King Saud University

PHYSICS FOR ENGINEERING I

(PHYS 1210)

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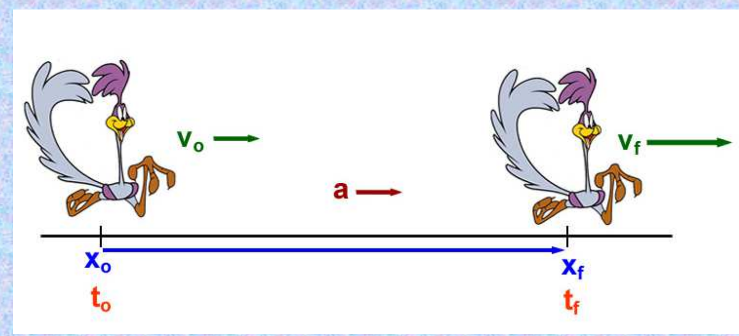
Course Contents

Chapter 2:

Motion in One Dimension



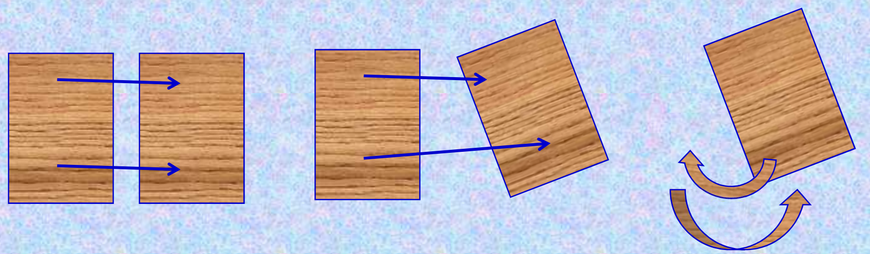
Chapter 2:

Motion in One Dimension

- ✓ Position, Velocity, and Speed
- ✓ Acceleration
- ✓ Motion diagrams
- ✓ Constant acceleration motion

Motion

- Study the kinematics (motion in space and time) of particles (having mass and negligible size) without studying the cause of motion.
- Motion types: translational, rotational, and vibrational motions.



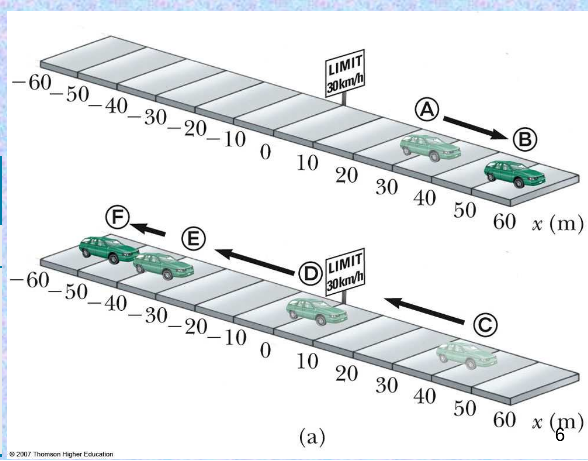
Translational **Rotational** **Vibrational Motion**

Position, Velocity, and Speed

- **The object's position is** *its location with respect to a chosen reference point*
- **Consider the point to be the origin of a coordinate system**
- In the diagram, allow the road sign to be the *reference point*

Position of the Car at Various Times

Position	t(s)	x(m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53



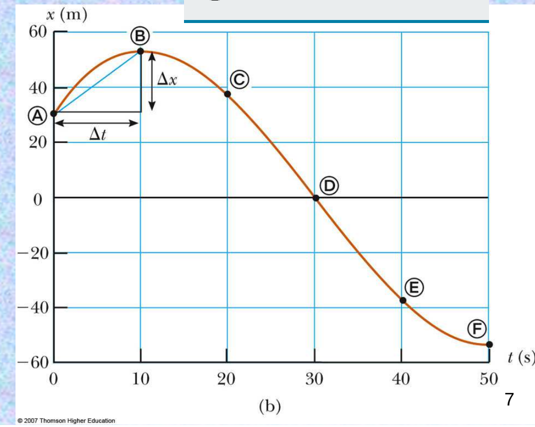
(a)

Position-Time Graph

Position of the Car at Various Times

Position	t(s)	x(m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

- **The position-time graph shows the motion of the particle (car)**
- **The smooth curve is a guess as to what happened between the data points**

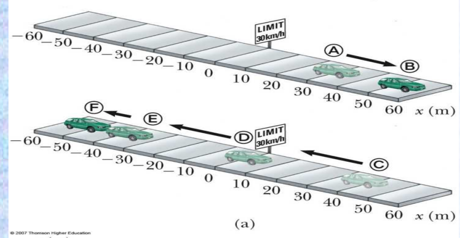


(b)

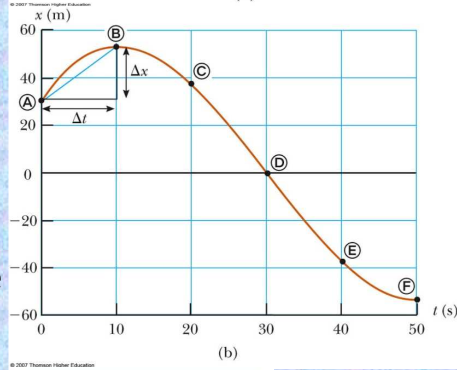
position–time graph.

