PHYS 507
Lecture 8: Electric Fields in Matter
Part A

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Dielectrics

- Insulators are dielectric materials.
- In dielectrics there are no free electrons, as in conductors. On the contrary, all charges are attached to specific atoms or molecules.
- The electrons in dielectrics can make only microscopic motions inside the molecules or atoms under the action of external electric fields.
- There are two principal mechanisms by which electric fields can distort the charge distribution of a dielectric atom or molecule: **stretching** and **rotating**.

Induced dipoles-a

- What happens to a neutral atom when it is placed in an electric field?
- The atom is made up by a positive “core”: the nucleus while the whole negative charge, carried by electrons, is distributed around it.
- The positive nucleus is pushed in the direction of the field. The electrons in the opposite way.
- If the field is strong enough it can pull the atom apart completely, “ionizing” it.

Induced dipoles-b

- When the electrostatic external forces balance with the Coulomb attraction between the nucleus and the electrons the atom is **polarized**.
- The atom has a tiny dipole moment **p**, which points in the same direction as **E**.
- Typically, this induced dipole moment is approximately proportional to the field (as long as the field is not very strong) and we have: $\mathbf{p}=\alpha\mathbf{E}$. The quantity α is called **atomic polarizability**. The value of α depends on the detailed structure of the atom in question.

Induced dipoles-c

- For molecules the situation is more complicated because they show stronger polarization in some directions than others. The molecules have a symmetry axis. When the external electric field is at some angle to the axis, you must resolve it into parallel and perpendicular components and multiply it by the corresponding polarizability:

$$\mathbf{p} = \alpha_{\perp}\mathbf{E}_{\perp} + \alpha_{\parallel}\mathbf{E}_{\parallel}$$

- In this case the induced dipole moment may not even be in the same direction as **E**.

Induced dipoles-d

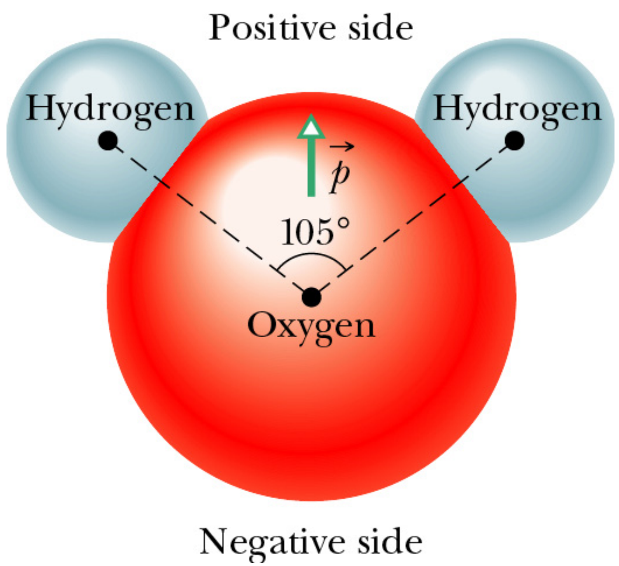
For a completely asymmetrical molecule is replaced by the most general relation between **E** and **p**:

$$p_x = \alpha_{xx}E_x + \alpha_{xy}E_y + \alpha_{xz}E_z$$
$$p_y = \alpha_{yx}E_x + \alpha_{yy}E_y + \alpha_{yz}E_z$$
$$p_z = \alpha_{zx}E_x + \alpha_{zy}E_y + \alpha_{zz}E_z$$

The set of nine constants α_{ij} constitute the **polarizability tensor** for the molecule. Their actual values depend on the orientation of the axes we chose. Though it is always possible to choose "principal axes such that all off-diagonal terms vanish, leaving just three nonzero polarizabilities: α_{xx}, α_{yy}, α_{zz}.

Alignment of polar molecules-a

- Some molecules have permanent dipole moments. Like, for example, the water molecule shown in the figure.
- What happens when such molecules (called **polar molecules**) are placed in the electric field?



Alignment of polar molecules-b

- In this case the field exerts a net torque on the molecule given by:

$$\mathbf{N} = \mathbf{p} \times \mathbf{E}$$

- This torque will rotate the molecule until **p** is line up parallel to **E.**
- If the electric field is not uniform then a net force also will act on the molecule given by:

$$\mathbf{F} = (\mathbf{p} \cdot \nabla)\mathbf{E}$$

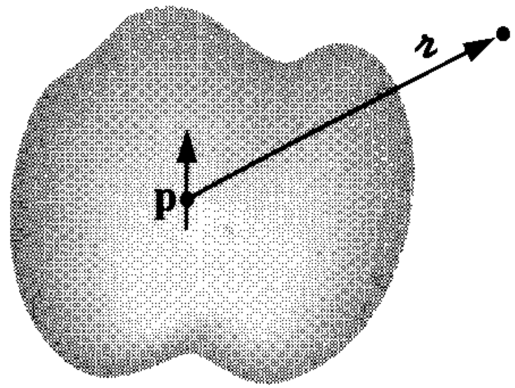
Polarization

- What happens to a piece of dielectric material when it is placed in an electric field?
- Either the substance is made up by neutral atoms or polar molecules the result will be the same: **a lot of little dipoles will end up to point along the direction of the field**.
- The material will become **polarized**.
- A convenient measure if this effect is the **polarization**, which is defined as:

$$\mathbf{P} = \textit{dipole moment per unit volume}$$

The field of a polarized object-a

- The question now is: What is the field produced by a polarized object? (Be careful, we do not mean the external field that polarized the object).
- Since we know what the field of a dipole is, in this case we can consider our material as made up by a large number of tiny dipoles.



