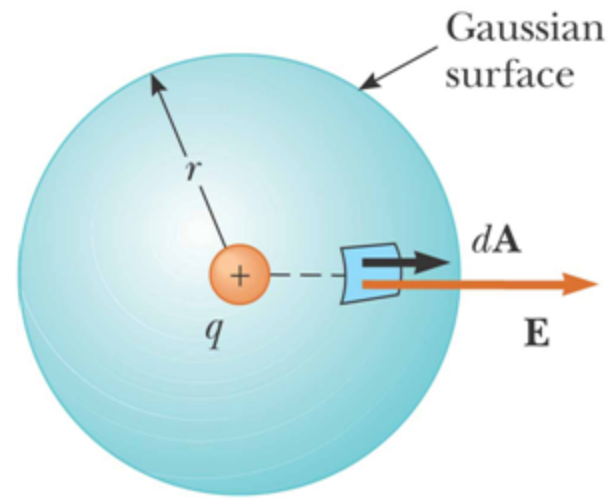
Phys 104

Chapter 24 Gauss's Law

24.2 Gauss's Law

- Let us again consider a **positive point charge** q located at the center of a sphere of radius r . The net flux through the closed surface (Gaussian surface).

$$\Phi = \oint \vec{E} \bullet d\vec{A} = \oint E \cos\theta dA = \oint EdA$$



E is constant over the surface and given by

$$E = k_e q/r^2$$

$$\Phi = \oint EdA = \oint kq/r^2 dA$$

$$\Phi = kq/r^2 \oint dA = kq/r^2 (4\pi r^2)$$

because the surface is spherical

$$\Phi = 4\pi kq$$

$$4\pi k = 1/\varepsilon_0 \text{ where } \varepsilon_0 = 8.85\text{x}10^{-12} \frac{C^2}{Nm^2}$$

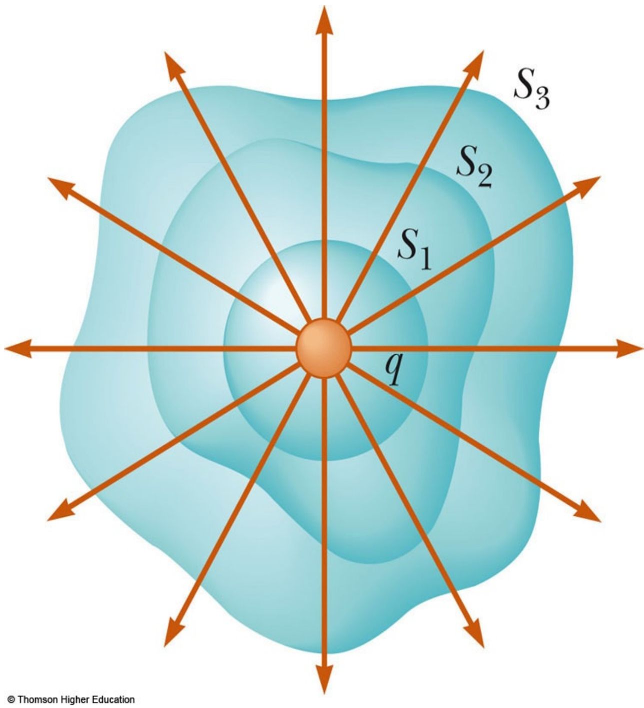
$$\boxed{\Phi_{net} = \frac{q_{enc}}{\varepsilon_0}}$$

Gauss's Law

Gauss's Law
The net flux through the spherical surface is proportional to the charge inside. The flux is independent of the radius r .