Displacement

- **Defined as** *the change in position during some time interval*
  - **Represented as Δx**
    $\Delta x \equiv x_f - x_i$
  - **SI units are meters (m)**
  - **Δx can be positive or negative**
- **Different than** *distance* **– the length of a path followed by a particle**



(a)



(b)

$$\Delta x \equiv x_f - x_i$$

Average Velocity

- The average velocity is rate at which the displacement occurs.

i.e., The average velocity V_{av} of a particle is defined as particle's displacement Δx divided by the time interval Δt during which that displacement occurs:

$$v_{x,avg} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

- The x indicates motion along the x-axis

- The dimensions are length / time [L/T]
- The SI units are m/s
- Is also the slope of the line in the position – time graph

The average speed

The average speed of a particle, a scalar quantity, is defined as the total distance traveled divided by the total time interval required to travel that distance:

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}}$$

Distance is the length of a path followed by a particle.

- Speed is a scalar quantity
  - same units as velocity
  - total distance / total time: $v_{avg} \equiv \frac{d}{t}$
- The speed has no direction and is always expressed as a positive number
- Neither average velocity nor average speed gives details about the trip described

Notes:

Position

Displacement

Distance

Velocity

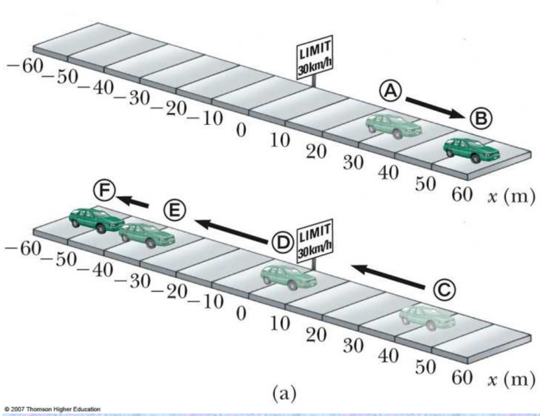
Speed

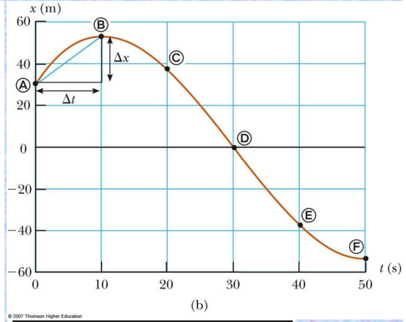
Acceleration

- "Velocity" and "Speed" will indicate *instantaneous* values
- *Average* will be used when the average velocity or average speed is indicated

Example 2.1 Calculating the Average Velocity and Speed

Find the displacement, average velocity, and average speed of the car in Figure 2.1a between positions Ⓐ and Ⓕ.





Position of the Car at Various Times

Position	t (s)	x (m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53

Example 2.1 Calculating the Average Velocity and Speed

Find the displacement, average velocity, and average speed of the car in Figure 2.1a between positions Ⓐ and Ⓕ.

Solution From the position–time graph given in Figure 2.1b, note that $x_A = 30$ m at $t_A = 0$ s and that $x_F = -53$ m at $t_F = 50$ s. Using these values along with the definition of displacement, Equation 2.1, we find that

$$\Delta x = x_F - x_A = -53\text{ m} - 30\text{ m} = \boxed{-83\text{ m}}$$

This result means that the car ends up 83 m in the negative direction (to the left, in this case) from where it started. This number has the correct units and is of the same order of magnitude as the supplied data. A quick look at Figure 2.1a indicates that this is the correct answer.

It is difficult to estimate the average velocity without completing the calculation, but we expect the units to be meters per second. Because the car ends up to the left of where we started taking data, we know the average velocity must be negative. From Equation 2.2,

$$\bar{v}_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{x_F - x_A}{t_F - t_A}$$

$$= \frac{-53\text{ m} - 30\text{ m}}{50\text{ s} - 0\text{ s}} = \frac{-83\text{ m}}{50\text{ s}}$$

$$= \boxed{-1.7\text{ m/s}}$$

We cannot unambiguously find the average speed of the car from the data in Table 2.1, because we do not have information about the positions of the car between the data points. If we adopt the assumption that the details of the car's position are described by the curve in Figure 2.1b, then the distance traveled is 22 m (from Ⓐ to Ⓑ) plus 105 m (from Ⓑ to Ⓕ) for a total of 127 m. We find the car's average speed for this trip by dividing the distance by the total time (Eq. 2.3):

$$\text{Average speed} = \frac{127\text{ m}}{50\text{ s}} = \boxed{2.5\text{ m/s}}$$

Instantaneous Velocity

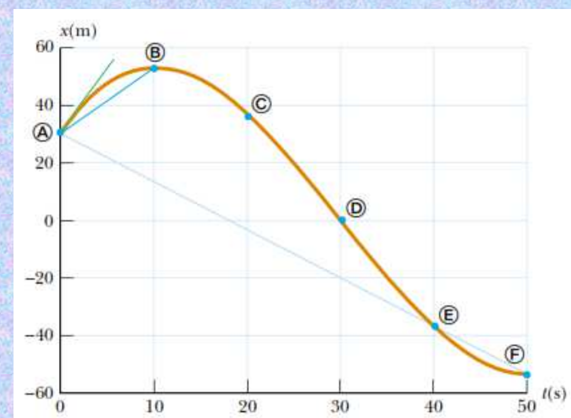
- **The limit of the average velocity as the time interval becomes infinitesimally short, or as the time interval approaches zero**
- **The instantaneous velocity indicates what is happening at every point of time**
- **The general equation for instantaneous velocity is**

$$v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- **The instantaneous velocity can be positive, negative, or zero.**
- **The term *velocity* will be used to denote for instantaneous velocity**

Instantaneous Velocity, graph

- **The instantaneous velocity is the slope of the line tangent to the x vs. t curve**
- **This would be the green line**



Instantaneous Speed

- The instantaneous speed is the magnitude of the instantaneous velocity.
- The instantaneous speed has no direction associated with it.