The field of a polarized object-b

- We can prove that the field produced by the polarized object has two components:
- a) the field produced by a volume charge density

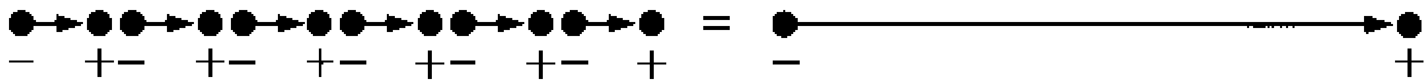
  $$\rho_b = -\nabla \cdot \mathbf{P}$$

  b) the field produced by a surface charge density $\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}$

The question now arises: Are these bound charges "fictitius" or correspond to genuine accumulation of electric charge?

A physical interpretation

- Suppose we have a line series of dipoles in a dielectric material. Along the line the head of one effectively cancels the tail of the next. There are only two charges left: the negative "tail" of the first and the positive "head" of the last one.



- We call the net charge at the ends **bound** charge

The field inside a dielectric-a

- Polarization **P** is a continuous function. The molecular dipoles, though, are discrete entities. How is this justified?
- Outside the dielectric there is no real problem: we are far away from the molecules so the dipole potential dominates and the detailed structure of the source is "washed out" from the distance.
- **What about inside the dielectric?**

The field inside a dielectric-b

- It is impossible to define the **microscopic** electric field inside a molecule.
- What we are interested in, is the **macroscopic** electric field, i.e. the average field over regions **large** enough to contain many thousands of atoms and **small** enough to ensure that we do not wash out any significant large-scale variations in the field.

The Electric Displacement

- We saw that the field due to polarization of the medium is just the field of the bound charge. We need now to take into account all the charge: bound and free. The total charge density is: $\rho=\rho_b+\rho_f$. The Gauss Law gives:

$$\varepsilon_0 \nabla \cdot \mathbf{E} = \rho = \rho_b + \rho_f = -\nabla \cdot \mathbf{P} + \rho_f \Rightarrow$$

$$\nabla \cdot \left(\varepsilon_0 \mathbf{E} + \mathbf{P}\right) = \rho_f$$

- We define as **electric displacement D** the quantity:

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$$

The Gauss Law

- The Gauss Law assumes the form:

$$\nabla \cdot \mathbf{D} = \rho_f$$

- Or in an integral form:

$$\oint \mathbf{D} \cdot d\mathbf{a} = Q_{f,enc}$$

- These forms make reference only to **free** charges.

Some remarks on Electric Displacement

- The form of Gauss law may give you the impression that **D** is just like **E.**
- No! There is no Coulomb law for **D**.
- The **curl** of **D** is not always zero:

$$\nabla \times \mathbf{D} = \varepsilon_0 \left(\nabla \times \mathbf{E} \right) + \left(\nabla \times \mathbf{P} \right) = \left(\nabla \times \mathbf{P} \right)$$

- Thus you cannot express **D** as a gradient of a scalar. There is no potential for **D**!

Boundary conditions

- The boundary conditions for the electric displacement take the form:

$$D^{\perp}_{\text{above}} - D^{\perp}_{\text{below}} = \sigma_f$$

$$\mathbf{D}^{\parallel}_{\text{above}} - \mathbf{D}^{\parallel}_{\text{below}} = \mathbf{P}^{\parallel}_{\text{above}} - \mathbf{P}^{\parallel}_{\text{below}}$$

Linear Dielectrics-a

- For many substances, the polarization is proportional to the applied electric field (provided **E** is not too strong):

$$\mathbf{P} = \varepsilon_0 \chi_e \mathbf{E}$$

- The constant of proportionality χ_e , is called the **electric susceptibility** of the medium. The value of this constant depends on the microscopic structure of the substance in question and on external conditions such as temperature. These materials are called **linear dielectrics**.

Note that **E** is the total field!

Linear Dielectrics-b

- In linear media we have:

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} = \varepsilon_0 \mathbf{E} + \varepsilon_0 \chi_e \mathbf{E} = \varepsilon_0 \left(1 + \chi_e\right) \mathbf{E}$$

- We see that **D** is proportional to **E**

$$\mathbf{D} = \varepsilon \mathbf{E} \quad \text{with } \varepsilon = \varepsilon_0 \left(1 + \chi_e\right)$$

- The constant ε is called the **permittivity** of the material. While the quantity $1+\chi_e$ is called the **relative permittivity** or **dielectric constant**.

When all the space is filled with a homogeneous linear dielectrics the field everywhere is simply reduced by a factor of one over the dielectric constant.

Material	Dielectric Constant	Material	Dielectric Constant
Vacuum	1	Benzene	2.28
Helium	1.000065	Diamond	5.7
Neon	1.00013	Salt	5.9
Hydrogen	1.00025	Silicon	11.8
Argon	1.00052	Methanol	33.0
Air (dry)	1.00054	Water	80.1
Nitrogen	1.00055	Ice (-30° C)	99
Water vapor (100° C)	1.00587	$KTaNbO_3$ (0° C)	34,000

Energy in the presence of dielectrics

- We can prove that the total work done to build the free charge is given by:

$$W = \frac{1}{2}\int \mathbf{D}\cdot\mathbf{E}\, d\tau$$