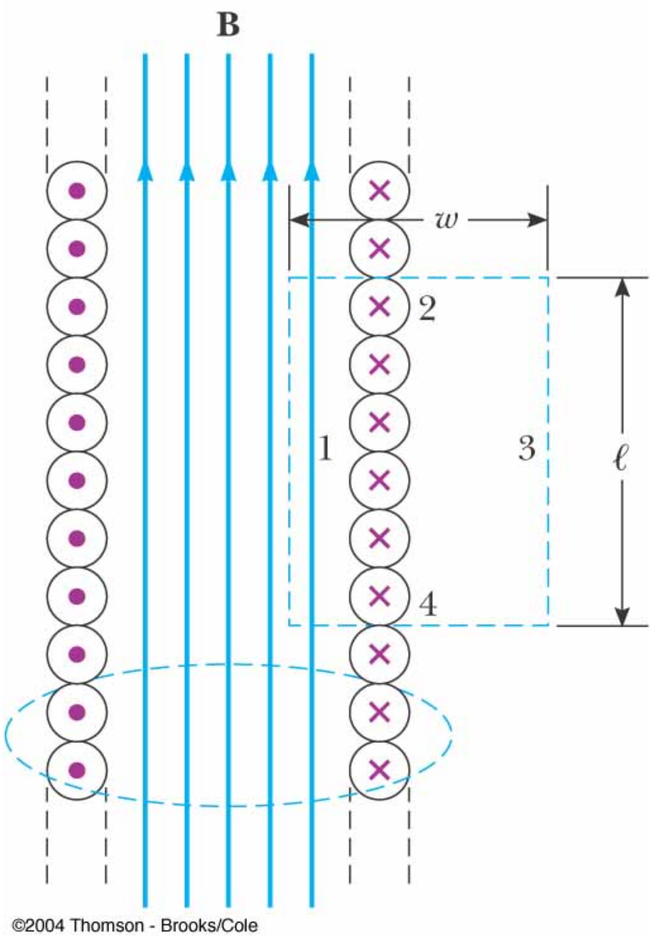
- As the **length** of the solenoid increases, the interior field becomes more uniform and the exterior field becomes **weaker**.

30.4 The Magnetic Field of a Solenoid

- An ***ideal solenoid*** is approached when the turns are closely spaced and the length is much greater than the radius of the turns.
- A longitudinal cross section of part of such a solenoid carrying a current *I*. In this case, the external field is close to zero, and the interior field is uniform over a great volume.
- The magnetic field inside the solenoid

$$B = \mu_0 \frac{N}{\ell} I = \mu_0 n I$$

- Where $n = N/\ell$ is the number of turns per unit length.



30.5 Magnetic Flux

- The total magnetic flux Φ_B through the surface is

$$\Phi_B = \int \mathbf{B} \cdot d\mathbf{A}$$

- Consider the special case of a plane of area *A* in a uniform field **B** that makes an angle θ with *d***A**. The magnetic flux through the plane is

$$\Phi_B = BA \cos\theta$$

- If the magnetic field is **parallel** to the plane, then θ = 90° and the flux through the plane is **zero**.
- If the field is **perpendicular** to the plane, then θ = 0 and the flux through the plane is *BA* (the maximum value).

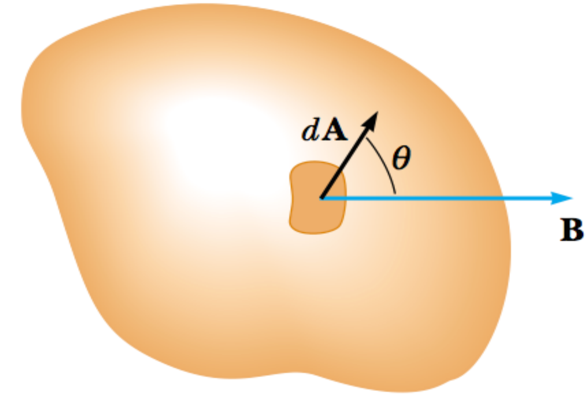
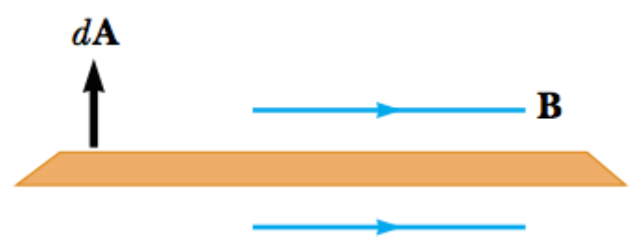
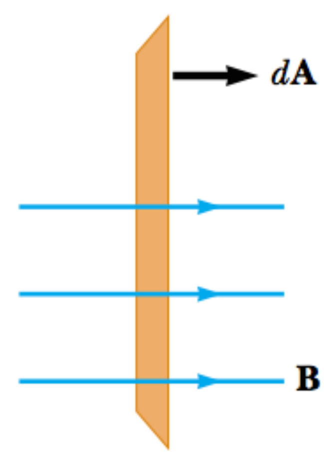


Figure 30.20 The magnetic flux through an area element dA is $\mathbf{B} \cdot d\mathbf{A} = B\,dA \cos\theta$, where $d\mathbf{A}$ is a vector perpendicular to the surface.





30.5 Magnetic Flux

- The unit of magnetic flux is $T.m^2$, which is defined as a *weber* (Wb); $1Wb = 1\ T\ .\ m^2$.