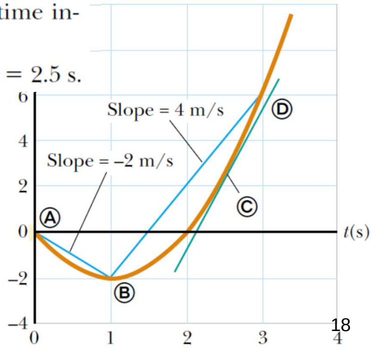
Example 2.3 Average and Instantaneous Velocity

A particle moves along the x axis. Its position varies with time according to the expression $x = -4t + 2t^2$ where x is in meters and t is in seconds.[3] The position–time graph for this motion is shown in Figure 2.4. Note that the particle moves in the negative x direction for the first second of motion, is momentarily at rest at the moment $t = 1$ s, and moves in the positive x direction at times $t > 1$ s.

(A) Determine the displacement of the particle in the time intervals $t = 0$ to $t = 1$ s and $t = 1$ s to $t = 3$ s.

(B) Calculate the average velocity during these two time intervals.

(C) Find the instantaneous velocity of the particle at $t = 2.5$ s.



(A) Determine the displacement of the particle in the time intervals $t = 0$ to $t = 1$ s and $t = 1$ s to $t = 3$ s.

Solution During the first time interval, the slope is negative and hence the average velocity is negative. Thus, we know that the displacement between Ⓐ and Ⓑ must be a negative number having units of meters. Similarly, we expect the displacement between Ⓑ and Ⓓ to be positive.

In the first time interval, we set $t_i = t_A = 0$ and $t_f = t_B = 1$ s. Using Equation 2.1, with $x = -4t + 2t^2$, we obtain for the displacement between $t = 0$ and $t = 1$ s,

$$\Delta x_{A\to B} = x_f - x_i = x_B - x_A$$
$$= [-4(1) + 2(1)^2] - [-4(0) + 2(0)^2]$$
$$= \boxed{-2\text{ m}}$$

To calculate the displacement during the second time interval ($t = 1$ s to $t = 3$ s), we set $t_i = t_B = 1$ s and $t_f = t_D = 3$ s:

$$\Delta x_{B\to D} = x_f - x_i = x_D - x_B$$
$$= [-4(3) + 2(3)^2] - [-4(1) + 2(1)^2]$$
$$= \boxed{+8\text{ m}}$$

These displacements can also be read directly from the position–time graph.

(B) Calculate the average velocity during these two time intervals.

Solution In the first time interval, $\Delta t = t_f - t_i = t_B - t_A = 1$ s. Therefore, using Equation 2.2 and the displacement calculated in (a), we find that

$$\bar{v}_{x(A\to B)} = \frac{\Delta x_{A\to B}}{\Delta t} = \frac{-2\text{ m}}{1\text{ s}} = \boxed{-2\text{ m/s}}$$

In the second time interval, $\Delta t = 2$ s; therefore,

$$\bar{v}_{x(B\to D)} = \frac{\Delta x_{B\to D}}{\Delta t} = \frac{8\text{ m}}{2\text{ s}} = \boxed{+4\text{ m/s}}$$

These values are the same as the slopes of the lines joining these points in Figure 2.4.

(C) Find the instantaneous velocity of the particle at $t = 2.5$ s.

Solution We can guess that this instantaneous velocity must be of the same order of magnitude as our previous results, that is, a few meters per second. By measuring the slope of the green line at $t = 2.5$ s in Figure 2.4, we find that

$$v_x = \boxed{+6\ \mathrm{m/s}}$$

[3] Simply to make it easier to read, we write the expression as $x = -4t + 2t^2$ rather than as $x = (-4.00\ \mathrm{m/s})t + (2.00\ \mathrm{m/s^2})t^{2.00}$. When an equation summarizes measurements, consider its coefficients to have as many significant digits as other data quoted in a problem. Consider its coefficients to have the units required for dimensional consistency. When we start our clocks at $t = 0$, we usually do not mean to limit the precision to a single digit. Consider any zero value in this book to have as many significant figures as you need.

Acceleration

- When the velocity of a particle changes with time, the particle is said to be *accelerating*.
-
- Average acceleration is defined as ratio of the change in velocity (Δv) of a particle in a time interval (Δt).
- Average acceleration can be positive or negative.

$$\bar{a}_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

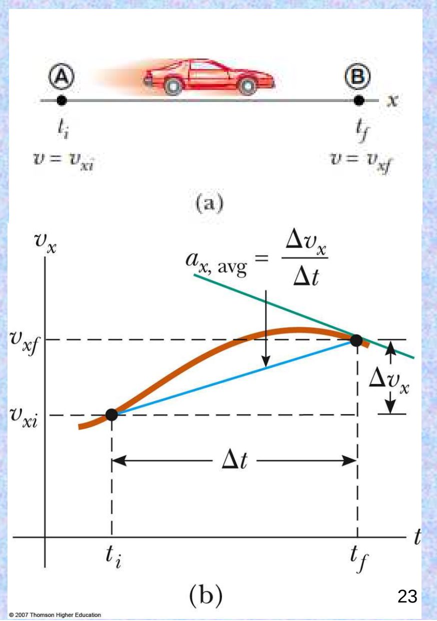
- The instantaneous acceleration is given by