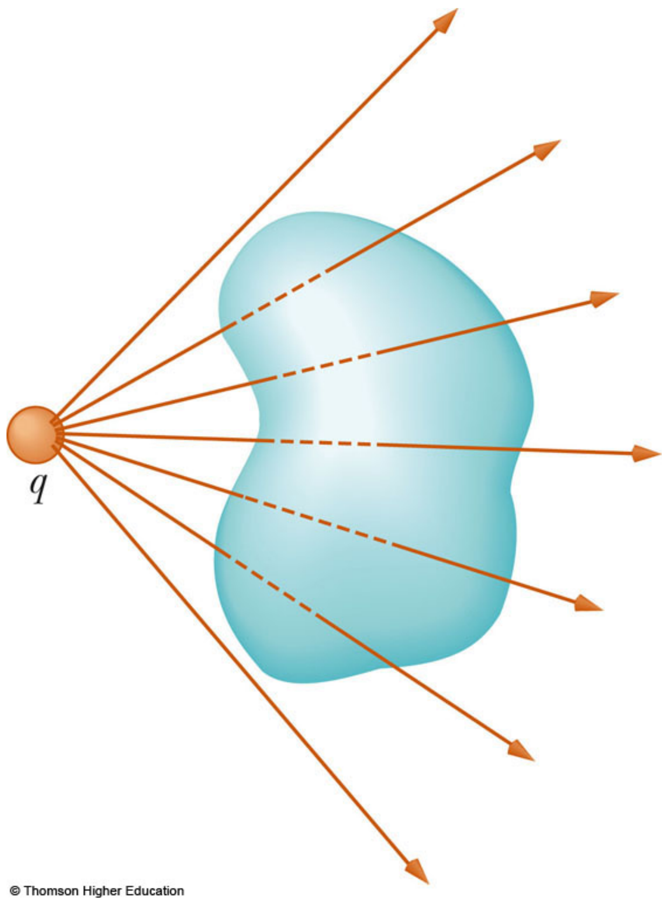
24.2 Gauss's Law

- Gaussian surfaces of various shapes can surround the charge q (only S_1 is spherical).
- The electric flux is proportional to the number of electric field lines penetrating these surfaces, and this number is the same.
- Thus the net flux through any closed surface surrounding a point charge q is given by q/ε_o and is independent of the shape of the surface



24.2 Gauss's Law

- If the charge is outside the closed surface of an arbitrary shape, then any electric field line that enters the surface leaves the surface at another point.
- The number of electric field lines entering the surface equals the number leaving the surface.
- Thus, the net electric flux through a closed surface that surrounds no charge is zero.



24.2 Gauss's Law

- Since the electric field due to many charges is the vector sum of the electric fields produced by the individual charges, the flux through any closed surface can be expressed as

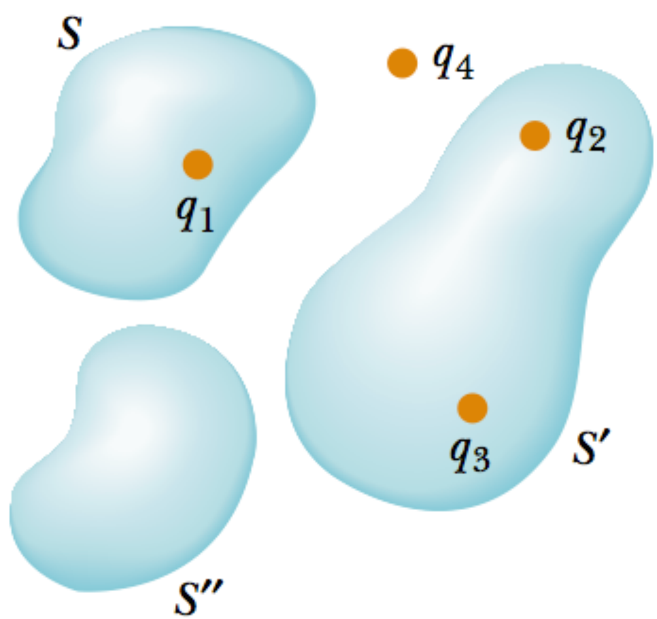
$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint \left(\vec{E}_1 + \vec{E}_2 + \ldots\right) \cdot d\vec{A}$$

- Although Gauss' s law can, in theory, be solved to find **E** for any charge configuration, in practice it is limited to symmetric situations.

- One should choose a Gaussian surface over which the surface integral can be simplified and the electric field determined.

Quick Quiz 24.4 Consider the charge distribution shown in Figure 24.9. The charges contributing to the total electric *flux* through surface S' are (a) q_1 only (b) q_4 only (c) q_2 and q_3 (d) all four charges (e) none of the charges.

Quick Quiz 24.5 Again consider the charge distribution shown in Figure 24.9. The charges contributing to the total electric *field* at a chosen point on the surface S' are (a) q_1 only (b) q_4 only (c) q_2 and q_3 (d) all four charges (e) none of the charges.



