PHYS 301
HANDOUT 4
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1. Show that $\frac{d}{dz}e^z = e^z$ (Sch.75)

2. Show that a) $\frac{d}{dz}\sin z = \cos z$ b) $\frac{d}{dz}\cos z = -\sin z$ (Sch 75).

3. Show that $\frac{d}{dz}z^{1/2} = \frac{1}{2z^{1/2}}$, having in mind that the function $z^{1/2}$ is a multivalued one (Sch. 76).

4. Show that $\frac{d}{dz}\ln f(z) = \frac{f'(z)}{f(z)}$ (Sch.15)

5. Find the derivative of the function $\cos^2(2z+3i)$ (Sch.77)

(Ans: $-4\cos(2z+3i)\sin(2z+3i)$)

6. Find the derivative of the function $(z-3i)^{4z+2}$ (Sch.77)

(Ans: $(z-3i)^{4z+1}(4z+2)+4(z-3i)^{4z+2}\ln(z-3i)$)

7. If $w^3 - 3z^2w + 4\ln z = 0$, calculate the derivative dw/dz. (Sch. 78)

(Ans: $(6zw - 4/z)/(3w^2 - 3z^2)$)

8. Show that the function e^z is periodic and find its period.

9. Show that $\frac{\exp z_1}{\exp z_2} = \exp(z_1 - z_2)$.

10. Show the following relations: a) $|\sin z|^2 = \sin^2 x + \sinh^2 y$, b) $|\cos z|^2 = \cos^2 x + \sinh^2 y$.

11. Show the following relations: a) $\frac{d}{dz}(\sinh z) = \cosh z$, b) $\frac{d}{dz}(\cosh z) = \sinh z$.

12. Show that $\mathrm{Log}(e^z) = z \qquad (-\pi < \mathrm{Im}\, z \le \pi)$.

13. Study the relation $\log(z_1 z_2) = \log z_1 + \log z_2$

14. Study the relation $z^{1/n} = \exp\left(\frac{1}{n}\log z\right)$.

15. Calculate i^{-2i}.

16. What is the principal value of $(-i)^i$.

17. Find the function $\sin^{-1} z$ and study its multivalue character.

18. Find function $\cos^{-1} z$ and study its multivalue character.

19. Discuss the Caychy-Riemann conditions for the function $f(z) = \overline{z}$

20. Show tha if a complex function is differentiable and has constant modulus then it is a constant function.

21. Define the points where the function $f(z) = (x + ay)^2 + 2i(x - ay)$ is analytic. Assume that *a* is real and constant.