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt} \qquad a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

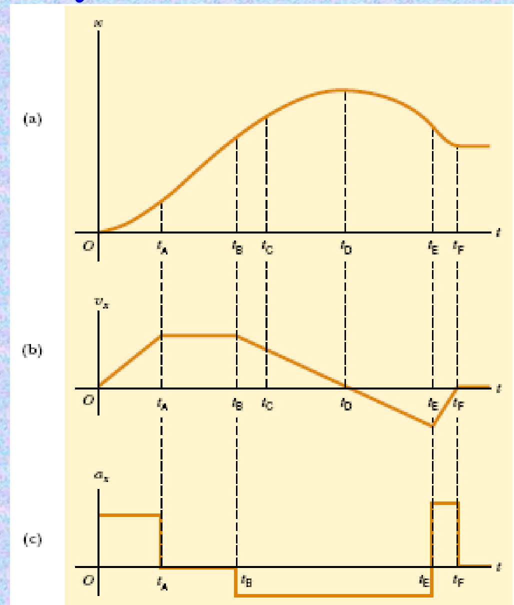
- Dimensions are L/T^2
- SI units are m/s^2

Instantaneous Acceleration - graph

- The slope of the velocity-time graph is the acceleration
- The green line represents the instantaneous acceleration
- The blue line is the average acceleration



Graphical representation of position, velocity and acceleration with time



Example 2.5 Average and Instantaneous Acceleration

The velocity of a particle moving along the x axis varies in time according to the expression $v_x = (40 - 5t^2)$ m/s, where t is in seconds.

(A) Find the average acceleration in the time interval $t = 0$ to $t = 2.0$ s.

Solution Figure 2.8 is a v_x-t graph that was created from the velocity versus time expression given in the problem statement. Because the slope of the entire v_x-t curve is negative, we expect the acceleration to be negative.

We find the velocities at $t_i = t_A = 0$ and $t_f = t_B = 2.0$ s by substituting these values of t into the expression for the velocity:

$$v_{xA} = (40 - 5t_A^2)\text{ m/s} = [40 - 5(0)^2]\text{ m/s} = +40\text{ m/s}$$

$$v_{xB} = (40 - 5t_B^2)\text{ m/s} = [40 - 5(2.0)^2]\text{ m/s} = +20\text{ m/s}$$

Therefore, the average acceleration in the specified time interval $\Delta t = t_B - t_A = 2.0$ s is

$$\bar{a}_x = \frac{v_{xf} - v_{xi}}{t_f - t_i} = \frac{v_{xB} - v_{xA}}{t_B - t_A} = \frac{(20 - 40)\text{ m/s}}{(2.0 - 0)\text{ s}}$$

$$= \boxed{-10\text{ m/s}^2}$$

The negative sign is consistent with our expectations—namely, that the average acceleration, which is represented by the slope of the line joining the initial and final points on the velocity-time graph, is negative.

(B) Determine the acceleration at $t = 2.0$ s.

Solution The velocity at any time t is $v_{xi} = (40 - 5t^2)$ m/s and the velocity at any later time $t + \Delta t$ is

$$v_{xf} = 40 - 5(t + \Delta t)^2 = 40 - 5t^2 - 10t\,\Delta t - 5(\Delta t)^2$$

Therefore, the change in velocity over the time interval Δt is

$$\Delta v_x = v_{xf} - v_{xi} = [-10t\,\Delta t - 5(\Delta t)^2]\text{ m/s}$$

Dividing this expression by Δt and taking the limit of the result as Δt approaches zero gives the acceleration at *any* time t:

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \lim_{\Delta t \to 0} (-10t - 5\Delta t) = -10t\text{ m/s}^2$$

Therefore, at $t = 2.0$ s,

$$a_x = (-10)(2.0)\text{ m/s}^2 = \boxed{-20\text{ m/s}^2}$$

Because the velocity of the particle is positive and the acceleration is negative, the particle is slowing down.

Note that the answers to parts (A) and (B) are different. The average acceleration in (A) is the slope of the blue line in Figure 2.8 connecting points Ⓐ and Ⓑ. The instantaneous acceleration in (B) is the slope of the green line tangent to the curve at point Ⓑ. Note also that the acceleration is *not* constant in this example. Situations involving constant acceleration are treated in Section 2.5.

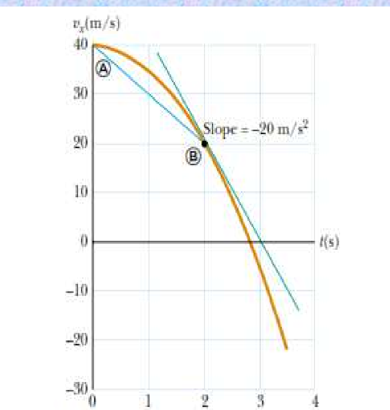


Figure 2.8 (Example 2.5) The velocity–time graph for a particle moving along the x axis according to the expression $v_x = (40 - 5t^2)$ m/s. The acceleration at $t = 2$ s is equal to the slope of the green tangent line at that time.