Conceptual Example 24.3 Flux Due to a Point Charge

A spherical gaussian surface surrounds a point charge q. Describe what happens to the total flux through the surface if

(A) the charge is tripled,

(B) the radius of the sphere is doubled,

(C) the surface is changed to a cube, and

(D) the charge is moved to another location inside the surface.

Solution

(A) The flux through the surface is tripled because flux is proportional to the amount of charge inside the surface.

(B) The flux does not change because all electric field lines from the charge pass through the sphere, regardless of its radius.

(C) The flux does not change when the shape of the gaussian surface changes because all electric field lines from the charge pass through the surface, regardless of its shape.

(D) The flux does not change when the charge is moved to another location inside that surface because Gauss's law refers to the total charge enclosed, regardless of where the charge is located inside the surface.

Problem 24.9

The following charges are located inside a submarine: 5.00 μC, –9.00 μC, 27.0 μC, and –84.0 μC.

(a) Calculate the net electric flux through the hull of the submarine.

(b) Is the number of electric field lines leaving the submarine greater than, equal to, or less than the number entering it?

$$\Phi_E = \frac{q_{\text{in}}}{\epsilon_0} = \frac{(+5.00\ \mu\text{C} - 9.00\ \mu\text{C} + 27.0\ \mu\text{C} - 84.0\ \mu\text{C})}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = -6.89\times10^{6}\ \text{N}\cdot\text{m}^2/\text{C}^2$$

(b) Since the net electric flux is negative, more lines enter than leave the surface.

24.3 Application of Gauss's Law to Various Charge Distributions

Example 24.4 The Electric Field Due to a Point Charge

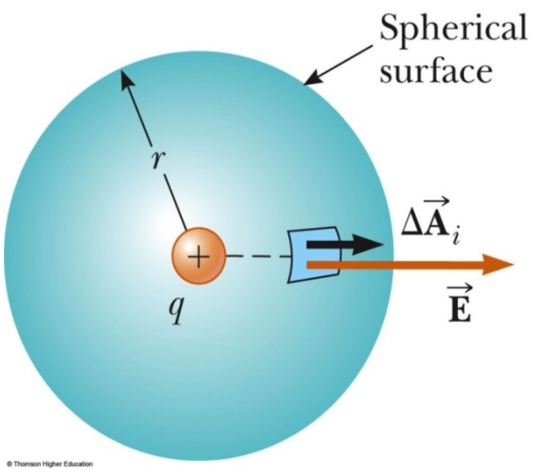
- Starting with Gauss's law, calculate the electric field due to an isolated point charge *q*.
- The field lines are directed radially outwards by symmetry, and **E** is parallel to *d***A** at each point. Therefore, **E**.*d***A** = *E dA* and Gauss's law gives

$$\Phi_E = \oint \mathbf{E}\cdot d\mathbf{A} = \oint E\,dA = \frac{q}{\epsilon_0}$$

$$\oint E\,dA = E\oint dA = E(4\pi r^2) = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{4\pi\epsilon_0 r^2} = \boxed{k_e\frac{q}{r^2}}$$

Coulomb's Law



A spherical Gaussian surface centered on a point charge q

Example 24.5 A Spherically Symmetric Charge Distribution

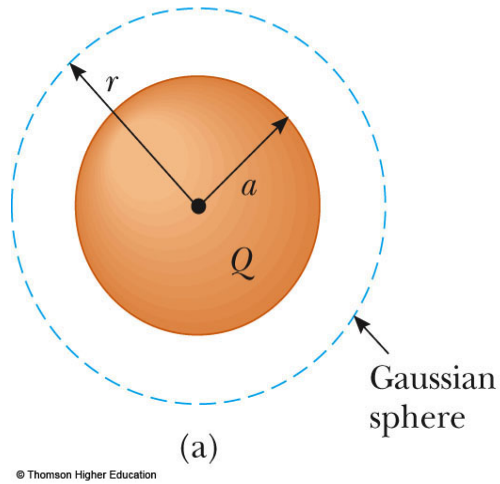
An insulating solid sphere of radius ***a*** has a uniform volume charge density ρ and carries total charge Q.

