PHYS 301
HANDOUT 3
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1. Find the derivative of the complex function $f(z) = z^2$.

2. Check if the function $f(z) = z + \left(z - z^*\right)/2$ is differentiable.

3. Check if the function $f(z) = |z|^2$ is differentiable.

4. The complex function $f(z) = z^2$ is differentiable. Verify the Cauchy-Riemann conditions.

5. Discuss the Cauchy-Riemann conditions for the complex function $f(z) = |z|^2$.

6. For the complex function $f(z) = e^x\left(\cos y + i\sin y\right)$, show that $f^{'}(z) = f(z)$.

7. Find the derivative of the complex function $f(z) = \frac{1}{z} = \frac{1}{r}e^{-i\theta}$.

8. The function $f(z) = x + iy$ is analytic. What happens with the function $f(z) = z^*$? (Ver. 29)

9. The function $f(z) = u(x,y) + iv(x,y)$ is analytic. What happens with the function $f^*(z^*)$?

10. If there is some common region in which $w_1 = u(x,y) + iv(x,y)$ and $w_2 = w_1^* = u(x,y) - iv(x,y)$ are both analytic, prove that $u(x,y)$ and $v(x,y)$ are constants. (Arf. 364)

11. The function $f(z)$ is analytic. Show that the derivative of $f(z)$ with respect to z^* vanishes.
Hint. Use the chain rule and take $x = \left(z + z^*\right)/2,\ y = \left(z - z^*\right)/2i$.
Note: This result emphasizes that our analytic function $f(z)$ is not just a complex function of two real variables *x* and *y*. It is a function of the complex variable $x + iy$. (Arf. 365)

12. Show that the function $u = e^{-x}\left(x\sin y - y\cos y\right)$ is harmonic. (b) What is the function *v*, such that $f(z) = u + iv$ is to be analytic? (Sch. 73)