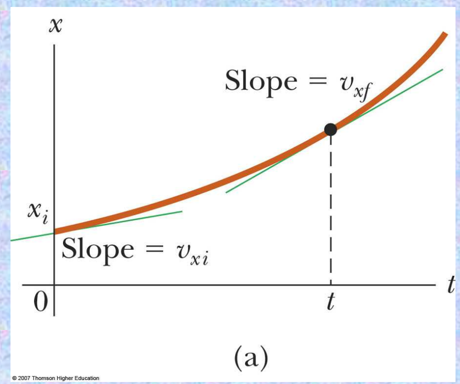
Kinematic Equations

- The kinematic equations can be used with any particle under uniform acceleration.
- The kinematic equations may be used to solve any problem involving *one-dimensional motion with a constant acceleration*
- You may need to use two of the equations to solve one problem
- Many times there is more than one way to solve a problem

One-Dimensional Motion with Constant Acceleration

- The slope of the curve is the velocity
- The curved line indicates the velocity is changing
  - Therefore, there is an acceleration

$$a_x = \frac{v_{xf} - v_{xi}}{t - 0}$$



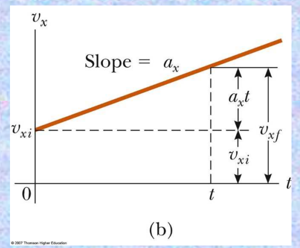
Velocity as a function of time

(a) the position–time graph,

$$v_{xf} = v_{xi} + a_x t \quad \text{(for constant } a_x\text{)}$$

Constant-acceleration motion

- The slope gives the acceleration
- The graph is a straight line, the (constant) slope of which is the acceleration $a_x = dv_x/dt$



b) The velocity–time graph

- The zero slope indicates a constant acceleration

$$v_{x,\text{avg}} = \frac{v_{xi} + v_{xf}}{2} \quad (\text{for constant } a_x)$$

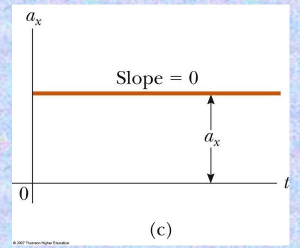
➢ Velocity as a function of position

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i) \quad (\text{for constant } a_x)$$

➢ Position as a function of time

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})t \quad (\text{for constant } a_x)$$

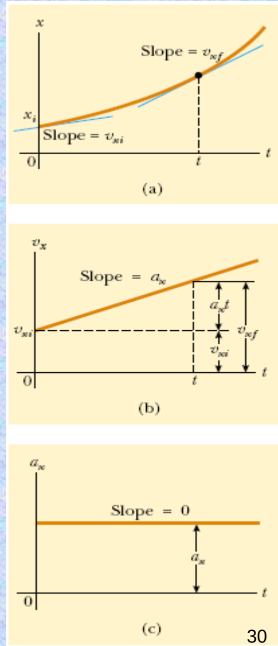
$$x_f = x_i + v_{xi}t + \tfrac{1}{2}a_x t^2 \quad (\text{for constant } a_x)$$



(c) The acceleration–time graph

Kinematic Equations for Motion of a Particle Under Constant Acceleration

Equation	Information Given by Equation
$v_{xf} = v_{xi} + a_x t$	Velocity as a function of time
$x_f = x_i + \frac{1}{2}(v_{xi} + v_{xf})t$	Position as a function of velocity and time
$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$	Position as a function of time
$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$	Velocity as a function of position



Example 2.7 Carrier Landing

A jet lands on an aircraft carrier at 140 mi/h ($\approx$63 m/s).

(A) What is its acceleration (assumed constant) if it stops in 2.0 s due to an arresting cable that snags the airplane and brings it to a stop?

(B) If the plane touches down at position $x_i = 0$, what is the final position of the plane?

What If? **Suppose the plane lands on the deck of the aircraft carrier with a speed higher than 63 m/s but with the same acceleration as that calculated in part (A). How will that change the answer to part (B)?**

Example 2.7 Carrier Landing

A jet lands on an aircraft carrier at 140 mi/h ($\approx$63 m/s).

(A) What is its acceleration (assumed constant) if it stops in 2.0 s due to an arresting cable that snags the airplane and brings it to a stop?

Solution We define our x axis as the direction of motion of the jet. A careful reading of the problem reveals that in addition to being given the initial speed of 63 m/s, we also know that the final speed is zero. We also note that we have no information about the change in position of the jet while it is slowing down. Equation 2.9 is the only equation in Table 2.2 that does not involve position, and so we use it to find the acceleration of the jet, modeled as a particle:

$$a_x = \frac{v_{xf} - v_{xi}}{t} \approx \frac{0 - 63\text{ m/s}}{2.0\text{ s}}$$

$$= -31\text{ m/s}^2$$

(B) If the plane touches down at position $x_i = 0$, what is the final position of the plane?

Solution We can now use any of the other three equations in Table 2.2 to solve for the final position. Let us choose Equation 2.11:

$$x_f = x_i + \tfrac{1}{2}(v_{xi} + v_{xf})t = 0 + \tfrac{1}{2}(63\text{ m/s} + 0)(2.0\text{ s})$$

$$= 63\text{ m}$$

If the plane travels much farther than this, it might fall into the ocean. The idea of using arresting cables to slow down landing aircraft and enable them to land safely on ships originated at about the time of the first World War. The cables are still a vital part of the operation of modern aircraft carriers.

What If? **Suppose the plane lands on the deck of the aircraft carrier with a speed higher than 63 m/s but with the same acceleration as that calculated in part (A). How will that change the answer to part (B)?**

Answer If the plane is traveling faster at the beginning, it will stop farther away from its starting point, so the answer to part (B) should be larger. Mathematically, we see in Equation 2.11 that if v_{xi} is larger, then x_f will be larger.