(A) Find the magnitude of the E-field at a point **outside** the sphere

(B) Find the magnitude of the E-field at a point **inside** the sphere

(A) For $r > a$

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = E \oint dA = E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$$

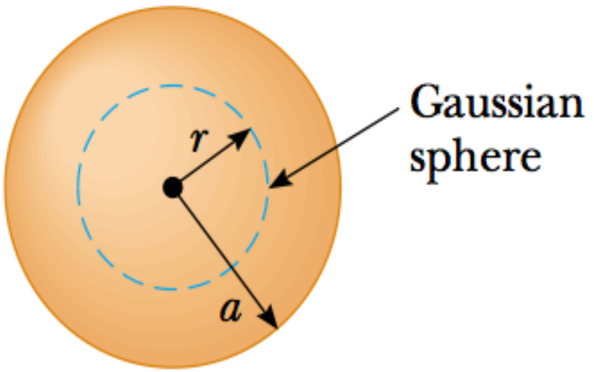


$$E = \frac{Q}{4\pi\varepsilon_0 r^2} = k_e \frac{Q}{\boxed{r^2}}$$

(B) for r < a

Now we select a spherical Gaussian surface with radius $r < a$. Let us denote the volume of this smaller sphere by V' .
The charge q in within the gaussian surface of volume V' is less than Q.

$$q_{\text{in}} = \rho V' = \rho\left(\tfrac{4}{3}\pi r^3\right)$$



(b)

By symmetry, the magnitude of the electric field is constant everywhere on the spherical gaussian surface and is normal to the surface at each point.
Gauss's law in the region $r < a$ gives

$$\oint E\,dA = E\oint dA = E\,(4\pi r^2) = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E = \frac{q_{\text{in}}}{4\pi\epsilon_0 r^2} = \frac{\rho\left(\frac{4}{3}\pi r^3\right)}{4\pi\epsilon_0 r^2} = \frac{\rho}{3\epsilon_0}\,r$$

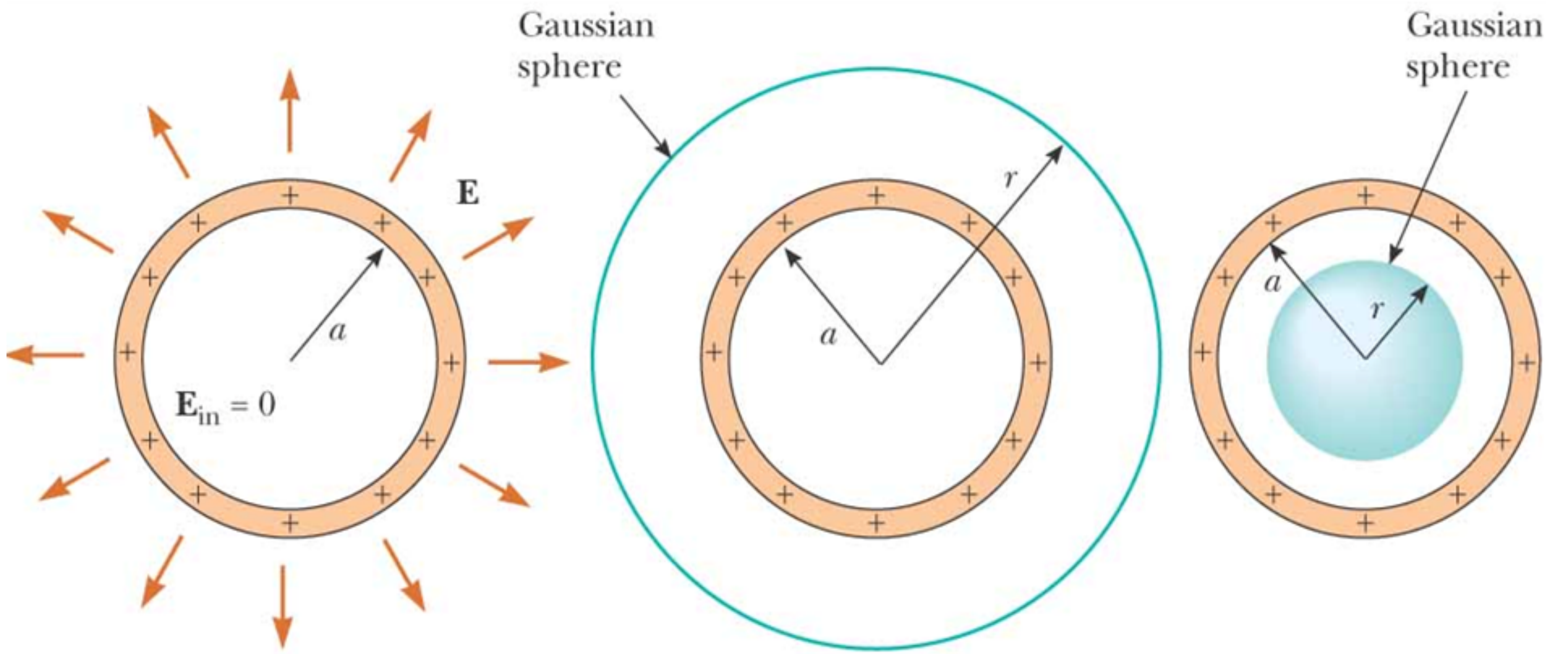
$$E = \frac{Qr}{4\pi\epsilon_0 a^3} = \boxed{k_e\frac{Q}{a^3}\,r}$$