If the landing deck has a length of 75 m, we can find the maximum initial speed with which the plane can land and still come to rest on the deck at the given acceleration from Equation 2.13:

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

$$\rightarrow v_{xi} = \sqrt{v_{xf}^2 - 2a_x(x_f - x_i)}$$

$$= \sqrt{0 - 2(-31\text{ m/s}^2)(75\text{ m} - 0)}$$

$$= 68\text{ m/s}$$

Freely Falling Objects

- A ***freely falling object*** is any object moving freely under the influence of gravity alone.
- It does not depend upon the initial motion of the object
  - ✓ Dropped – released from rest
  - ✓ Thrown downward
  - ✓ Thrown upward
- The acceleration of an object in free fall is directed downward, regardless of the initial motion
- **The magnitude of free fall acceleration is g = 9.80 m/s²**
  - g decreases with increasing altitude
  - g varies with latitude
  - **9.80 m/s²** is the average at the Earth's surface
  - The italicized g will be used for the acceleration due to gravity
    - Not to be confused with g for grams

Freely Falling Objects

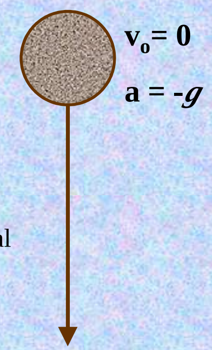
- We will neglect air resistance
- Free fall motion is constantly accelerated motion in one dimension
- Let upward be positive
- Use the kinematic equations with

  $a_y = -g = -9.80\ \text{m/s}^2$

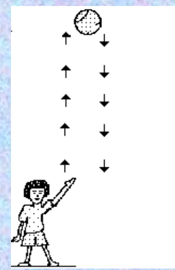
- **Object Dropped**
  - ❑ Initial velocity is zero
  - ❑ Let up be positive
  - ❑ Use the kinematic equations
  - ❑ Generally use **y** instead of **x** since vertical
  - ❑ Acceleration is

    $a_y = -g = -9.80\ \text{m/s}^2$
  - ❑ **If initial velocity ≠ 0**

    With upward being positive, initial velocity will be negative



Freely Falling Objects

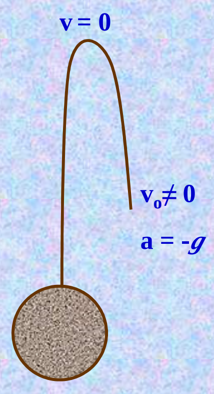


- **Object thrown upward**
  - ❑ **Initial velocity is upward, so positive**
  - ❑ **The instantaneous velocity at the maximum height is zero**
  - ❑ **Acceleration is**

    $a_y = -g = -9.80\ \text{m/s}^2$
  - ❑ **The motion may be symmetrical**

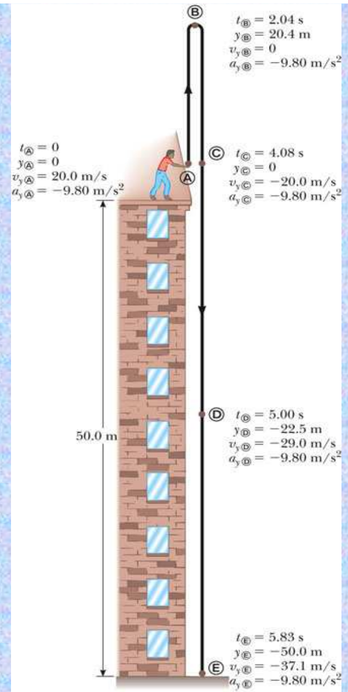
    Then $t_{up} = t_{down}$

    Then $v = -v_o$
  - ❑ **The motion may not be symmetrical**
    - **Break the motion into various parts**
    - **Generally up and down**



Free Fall Example

- **Initial velocity at A is upward (+) and acceleration is $-g$ (-9.8 m/s²)**
- **At B, the velocity is 0 and the acceleration is $-g$ (-9.8 m/s²)**
- **At C, the velocity has the same magnitude as at A, but is in the opposite direction**
- **The displacement is –50.0 m (it ends up 50.0 m below its starting point)**



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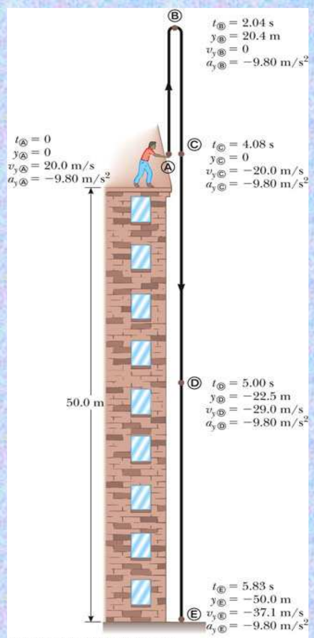
Example 2.12 Not a Bad Throw for a Rookie!

A stone thrown from the top of a building is given an initial velocity of 20.0 m/s straight upward. The building is 50.0 m high, and the stone just misses the edge of the roof on its way down, as shown in Figure 2.14. Using $t_A = 0$ as the time the stone leaves the thrower's hand at position Ⓐ, determine **(A)** the time at which the stone reaches its maximum height, **(B)** the maximum height, **(C)** the time at which the stone returns to the height from which it was thrown, **(D)** the velocity of the stone at this instant, and **(E)** the velocity and position of the stone at $t = 5.00$ s.



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Solution (A) As the stone travels from Ⓐ to Ⓑ, its velocity must change by 20 m/s because it stops at Ⓑ. Because gravity causes vertical velocities to change by about 10 m/s for every second of free fall, it should take the stone about 2 s to go from Ⓐ to Ⓑ in our drawing. To calculate the exact time t_B at which the stone reaches maximum height, we use Equation 2.9, $v_{yB} = v_{yA} + a_y t$, noting that $v_{yB} = 0$ and setting the start of our clock readings at $t_A = 0$:

$$0 = 20.0 \text{ m/s} + (-9.80 \text{ m/s}^2)t$$

$$t = t_B = \frac{20.0 \text{ m/s}}{9.80 \text{ m/s}^2} = \boxed{2.04 \text{ s}}$$

Our estimate was pretty close.

(B) Because the average velocity for this first interval is 10 m/s (the average of 20 m/s and 0 m/s) and because it travels for about 2 s, we expect the stone to travel about 20 m. By substituting our time into Equation 2.12, we can find the maximum height as measured from the position of the thrower, where we set $y_A = 0$:

$$y_{max} = y_B = y_A + v_{xA}t + \tfrac{1}{2}a_y t^2$$

$$y_B = 0 + (20.0 \text{ m/s})(2.04 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(2.04 \text{ s})^2$$

$$= \boxed{20.4 \text{ m}}$$

Our free-fall estimates are very accurate.

(C) There is no reason to believe that the stone's motion from Ⓑ to Ⓒ is anything other than the reverse of its motion from Ⓐ to Ⓑ. The motion from Ⓐ to Ⓒ is symmetric. Thus, the time needed for it to go from Ⓐ to Ⓒ should be twice the time needed for it to go from Ⓐ to Ⓑ. When the stone is back at the height from which it was thrown (position Ⓒ), the y coordinate is again zero. Using Equation 2.12, with $y_C = 0$, we obtain

$$y_C = y_A + v_{yA}t + \tfrac{1}{2}a_y t^2$$

$$0 = 0 + 20.0t - 4.90t^2$$

This is a quadratic equation and so has two solutions for $t = t_C$. The equation can be factored to give

$$t(20.0 - 4.90t) = 0$$

One solution is $t = 0$, corresponding to the time the stone starts its motion. The other solution is $\boxed{t = 4.08 \text{ s},}$ which

(D) Again, we expect everything at Ⓒ to be the same as it is at Ⓐ, except that the velocity is now in the opposite direction. The value for t found in (c) can be inserted into Equation 2.9 to give

$$v_{yC} = v_{yA} + a_y t = 20.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(4.08 \text{ s})$$

$$= \boxed{-20.0 \text{ m/s}}$$

The velocity of the stone when it arrives back at its original height is equal in magnitude to its initial velocity but opposite in direction.

(E) For this part we ignore the first part of the motion (Ⓐ→Ⓑ) and consider what happens as the stone falls from position Ⓑ, where it has zero vertical velocity, to position Ⓓ. We define the initial time as $t_B = 0$. Because the given time for this part of the motion relative to our new zero of time is 5.00 s − 2.04 s = 2.96 s, we estimate that the acceleration due to gravity will have changed the speed by about 30 m/s. We can calculate this from Equation 2.9, where we take $t = 2.96$ s:

$$v_{yD} = v_{yB} + a_y t = 0 \text{ m/s} + (-9.80 \text{ m/s}^2)(2.96 \text{ s})$$

$$= \boxed{-29.0 \text{ m/s}}$$

We could just as easily have made our calculation between positions Ⓐ (where we return to our original initial time $t_A = 0$) and Ⓓ:

$$v_{yD} = v_{yA} + a_y t = 20.0 \text{ m/s} + (-9.80 \text{ m/s}^2)(5.00 \text{ s})$$
$$= -29.0 \text{ m/s}$$

To further demonstrate that we can choose different initial instants of time, let us use Equation 2.12 to find the position of the stone at $t_D = 5.00$ s (with respect to $t_A = 0$) by defining a new initial instant, $t_C = 0$:

$$y_D = y_C + v_{yC} t + \tfrac{1}{2} a_y t^2$$
$$= 0 + (-20.0 \text{ m/s})(5.00 \text{ s} - 4.08 \text{ s})$$
$$+ \tfrac{1}{2}(-9.80 \text{ m/s}^2)(5.00 \text{ s} - 4.08 \text{ s})^2$$
$$= \boxed{-22.5 \text{ m}}$$

Kinematic Equations derived from Calculus

- Displacement equals the area under the velocity – time curve

$$\lim_{\Delta t_n \to 0} \sum_n v_{xn} \Delta t_n = \int_{t_i}^{t_f} v_x(t)\,dt$$

- The limit of the sum is a definite integral

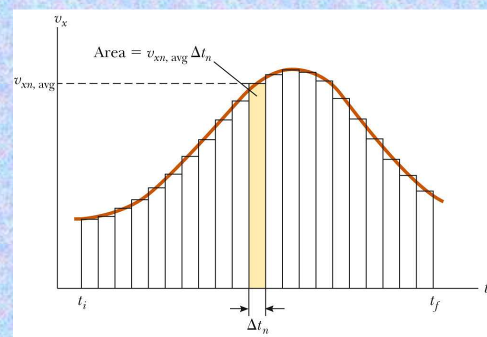

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Figure 2.15 Velocity versus time for a particle moving along the x axis. The area of the shaded rectangle is equal to the displacement Δx in the time interval Δt_n, while the total area under the curve is the total displacement of the particle.