Example 24.6 **The Electric Field Due to a Thin Spherical Shell**

A thin spherical shell of radius *a has a total charge Q distributed* uniformly over its surface Find the electric field at points

(A) outside and

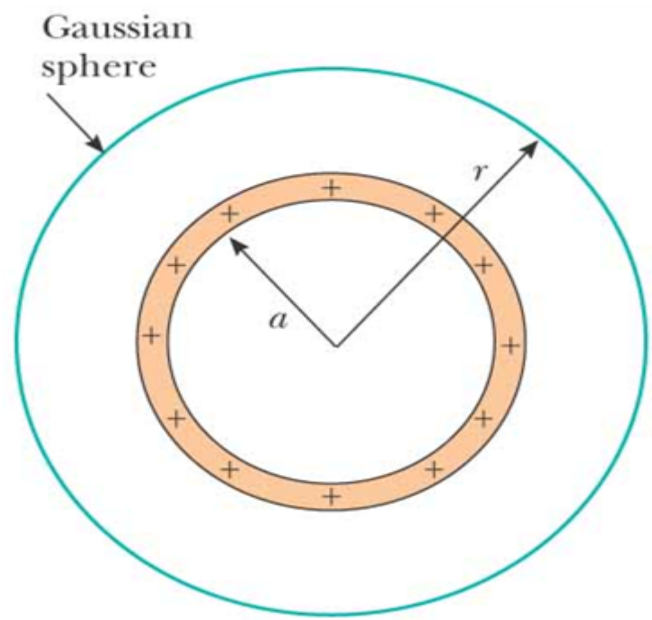
(B) inside the shell.



(a) (b) (c)

- **(A) Let's start with the Gaussian surface outside the sphere of charge, r > a**

The calculation for the field outside the shell is identical to that for the solid sphere.
The charge inside this surface is *Q*. Therefore, the field at a point outside the shell is equivalent to that due to a point charge *Q* located at the center:



$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$$

so the electric field is

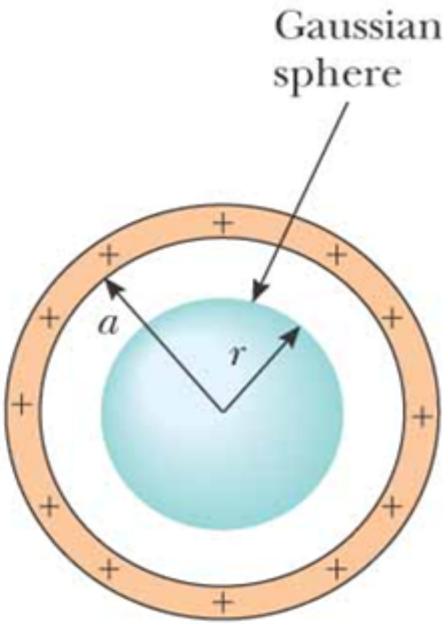
$$E = \boxed{k_e \frac{Q}{r^2}} \qquad (\text{for } r > a)$$

- (B) Let's take the Gaussian surface **inside** the sphere of charge, $r < a$
- We know that the enclosed charge is zero so

$$\text{Flux} = \Phi_E = EA = 0$$

- We find that the electric field is zero everywhere inside spherical shell of charge

$$E = 0$$



- Thus we obtain two results
  - The electric field outside a spherical shell of charge is the same as that of a point charge.
  - The electric field inside a spherical shell of charge is zero.