Displacement = area under the v_x-t graph

For example, suppose a particle moves at a constant velocity v_{xi}. In this case, the v_x-t graph is a horizontal line, as in Figure 2.16, and the displacement of the particle during the time interval Δt is simply the area of the shaded rectangle:

$$\Delta x = v_{xi}\,\Delta t \qquad (\text{when } v_x = v_{xi} = \text{constant})$$

As another example, consider a particle moving with a velocity that is proportional to t, as in Figure 2.17. Taking $v_x = a_x t$, where a_x is the constant of proportionality (the

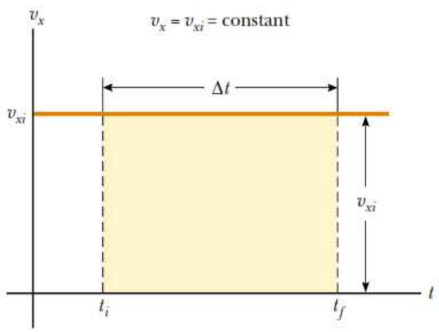

Figure 2.16 The velocity–time curve for a particle moving with constant velocity v_{xi}. The displacement of the particle during the time interval $t_f - t_i$ is equal to the area of the shaded rectangle.

As another example, consider a particle moving with a velocity that is proportional to t, as in Figure 2.17. Taking $v_x = a_x t$, where a_x is the constant of proportionality (the

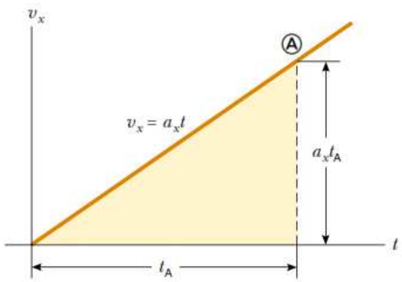

Figure 2.17 The velocity–time curve for a particle moving with a velocity that is proportional to the time.

acceleration), we find that the displacement of the particle during the time interval $t = 0$ to $t = t_A$ is equal to the area of the shaded triangle in Figure 2.17:

$$\Delta x = \tfrac{1}{2}(t_A)(a_x t_A) = \tfrac{1}{2}\,a_x t_A^{\,2}$$

Kinematic Equations – General Calculus Form

$$a_x = \frac{dv_x}{dt}$$

$$v_{xf} - v_{xi} = \int_0^t a_x\,dt$$

$$v_x = \frac{dx}{dt}$$

$$x_f - x_i = \int_0^t v_x\,dt$$

- **The integration form of $v_f - v_i$ gives**

$$v_{xf} - v_{xi} = a_x t$$

- **The integration form of $x_f - x_i$ gives**

$$x_f - x_i = v_{xi}t + \frac{1}{2}a_x t^2$$

Example

The speed of a bullet as it travels down the barrel of a rifle toward the opening is given by ***$v = (-5 \times 10^7)\,t^2 + (3 \times 10^5)\,t$, where v is in meters per second and t is in*** seconds. The acceleration of the bullet just as it leaves the barrel is zero.

(a) Determine the acceleration and position of the bullet as a function of time when the bullet is in the barrel.
(b) Determine how long the bullet is accelerated.
(c) Find the speed at which the bullet leaves the barrel.
(d) What is the length of the barrel?

a) $a = \dfrac{dv}{dt}$

$$a = -(10.0\times10^7\ \text{m/s}^3)t + 3.00\times10^5\ \text{m/s}^2$$

Take $x_i = 0$ at $t = 0$. Then $v = \dfrac{dx}{dt}$

$$x - 0 = \int_0^t v\,dt = \int_0^t \left(-5.00\times10^7\,t^2 + 3.00\times10^5\,t\right)dt$$

$$x = -5.00\times10^7\,\frac{t^3}{3} + 3.00\times10^5\,\frac{t^2}{2}$$

$$x = -(1.67\times10^7\ \text{m/s}^3)t^3 + (1.50\times10^5\ \text{m/s}^2)t^2$$

Example

The speed of a bullet as it travels down the barrel of a rifle toward the opening is given by ***$v = (-5 \times 10^7)\, t^2 + (3 \times 10^5)\, t$*, *where v is in meters per second and t is in*** seconds. The acceleration of the bullet just as it leaves the barrel is zero. **(a)** Determine the acceleration and position of the bullet as a function of time when the bullet is in the barrel. **(b)** Determine how long the bullet is accelerated. **(c)** Find the speed at which the bullet leaves the barrel. **(d)** What is the length of the barrel?

b) The bullet escapes when $a = 0$, at $-(10.0 \times 10^7\ \text{m/s}^3)t + 3.00 \times 10^5\ \text{m/s}^2 = 0$

$$t = \frac{3.00 \times 10^5\ \text{s}}{10.0 \times 10^7} = \boxed{3.00 \times 10^{-3}\ \text{s}}.$$

c) New $v = (-5.00 \times 10^7)(3.00 \times 10^{-3})^2 + (3.00 \times 10^5)(3.00 \times 10^{-3})$

$$v = -450\ \text{m/s} + 900\ \text{m/s} = \boxed{450\ \text{m/s}}.$$

d) $x = -(1.67 \times 10^7)(3.00 \times 10^{-3})^3 + (1.50 \times 10^5)(3.00 \times 10^{-3})^2$

$x = -0.450\ \text{m} + 1.35\ \text{m} = \boxed{0.900\ \text{m}}$